Application of Multibody System (MBS) Techniques to Automotive Vehicle Chassis Simulation for Motion Control Studies

by

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Para a Renata e o Lucas, com todo amor

" Philosophy is written in that great book which ever lies before our eyes - I mean the Universe - But we cannot understand it if we do not first learn the language and grasp the symbols in which it is written. This book is written in the mathematical language, and the symbols are triangles, circles and other geometric figures, without whose help it is impossible to comprehend a single word of it; without which one wanders in vain through dark labyrinth."

Galileo Galilei

Summary

The subject of this thesis is the application of multibody system techniques for vehicle chassis modelling aimed at the development of integrated vehicle control. This work revised the application of automatic control to automotive vehicles and it has observed that most applications to date adopt a piecemeal approach whereby individual vehicle subsystems are treated in isolation. The coordination and integration of automotive vehicle motion control requires that the interaction amongst the various subsystems is taken into consideration at the design stage. Integrated vehicle control is then proposed as the motion management concept. A revision about existing techniques for generating the equations of motion for vehicle chassis, using multibody systems (MBS) techniques is also carried out. It realises that resulting models can be complex and that simplifications in chassis description is recommendable. For this purpose, it has developed a technique for representing suspension geometry effects which by taking the MBS structure into account, results in small and fast runtime simulation models. Yet, the model is capable of describing the full range of normal operation of the automotive vehicle. Using the previously developed model, comprehensive analysis of all aspects of vehicle motion is carried out. The objectives of such analysis is the determination of a driving envelope in which the use of linearised models of the nonlinear chassis can be justified for control analysis and design. Finally, the numerical and control theoretical properties of the linearised models are addressed. State space and transfer function matrix representations are used for such purposes.

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Contents

List of Figures

List of Tables

Chapter 1

Introduction

The aim of Automotive Engineering is to make the motor vehicle safer, easier to drive, with improved performance to cost ratio and minimised pollutant emissions. Along the years, the techniques used to obtain compromise solutions amongst these conflicting objectives have increased in number and complexity. Important breakthroughs in vehicle development have been reached through its history. Some examples include developments in pneumatic tyre technology and in powertrains, the development of suspension systems, the use of new materials for weight reduction, the extensive application of microelectronics and the influence of aerodynamics in body styling.

The period following the early 1970's has seen a rapid increase in the application of automatic control techniques in automotive vehicles in order to achieve those objectives. These past and current developments in automotive vehicle control have progressed mostly in a piecemeal fashion, whereby individual vehicle subsystems, such as the engine, suspension and braking systems have been studied in isolation. Future applications of control in automotive vehicles will follow a trend towards system integration, leading ultimately to the development of integrated vehicle control systems capable of coordinating the action of the various subsystems. The coordination and integration of automotive vehicle subsystem control requires the interaction amongst the various subsystems to be taken into consideration at the control design stages, i.e., a total system approach to automotive vehicle control is needed.

The study of vehicle control viewed as a total system composed of interacting subsystems is introduced by this work under the concept of *Motion Management.*

Therefore, the motion management concept is taken as the focus for the development of *integrated* vehicle control. At its highest level, motion management is concerned with ensuring that a vehicle is capable of tracking in a safe and stable manner any reasonable speed and direction profile demanded by the driver whilst providing a comfortable ride for the passengers.

In order to provide support for these ideas, this work presents an overview of developments in automotive vehicle control from the early engine control to the more recent active suspension and 4-wheel steering systems. As a result it shows that there is a trend in vehicle dynamics research in the direction of addressing the various aspects of vehicle motion in a more complex way which considers the interactions between the various subsystems in the vehicle.

A discussion is carried out analysing the various possibilities for deriving the system's representation, and which are the most appropriate tools for the task of obtaining them, bearing the motion management problem in mind. Hence, for the derivation of the system's equations, a modular methodology is adopted, which allows for all aspects of normal vehicle operation to be considered and the increased complexity of the vehicle's subsystems to be added with minimum effort. This methodology is based on the use of computer based mathematical modelling and support tools.

Afterwards, a nonlinear vehicle modelling technique is developed which introduces simplifications necessary to make the motion management concept feasible, but that is still capable of describing all aspects of vehicle motion.

Extensive simulation and analysis are performed with the developed model in order to evaluate the possible ranges of operation and parameters variation in which linear approximations would still be valid for the design of integrated control systems.

Finally, a control design oriented analysis is carried out with linearised vehicle models, in order to evaluate the theoretic control properties and system's characteristics for the study and implementation of the motion management concept.

The introduction of integrated vehicle control under the motion management concept can be justified by the present stage of development and existence of quick and flexible mathematical modelling and simulation techniques, the widespread research in the area of active systems, present day development of microelectronics and a theoretical background which allows these studies to be performed through the use of system and control theories.

1.1 Motion Management

This section introduces the concept of motion management in a more detailed form and it intends to serve as a guideline for future developments of this work. It begins with illustrations of past and present levels of control use in automotive vehicles and it proceeds to propose an approach to the development of future implementations of integrated vehicle control under the heading of motion management. Two levels of approach can be devised for the implementation and study of the vehicle motion control. The first one is concerned with the control of vehicle motion per se, without considering the dynamics of the processes which generated the control actions, while the second takes these dynamics into consideration. They can be referred to as the generic and the extended motion management problems.

The normal operation of a conventional automotive vehicle, involves the interactions between the vehicle, the driver and the road environment, as illustrated in figure 1.1. It can be observed the closed loop characteristics of the total system, in that vehicle responses caused by the driver are also sensed by the driver and influence further actions. Other feedback loops include the braking, steering and power systems. For example, with manual steering, one gets significant 'road-feel' cues through the hands on the steering wheel.

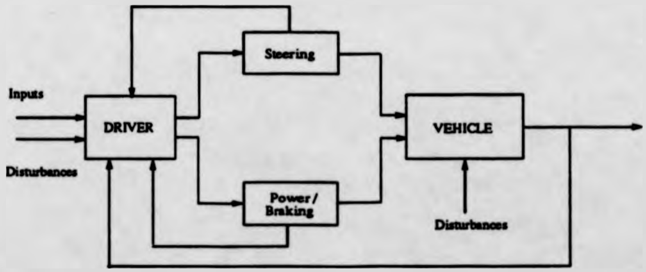

Figure 1.1: Conventional vehicle/driver/environment representation

The following observation should be made concerning the block diagrams that will follow. It is recognised that the powertrain and braking systems constitute each one of them a system of their own. Their grouping together has the purpose of simplifying the diagram and the criteria used for this grouping is related to the nature of energy manipulation in these systems. In this sense, the powertrain and the braking systems are mainly responsible for the changes in the total energy level within a vehicle, while the steering and suspension subsystems are primarily concerned with the distribution of energy forms stored in the motor vehicle. Also, within the "VEHICLE" block of the diagram of figure 1.1 is included the feedback characteristic of the tyre/road contact. It is represented in greater details by the diagram depicted in figure 1.2. Later on, when necessary, this representation is used.

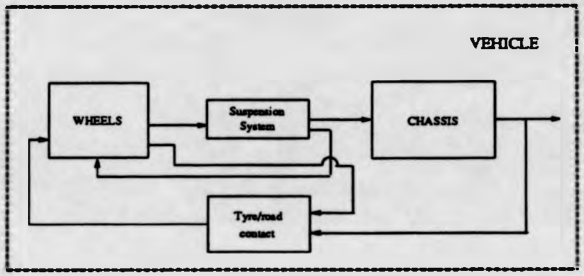

Figure 1.2: Detail of "VEHICLE" block

The level of control utilisation in automotive vehicles through their history presents various distinct phases. In the past there was no intentional control utilisation, besides the already mentioned feedback loops. In that case, the automotive vehicle was constituted only of passive systems and the natural interaction amongst its main subsystems offered no possibility to be acted upon, during vehicle operation. If changes in vehicle behaviour were desired, they had to be performed at the design stages.

Nowadays, although the majority of vehicles still operate in the way previously described, there are an increasing number of automotive vehicles incorporating a significant amount of control in various of their subsystems. This control utilisation has been initially introduced in the engine due to the fuel crisis and strict emission control laws. Stimulated by competition amongst different car manufacturers and using system approaches together with current state-of-the-art microelectronics, as well as active system concepts, significant overall performance enhancement, as well as in many subsystems, have also been achieved. However, present day control applications are restricted to a local basis. Besides this local basis characteristic, the control strategies adopted are mostly I/O and mode selection oriented and there is also a high level of table-oriented or scheduled and feed-forward control. This situation is due to present day state of development and cost of sensing devices, communications wiring and appropriate process models [106]. This present stage can be illustrated in the diagram of figure 1.3.

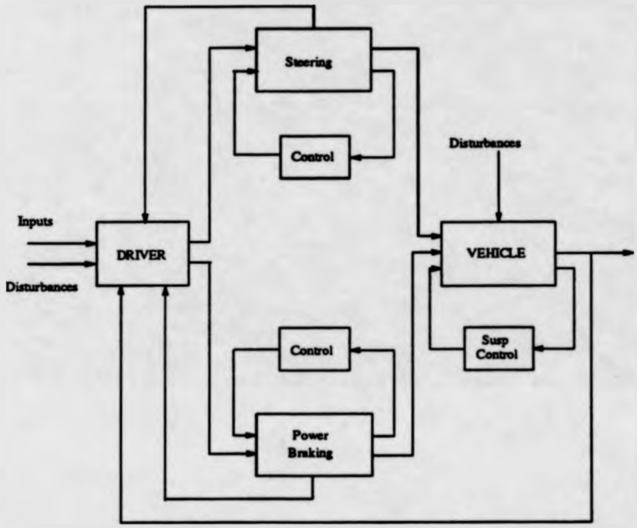

Figure 1.3: Present day control applications in the automotive vehicle

The tendency in future vehicles, as more knowledge is accumulated and new concepts are implemented using more advanced technologies and sophisticated mathematical and computational tools, is to have new control strategies adopted.

In this work a new approach for the development of vehicle control is proposed. In such approach it is necessary to take into consideration the highly interacting effects amongst the various subsystems in a motor vehicle, in order to generate a so called motion policy for its behaviour. This overall approach to integrated automotive control which has been defined as motion management can be represented by the diagram depicted in figure 1.4.

Although the driver is an active part of the system represented in the diagrams of figures 1.1 to 1.4, this work will not address the question of human operator dynamics. Some sources in the literature describe different techniques to the modelling of human response to vibrations [52, 197, 112] and tracking ability [52, 104, 115, 200, 295]. However, because of varied driver skill levels, it is very

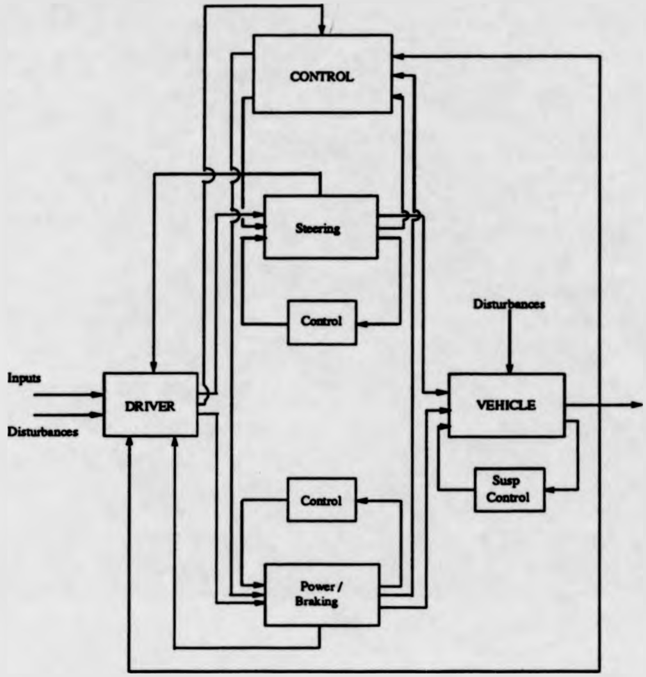

Figure 1.4: Motion management concept diagram

difficult to quantify vehicle response using this closed loop arrangement, and therefore the solution normally adopted in vehicle dynamics is to ignore the driver and analyse the motor vehicle as a dynamic system on its own, that is an open loop type of arrangement [62, 131, 255].

When automotive vehicle motion is analysed, it can be recognised that there are two types of inputs involved. Driver related inputs and environmental disturbances. Driver inputs include acceleration, braking and steering wheel inputs, while environmental inputs comprise road and aerodynamics disturbances. From a control point of view, there is a distinction between these two types of inputs, since it is desired that the system follows the driver's inputs while rejecting measurement noises, and also rejects the external disturbances. Concerning the steering input, a further subdivision is yet possible, by considering either steering wheel displacement or steering wheel torque as an input. Although the driver steers her/his vehicle by a combination of displacement and feel, it is possible to study these two types of inputs separately [255]. The response produced by a steering wheel rotary *displacement* have been called the *fixed control.* On the other hand, if the driver input is a steering wheel *torque*, the model is referred to as *free control.* In the study to be carried out in this work a fixed control type of input model is adopted.

Apart from aerodynamic resistances and gravitational effects, all external forces acting on a vehicle are applied through the wheels. Consequently, the motion management will entail the application of control action at the wheels, and will be based on a combination of

1. Propulsion control.

2. Steering control.

3. Suspension control.

Propulsion control is primarily concerned with the control of the fore/aft motion of a vehicle through the manipulation of torque at the driven/braked wheels. Propulsion control is achieved through a combination of powertrain control and braking control. This entails the prevention of wheel spin, during acceleration manoeuvres, and wheel lock, during braking manoeuvres. The propulsion control problem can be illustrated by the diagram of figure 1.5.

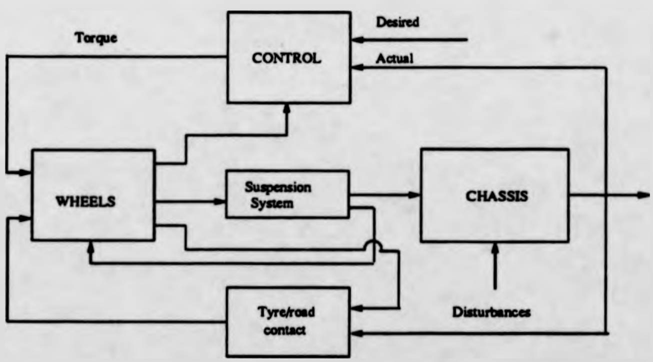

Figure 1.5: Propulsion control problem.

Steering control involves mainly the control of the lateral, yaw and roll motions of a vehicle through the manipulation of the steering angle of the steered wheels in order to obtain certain values of lateral forces at the tyres, which optimise vehicle handling behaviour. It can be represented by the diagram of figure 1.6.

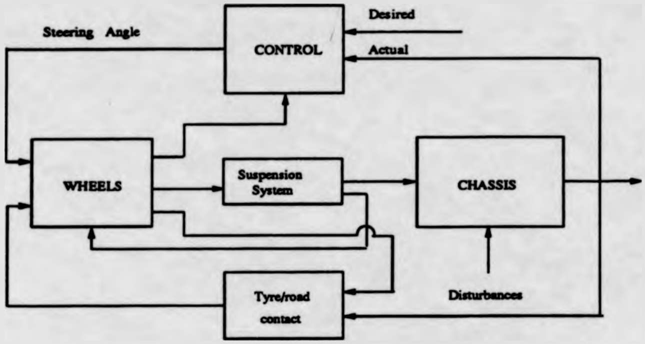

Figure 1.6: Steering control problem.

Suspension control ultimately deals with car body positioning. It entails principally the control of the vertical, roll and pitch motions of a vehicle through the manipulation of forces within the suspension. The suspension control scheme can be seen in the diagram of figure 1.7.

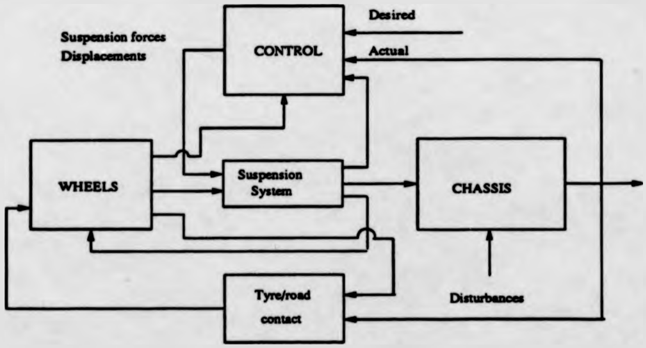

Figure 1.7: Suspension control problem.

An integrated vehicle control includes the effects of the propulsion, steering and suspension systems simultaneously considering the coupling effect amongst different aspects of vehicle motion.

The propulsion, steering and suspension subsystems are physically coupled via inertia and tyre forces. The coupling due to inertia forces results from the motion of the wheels around the wheel axis, wheel assembly around the king pin and from the suspension elements around suspension pins. This coupling is weak compared to the strong tyre force coupling; however, the forward, lateral, vertical, toe, camber and castor motion of the wheels are essential for a good approximation to real tyre forces and torques. These tyre forces depend upon a variety of quantities, but mainly on vertical displacement for the vertical component and wheel's longitudinal slip, sideslip angle and tyre's normal load for the longitudinal and lateral terms. They are also influenced by road/tyre adhesion properties and road surface profile.

If the propulsion, steering and suspension systems are acted together, the vehicle can be optimally adapted to any driving condition. This means, for instance, that with optimised longitudinal and lateral forces as well as complete body positioning compensation, the vehicle chassis can experience a cornering inclination which is driving condition dependent. By this previous example it can be seen that the interactions between the propulsion, steering and suspension subsystems in an automotive vehicle can be highly complex, and the need for dynamic simulation models capable of supporting vehicle motion control studies is apparent.

The mathematical model of the vehicle should thus include all aspects of chassis motion, that is, its forward, lateral, vertical, yaw, roll and pitch motions, as well as the movements of steering and suspension systems and also the wheels. It could be obtained through the conjunction of the previous modular models, plus the coupling effects previously neglected, or it can be obtained using present day simulation aid tools in a way that these subproblems can be studied separately, if desired. A diagram of the motion management model can be seen in figure 1.8. It can be observed that the motion management problem in its generic form is mainly concerned with vehicle motion.

On the other hand, the dynamics of the vehicle subsystems, which can be considered to be vehicle design dependent, can also be taken into consideration as an extension of the previous models. It comprises the 4 major subsystems which are going to exert the control actions determined by the motion management problem solution. They are

- Powertrain system.
- Braking system.
- Steering system.
- Suspension system.

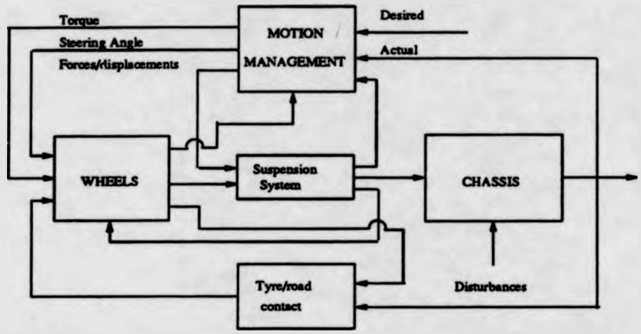

Figure 1.8: Generic motion management problem.

The powertrain includes the engine, transmission and driveline. Each one of them may operated based on different principles. For example, the engine can be petrol or diesel powered. The transmission can be manual, automatic, continuous or discrete and the driveline characteristics depend on whether the vehicle is 2, front or rear, or 4 wheel drive and on the type of differential used.

In the same way, the models of the other subsystems describe their own particularities. For instance, the vehicle can be 2 or 4 wheel steering incorporating a passive, semi-active or active suspension system.

Therefore, these extended models would also consider the dynamics due to the specific characteristics of these subsystems in the vehicle motion control problem. For this extended motion management problem the resulting block diagram can be seen in figure 1.9.

Through a structured and modular development of the mathematical models of the subsystems, different car designs and actually 'assemblies' can be obtained in order to evaluate the effect of different concepts in vehicle motion control.

With such a modular approach, where quick generation and modification of the vehicle model is possible, minimised time of optimum solution derivation can be achieved. Another important aspect of the motion management concept, in taking a systemic approach to the modelling of vehicles is the adoption of a uniform and well known system's terminology to describe vehicle's performance. Based on previous experience in automotive research, evaluation parameters or performance index can be established, in order to provide objective and quantitative results for vehicle performance analysis encompassing all areas of vehicle behaviour.

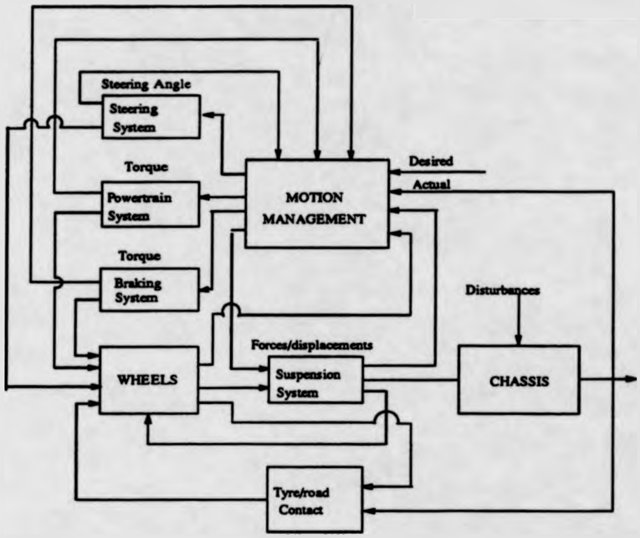

Figure 1.9: Extended motion management problem.

1.2 Mathematical Modelling

A brief discussion about mathematical modelling of automotive vehicles is carried out in this section in order to give an overview of techniques currently being used. It also intends to identify possible approaches which can be used in the development of models in which the motion management concept previously introduced could be implemented. A more detailed discussion is included in chapters 3 and 4. Also, the steps and procedures necessary to implement the motion management concept are discussed. For this purpose, extensive simulations are performed with the nonlinear model in order to determine conditions under which linear control design techniques could be applied. This is done in chapter 5. Using the results of the previous analysis of the nonlinear model, a further discussion about the analysis of linearised models is carried out in chapter 6 in which the properties of the resulting linear models are addressed.

The use of mathematical modelling in automotive vehicle design is a widespread practice which allows the development of vehicle systems at reduced costs and time and near optimised performance characteristics. This use embraces all aspects of vehicle design and in particular, vehicle motion. Through the establishment of simplifying assumptions and the application of the appropriate physical laws, equations are obtained to represent the relevant aspects of the automobile behaviour intended to be studied.

Those aspects of vehicle motion control which are of primary interest span a dynamic range with a frequency cut-off below 50 Hz. Up to this frequency, simulation studies of automotive vehicle dynamics can justifiably be based on lumped parameter models [242]. Current approaches to vehicle dynamics simulation fall into 2 distinct categories. The first approach involves the use of simple mathematical models which are derived from first principles and which are usually assembled manually. These models are used for investigations into propulsion control [220, 114, 47, 22, 251], steering control [147, 235, 281] and suspension control [120, 161, 145, 158, 265] when these subsystems are considered in isolation. However, the complexity of an automotive vehicle when viewed as a total system in which the interaction between the propulsion, steering and suspension, subsystems is represented prevents the use of these models for motion control studies.

The second approach to vehicle dynamics simulation involves mathematical models which incorporate representations of the vehicle kinematics, tyre characteristics and suspension geometry effects. In principle, these models are capable of supporting vehicle motion control studies since, to a varying degree, the interaction between the propulsion, steering and suspension subsystems are present. This approach to vehicle dynamics simulation can result in mathematical models which are highly complex. Consequently, automated model generation facilities based on multibody systems (MBS) modelling techniques are widely employed [242, 57, 209, 283, 237, 225, 137]. The use of bond graphs as model generation aids [164] are not wholly appropriate since they do not provide a means of describing the kinematics of the multibody system.

The use of MBS modelling techniques to vehicle dynamics simulation involves two essential considerations, viz. the way in which the MBS model is formulated and the particular MBS program which is to be employed. The MBS model formulation determines how the equations of motion are derived. The MBS program determines the ultimate form of the dynamic equations, as implemented within the computer simulation program. Approaches to MBS modelling based on a combination of Kane's Method [116] and symbolic computation has been shown [243, 237, 230, 257] to generate the most efficient (run time) simulation code.

While the use of these complex models makes the idea of an integrated control system very difficult due to the high nonlinearities of the equations of motion as well as the great number of states; it has not been answered yet whether a simplified model, however taking some relevant interactions from a control point of view, into consideration, could result in improved vehicle motion.

A positive answer to such a question would possibly mean, for example, a reduced number of transducers and their related apparatus necessary to be implemented in a future vehicle in which these concepts might be applied.

This thesis describes a pioneering application of a MBS technique to the modelling of an automotive vehicle for the development of motion control studies. MBS simulation models of chassis systems usually include a complete representation of the suspension [209, 143] in which individual suspension elements are detailed. The modelling of this work is based on an alternative approach [41], in which suspension geometry effects are incorporated in a way which does not involve a detailed representation of the suspension system. The basis of the approach is an empirical black-box representation of the suspension kinematics derived from readily available experimental data describing the suspension trajectory at each wheel hub. In this way, the resulting model is still adequate to cater for all aspects of vehicle motion, while resulting in small and fast computer code, which is appropriate for the study and implementation of the motion management concept.

However, the resulting model is still highly nonlinear and an extensive analysis of the nonlinear model is necessary in order to define an operating range of the model in which linear approximations are still adequate. For that purpose simulation runs which exercise all aspects of vehicle motion are performed and a linear driving envelope or conditions for linear approximations are presented and discussed.

In chapter 6 the resulting linearised model is discussed from a control theoretic point of view and model properties, as well as the techniques which can be used to analyse it are considered. Alternative model representations for control design purposes are also addressed. In this respect, the automotive vehicle, which from a system's point of view is a multiple input, multiple output (MIMO) system, are discussed in state space and transfer function matrix representations. In the state space representation topics such as conditioning, controllability, observability and structural properties are discussed and in the transfer function matrix representation topics such as poles and zeros, frequency response and singular values are addressed.

1.3 Conclusions

In order to provide better performance and driving characteristics, future vehicles will incorporate more integrated controls in various of their subsystems. Because the interactions amongst these subsystems are complex, a systematic approach to their development is necessary. With the aid of system theory concepts such an approach is proposed in this work under the motion management concept.

During the development of a new design, the aid of mathematical modelling is very important. However these models have to be able to cater for the aspects of the study intended to be carried out and yet be as simple as possible. This is a trade-off simulation engineers always have to face. These models should also be flexible in order to incorporate new design concepts with relative ease. The use of model building aid and simulation tools are necessary if a study such as the one proposed in this thesis is to be carried out.

With a modular approach to the study of the motor vehicle, using the aforementioned advanced computer tools, analysing the implications of model characteristics from a control point of view and representing the resultant model in 'standard' system theory form, the implementation of the motion management concept can be carried out.

Finally it must be said that while the technical feasibility of these proposed new automotive design approaches can prove satisfactory, their appearance in the market is not necessarily related, since economic factors might not be propitious for their introduction. The risk in carrying out such researches which may or may not pay off in business profits, must be viewed as a necessary part of technical progress.

Chapter 2

Overview of Control Applications to Vehicles

2.1 Introduction

Comparing present vehicles with those from the end of last century one might be surprised, as far as control and feedback control systems are concerned. Some of the vehicle's subsystems such as the powertrain, have attained enormous progress. Others, like the steering system, have gone through very little change and just recently have been subject to design proposals which incorporate some form of automatic control. Perhaps this is so because their early weaknesses were not as serious as those of the former's.

It is generally accepted that significant control applications in automotive vehicles started in the USA, in the early 1970's with the introduction of microprocessor based engine control, in order to meet the conflicting demands of high fuel economy resulting from the oil crisis and low exhaust emissions, imposed by legislative requirements. This use has now expanded to all areas of vehicle operation and they encompass topics which varies from the nearly standard braking system control (ABS) of present luxury vehicles to the exotic human sensorial feelings satisfaction within the vehicle's cabin by a combination of temperature, sound, visual and driver's position control in experimental vehicles.

This chapter provides an overview of automotive control applications related to vehicle motion. It embraces the early works on engine control to more recent studies on active suspensions and 4-wheel steering. As a matter of convenience, this overview is presented under the headings of propulsion control, suspension control and steering control. Propulsion control is further subdivided into powertrain and braking control.

This overview has the purpose of providing a foundation for the introduction of the motion management concept, where it is shown that most of the applications herein described have considered only those aspects of vehicle motion related to the specific control problem being addressed. In this sense, it is shown that most control applications to date in vehicles have adopted a piecemeal approach.

2.2 Propulsion Control

Propulsion control refers to vehicle behaviour when driving straight ahead or at small lateral acceleration values. It is suppose to embrace vehicle's performance characteristics, as well as lateral stability and steerability. In general terms, it is concerned with the torque control of the wheels and it is considered to encompass the powertrain and the braking system.

2.2.1 Powertrain Control

Powertrain control is taken to embrace control of the engine, clutch and transmission. The area of engine control appears to have attracted the greatest number of applications to date. Due to the complexity of the physical phenomena involved, the development of engine control systems has evolved from a mixture of empirical and theoretical modelling techniques and rule based and model based control systems [23, 220]. The ability to infer engine behaviour from related measurements plays a critical role in the synthesis of engine control concepts. It has included open loop engine calibration and feedback control of idle speed, air/fuel ratio and spark timing [279]. Recently reported applications of engine control [27, 106, 20, 76, 195, 290] make reference to the use of modern control methodologies, including multivariable, optimal and adaptive techniques.

A present day engine control system for idle speed control implemented in production vehicles is described by Hrovat and Powers [106]. It is based on a 5 state variables and 2 control inputs model and uses optimal control techniques. This control implementation was developed based on nonlinear model simulations comprised of 20 states and 5 control variables.

The purpose of the clutch is to make the engine and transmission separable. Clutch control was introduced in order to combine the ease of driving of automatic transmissions and the fuel economy characteristics of manual ones. Clutch control has received considerable attention recently and has seen several successful applications, both as a part of the powertrain [106, 292] and individually [308]. Usually clutch control involves logic control due to its discontinuous form of operation and pneumatic and hydraulic actuators are typically employed.

Transmission control can be divided in two major groups, namely stepped (discrete) transmissions and continuously variable transmissions (CVT). Discrete transmission control can be classified according to whether only the shift scheduling phase is controlled or whether both shift scheduling and shift execution phases are controlled. An application described by Hrovat [106] includes the control of shift scheduling and shift execution for which the control system was developed with the aid of a nonlinear simulation model incorporating submodels of engine, torque converter and clutch. The potential for the improvement in fuel economy arising from the introduction of CVT's into automotive powertrains has resulted in widespread interest of CVT control [35, 111, 222]. Recent work in this area includes the combined control of an engine and CVT within an integrated powertrain control system [155, 156] and the control of a powertrain incorporating a CVT and energy storage flywheel for regenerative braking purposes [84]. A systematic methodology for the study of integrated powertrain control systems has been employed by Jones et al. [113]. The methodology is based on a structured modular approach to powertrain modelling and has been applied to the study of clutch control in heavy good vehicles (HGV) [47], and the control of CVT in a car [114].

2.2.2 Braking and Traction Control

The application of automatic control techniques to vehicle braking systems has been concerned with the areas of antilock braking systems (ABS) and antislip regulation (ASR) [26]. Although the first patents concerning antiskid systems date back to 1905, actual development did not begin until 1957, and it was not before 1978 that a fully functional electronic antiskid braking system was introduced in a production vehicle [140].

The prevention of wheel lock during braking or wheel spin during acceleration has the purpose of maintaining vehicle directional stability and/or steerability and assuring that longitudinal forces are transmitted as effectively as the physical condition of the contact tyre/road permits. Generically speaking, the normalised difference between the wheel center and wheel peripherical (tangential) longitudinal velocities is called longitudinal slip. The tyre's longitudinal forces increase with longitudinal slip until a peak value is reached, thereafter suffering sudden reduction and causing the wheels to lock during braking or spin during acceleration. The main problem of locking or spinning the wheels is that lateral forces become negligible under these circumstances, therefore impairing steerability and possibly causing instability.

The avoidance of large wheel slip is done through brake torque control in ABS and through a combination of brake and engine torque control for ASR . Work on traction control has been more recent and follows developments in ABS control. Sensors initially designed for ABS have been used for ASR. In this case, ASR can be seen as an extension of ABS [47, 51, 157, 284].

Some of the main issues of ABS and ASR control are the computation of wheel slip and tyre/road friction coefficient estimates. A large number of different techniques are described in the literature for the purpose of obtaining these quantities [22, 291].

Brake torque control is implemented via brake pressure modulation. The practical implementation of brake torque control for ABS uses a varied number of pressure modulation schemes, using various types of hydraulic circuitry arrangements and sensors [22, 269, 66, 28, 64, 218]. Methods used to reduce brake pressure include pump activated valves (deboost), pump and dump systems and dynamic flow control [269]. Typical configurations are 4, 3 and 2 channels circuits with 2, 3 or 4 sensors [140, 269].

The controlled elements, the wheels, are subject to a variety of external disturbances such as variable inertia during clutch engagement or disengagement, additional engine braking torques, friction coefficient variations in the brake as well as in the tyre/road contact, etc. The regulation against all these unpredictable disturbances requires a considerable amount of complex nonlinear control loops [22] and most ABS controllers to date are largely empirical and rule based.

In ASR control the application of engine and braking system torque control also presents a wide variety of implementation concepts. BMW [291] has two types of approach, one which controls directly the wheel slip and another that exerts the control action based on vehicle dynamic responses. Their throttle control is handled more or less as a modified bang-bang control system. Within three different ranges of operation, a quasi-continuous slip controller with a proportional and differential type of behaviour is claimed to come into effect [284]. Sturm [276] describes the system used at Audi in which there are load and lateral acceleration sensitive devices which compensate braking and/or traction force distribution, therefore achieving yaw moment compensation. Maisch from Bosch, [157] presents a large number of engine torque control strategies, varying from throttle angle and ignition timing to fuel injection pulses. Schulze [251] presents Volkswagen's ASR approach in which the control action is based on a combination of engine and brake torque for low values of vehicle speed ($\leq 10 m/s$) and only engine torque, if otherwise. For the throttle versus engine power relationships look-up tables are used.

An interesting point to be observed is that fuel and ignition control are also dependent on engine states and are an integral part of most engine management systems (EMS) of present vehicles. In this way, it can be noticed the trend towards integrated control of vehicle forward motion, involving combined control of the powertrain and the braking systems.

The interaction of powertrain and braking control systems is inherent in four wheel drive (4WD) vehicles. Various different ways of managing torque split between front and rear, left and right wheels have been studied including many differential slip control techniques [67, 74, 77, 88, 188, 275]. However, developments of 4WD vehicles have progressed almost independently of wheel slip control systems [74, 291]. 4WD vehicles present additional problems due to the constraints imposed by central differentials. There is a large number of recent papers addressing the utilisation of both ABS and ASR in these vehicles with a large number of possibilities for wheel slip prevention strategies [276, 192, 34, 282, 306].

2.3 Suspension Control

The suspension system consists of a set of mechanical elements which connects the vehicle body and the wheels and whose general objective is to provide a comfortable and safe ride for the occupants (the driver and the passengers). In order to attain these objectives a number of functions have to be performed by the suspension system. For a comfortable ride it has to isolate the cabin from vibrations induced by road uneveness and suspension elements. The control of vehicle body and wheel attitudes are functions related mainly to safety aspects. Vehicle body attitude control can be thought to include supporting vehicle weight and controlling body inclinations. Wheel attitude control refers to controlling the relative position between the wheel and the road, which is related to the contact forces at the tyre/road contact. It must also be considered that these functions have to be performed within the boundaries of existing design constraints, such as available workspace for the movement of suspension elements, the existence of appropriate components and materials as well as their practical limitations. Typical vehicle body ride resonance frequencies are around 1 Hz and wheel hop frequencies are around 10 Hz, with one decade separation between them.

Typical road models are considered gaussian white noise and they have been described either in frequency or time domain [8, 29, 93]. Because road surface displacement approximates integrated white noise over the range of 0.01 to 10 cycles/m [264], various alternatives have been adopted to handle this fact and they include augmented state vector [296] and first order low pass filter approximations [86]. For road obstacles, deterministic time functions are used [197].

Conventional suspensions are a result of many years of research and ingenious concepts involving linkages, spring and dampers of fixed (very likely nonlinear) characteristics have been implemented in order to achieve these conflicting objectives. They are classified as passive systems since no external source of power is required for their operation. A variation of these systems are the so called adaptive suspensions in which the damping or spring stiffness are varied by means of a low power actuator in either continuous or discrete form in response to sensed variables.

With the development of microelectronics and using control theory, it has become clear that theoretical improvement in suspension performance is possible.

However, the progress of controlled suspensions has been relatively slower than initially expected. Karnopp [122] states that this is undoubtly due to the large forces involved in the suspension system, because large forces normally imply large amounts of control power and significant problems in assuring safety and reliability at reasonable costs. Nevertheless, a large number of papers have been devouted to this area and reviews have appeared with regularity in recent years [81, 94, 264].

Active systems consist of actuators which act as force producers according to some control law and they substitute the springs and dampers in passive suspensions. In these systems significant amounts of power are required to generate the suspension forces. They are classified as fast or slow depending on whether or not their frequency bandwidth covers the wheel hop modes resonance frequencies.

Semi-active systems have been introduced by Crosby and Karnopp in 1972 [15] and they are similar to active ones, the difference being that the actuators are replaced by controllable dampers, in which case the damper can be programmed to produce suspension forces similar to those produced by active force generators, at least during some of the time. In this circumstance, when work is demanded from the actuator, the force produced by the damper is zero. They have to include a parallel spring for weight support and thus the level of power required is much lower than those by active systems. It is worth noting that a large variation in suspension concepts exist within these general classification headings and which are not detailed in the description carried out in this work.

For controlled suspensions one of the control concepts used is one which attempts to separate the functions of attitude control and body isolation. This concept derives from the fact that in passive systems stiff springs and dampers control body and wheel attitudes well, but soft springs and dampers isolate the body better from road uneveness. This concept is presented by Karnopp [119] with the introduction of the skyhook damper concept.

Another control approach considers the use of stiff broad band actuators to generate the suspension forces. Ideally, this type of approach can result in effective body attitude control and wheel attitude and tyre/road contact forces. However, actuators in the size needed for automotive applications are not likely to approach this ideal behaviour. Also, the state-of-the-art actuators are not suitable for use in mass produced vehicles. Furthermore, there may be high power losses, due to the characteristics of these type of systems [122]. A number of other approaches to suspension design described in the literature fall in intermediary categories which try to realistically account for the limitations of these concepts [303].

In order to quantify the objective of passenger comfort, root mean square (rms) values of vertical acceleration of vehicle body at the passenger location and weighted in the frequency domain are nowadays widely accepted [112]. For the safety aspect, rms values of dynamic tyre load variations are normally employed, since it is related to the longitudinal and lateral forces generation by the tyres. Design constraints quantification involves rms values of relative displacement between sprung and unsprung masses and for active systems, power consumption and actuator force levels. Sharp [264] states that working space is the most basic conflict in suspension design and argues for comparisons which consider equal working space amongst different suspension designs.

Most theoretical studies to date use a single wheel, quarter car model with 2 degrees of freedom [168, 86, 120, 5, 107, 46, 264, 50]. This model contains the most basic features of the real problem and possesses the advantage of largely simplifying the modelling, analysis and design of the suspension. However, it does not take a number of important aspects into consideration such as the coupling between the various degrees of freedom, the correlation of the road inputs for all four wheels and load transfer effects for the tyre forces. Some more recent studies have taken this coupling effect into consideration in which it has been attempted to decouple some of the modes [158] and others have realised comparative analysis between passive, semi-active and active system with such coupling effects [16]. Fruhauf [75] has used approximations to cater for the time delay between front and rear road inputs. Most of these studies are based on models which have been generated by hand, in which linearisation for control system design was performed at the model generation stage. Recent approaches are adopting linearised models obtained through various linearisation techniques of nonlinear models derived using MBS programs [92, 271].

As far as control principles are concerned, linear quadratic (LQ) optimal control is the most used to date. This is due to suitability of this type of approach to suspension design because it allows the establishment of conflicting objectives in the cost function, which is the case of vehicle suspensions. Another facet is the ease of application of such techniques, however the choice of the weighting matrices seems somehow arbitrary. Performance indexes involving weighted sum of mean square values of body acceleration, dynamic tyre loads, suspension workspace and sometimes actuator forces are normally used [15, 16, 1, 178, 86, 54, 107]. Initial studies assumed full state availability [15, 158], but later ones [1, 87, 178] have supposed limited states and used state estimation techniques.

Recent applications include frequency domain multivariable design [92] and there is a large number of recent papers using preview information for the development of control algorithms [69, 87, 136, 264]. Michelberger [181] in a recent publication suggests an eigenstructure assignment approach for suspension design.

However, one trend which has been observed in controlled suspension design is a return to quarter car models and the attempt to develop new suspension concepts based on a mixture of active, semi-active and passive concepts with some adaptive characteristics. This might be attributed to limitations of fully active suspensions both on a theoretical basis as well as practical [122]. However, Wright from Lotus claims that the use of quarter car models hinders important benefits of fully active systems, making it compare unfavourably to other concepts [303]. One study about theoretical limitations of suspension systems is presented by Hedrick and Butsen [95] in which it is shown the existence of frequency response invariant points for some of the transfer functions derived from simple models. However, these theoretical limitations confirm practical difficulties already encountered especially for the control of the wheels attitude and vibrations at frequencies in the region of the wheel hop mode resonances.

When these issues which are mainly related to suspension concept are solved, more comprehensive models will be necessary. One important point to be made is that with the existence of active elements, the approach to suspension design can become much more audacious and it is a personal view of this author that current approaches are extremely conservative, in the sense that only the spring and damper elements of the suspension and most of the time, only in relation to ride aspects, have been dealt with. The influence upon other modes or degrees of freedom have been taken into consideration only in an indirect form, and the influence of other suspension elements, such as the linkages, have not yet been addressed at all. With the availability of MBS programs, however, tentative geometrical active suspensions should be possible, opening enormous possibilities for the design of completely innovative suspension systems.

2.4 Steering Control

Following the trend in the application of control to vehicle dynamics, one of the last areas to be addressed is the steering system. Steering control is aimed at improving vehicle handling behaviour, which is considered to comprise mainly the lateral, yaw and roll degrees of freedom of the vehicle body.

The automatically steered or guided vehicle is an important area of research and it could be classified under steering control. However it will not be treated in this paper. Extensive review and references on this subject are given by Goodall and Kortum [81] and more recently by Wickens [297].

An early work on steering control was developed by Kasselmann and Keranen and it is described in [52]. It was intended for the rejection of external disturbances such as wind gusts and rough roads, using the stability augmentation concept from the aircraft industry. In this case, driver's inputs were augmented using yaw rate feedback.

Although, its history dates back to the 1960's, it was not until the 1980's that four-wheel-steering (4WS) started to receive considerable attention from the scientific and motor industry communities. The main objectives of using additional rear wheel steering are to increase stability and decrease response times for high speed vehicle operation and to increase manoeuvreability at low speeds. The means adopted in the literature to achieve these objectives varies widely, but it can be said that the common characteristic of present day solutions, direct or indirectly, involve the control of the rear wheel steering angle such that in slow manoeuvres (usually involving large steering angles) the rear wheels steer in the opposite direction to the front wheels and in fast manoeuvres (usually with small front steering angles) the front and rear wheels are steered in phase. Some important drawbacks of these approaches are rear end projection at large steering angles, increase radius of turn at high speeds and as a consequence, increasing understeer characteristics.

One of the simplest rear wheel steering strategies is Honda's system, described by Sano et al. [235] in which the rear steering angle is a nonlinear function of the front steering angle only, which is practically implemented through the use of an ingenious mechanism. Such a strategy was adopted because it was found that large steering angles were associated with low speed manoeuvres, while small ones were related to high speed ones. Sano argues that proceeding in this way, there is no time delay in the lateral force build-up process of the rear wheels, and therefore the yaw rate and lateral acceleration response times improve.

Nissan's philosophy [267] is to control vehicle sideslip angle at the center of mass to zero, based on the fact that the sideslip angle is a basic information needed for the driver's controlling task and yet it is a difficult quantity to be perceived by her/him. A similar principle for controlling the rear wheels is adopted, in which the rear steering angle is varied out-of-phase for low speeds and in-phase at high speeds. Again large variation is observed in the turning radius with increasing speed resulting in a strong understeer characteristic. Also, the lateral acceleration and yaw rate steering response gain drops sharply, indicating deteriorated controllability. Later in another publication, [196], the addition of a time lag to the rear steering input is advocated in order to improve transient response.

Senger [260] proposes a similar approach, but with an additional objective of maintaining neutral steer behaviour and for this matter he adopts a rear steering angle which is a function of the product of yaw rate and forward velocity. Later in another publication, he also proposes the use of a linear Luenberger state observer in order to reduce the number of transducers necessary in such a system [262]. Williams of Jaguar [294] also reports the study of the performance of various rear wheel steering strategies and the use of a Kalman filter as a state observer.

Mazda's proposal [281] takes into consideration the yaw rate phase lag and lateral acceleration phase lag of the frequency response curves for those quantities in relation to front wheels steering angle and a subjective evaluation of driveability from a number of drivers. Because those frequency responses are speed dependent, it arrives at a similar control strategy. A very ingenious practical solution is presented to obtain speed dependent phase change at the rear wheels.

Lugner is progressing towards an integrated motion control approach in the sense that in a recent suggestion [150] for the control of the tyre forces he analyses the use of either additional rear wheel steering or braking force to improve vehicle cornering dynamics. In his early work [148, 149], a fully active 4WS is considered in which an additional steering angle is applied to both front and rear wheels as a function of the tyre's normal load. The objective of this control is to obtain equal force coefficients for the outer and inner wheels in a turn, since their balancing increases maximum driving velocity. Later [150] he attempts to improve vehicle cornering behaviour by using additional braking forces in order to create yaw moments which would restore the vehicle to an intended path. In this case, he analyses the loss of stability as the vehicle reaches a split μ situation while cornering. The action of the steering control system is to act only during the period in which the driver has not reacted yet, in the first second of the simulation. His aim is to provide a vehicle in a better position and more controllable to the driver under these emergency situations.

Nagai [199] proposes a model following control in which two different strategies are adopted. In the first one, front and rear wheels are steered by feedforward from the model, while in the other they are both calculated by feedforward from the model and feedback from the 'real' vehicle. The feedback path uses state feedback and the gain matrix is the solution to a LQR problem with cost function dependent on yaw rate, lateral acceleration and front and rear steering angle.

Ohnuma and Metz [207] present an interesting paper in which controllability and stability aspects of 4WS vehicles are discussed and which uses a highly control oriented approach. Issues such as observability, controllability and robustness are formally discussed and two types of controllers are suggested.

For steering control most of the work is performed based on a 2 degrees of freedom, the so called bicycle model [255]. However, there is a concern about the influence of other degrees of freedom on vehicle handling, insofar as they influence the tyre forces. Nalecz [202] approximates suspension characteristics in order to obtain roll information to account for roll steer in his model. Lugner adopts a four wheel vehicle model in which comprehensive tyre and powertrain models are included and approximate suspension characteristics are considered in order to obtain tyre normal force. However, because it is a hand derived model, a number of other approximations are made, such as not considering the wheel masses separately, assuming a roll axis, linearising trigonometric expressions involving small angle, and neglecting small terms. To date there has not been the use of a fully comprehensive model, or one that takes suspension geometric characteristics into consideration and the suspension masses separately. The reason for this is because this area has just recently become the focus of research interest and the present day state-of-the-art research is still at the exploratory stage. As more knowledge is accumulated, the need for more comprehensive models which considers other aspects of vehicle motion will be necessary.

2.5 Conclusions

The previous sections demonstrate that since the early 1970's there has been an increasing number of applications of automatic control techniques in automotive vehicles. In the majority of the cases to date these applications have developed in a piecemeal fashion, whereby individual vehicle subsystems have been treated in isolation. The limitations of these approaches have two main sources. One is related to modelling techniques and the other with the complexity of the control problem involved.

However, recent work in the area of powertrain control indicates a trend towards the combined control of the engine and transmission within an integrated powertrain control system.

Future applications of control in automotive vehicles will progress this trend along the road to system integration, with the ultimate aim of developing a co-ordinated vehicle control system capable of integrating the actions of the various vehicle subsystems. This progress toward integration of automotive vehicle systems control will require that the interaction amongst the various subsystems to be taken into consideration at the control design stage. To achieve this, a systematic approach to the study of automotive vehicle control will be necessary, in which the automotive vehicle is viewed as a total system made up of a number of interacting subsystems.

The integrability of vehicle subsystems which function under various operating principles such as the powertrain and braking system which include electronic, electrical, hydraulic and mechanical principles of operation, will require the use of flexible mathematical modelling tools. These tools should allow the quick assembly and execution of these complex simulation models. In this respect, the use of multibody systems techniques for the generation of part of the simulation model for control studies opens new possibilities for the development of more complex models capable of addressing vehicle motion control. Therefore, a simulation and design environment that is capable of dealing with these aspects of vehicle control is discussed, proposed, implemented and utilised in the work carried out in this thesis. The next chapter will address the theoretical background and the techniques currently available for obtaining a comprehensive mathematical model to describe vehicle chassis dynamics. In this analysis the emphasis will be placed at techniques which allow the quick generation of complex vehicle chassis models that perform in reasonable runtime and that are capable of integrating in a general simulation, analysis and design environment.

Chapter 3

Mathematical Modelling of Automotive Vehicle Dynamics

3.1 Introduction

The objective of the present chapter is to give an overview of Multibody Systems (MBS) modelling techniques with respect primarily to its historical developments, the mathematical formalisms used and the computer implementations of such formalisms. A specific aim of this chapter is also to present in a simplified way, Kane's Method, which is the formalism used in SD/FAST, the MBS software used in the development of the work presented in this thesis.

In the study of a modelling problem, various levels of distinct complexities are involved. The capacity of properly defining the relevant aspects of a modelling problem at each level is a quality required from scientists and engineers. Independently of which level it is referred to, the following steps are part of the modelling process:

1. describe a physical model of the system which contains the relevant aspects for the study intended, with its simplifying assumptions;

2. derive constitutive equations which describe mathematically the behaviour of the system;

3. solve the resulting equations either analytically or numerically in order to get the estimated behaviour of the system;

4. verify the the results of the model by comparison with the behaviour of the real system; and

5. modify the physical system if necessary or use it for analysis and design purposes.

The use of computers to perform tasks 2 and 3 is a widespread practice these days, for example, with finite elements methods, electrical network programs, MBS equation generation programs, general purpose simulation languages, etc., and they allow the scientists and engineers to concentrate on the more important aspects 1 and 5 of the study.

3.2 MBS Modelling Techniques

Due to the peculiar characteristics of space science projects and the increased degree of complexity needed to develop these projects adequately, mathematical modelling assumed a crucial role. Computers and programs were required to present improved performance and theories and methods had to be devised which would result in safe design within the established timetable.

It was not only MBS modelling which benefited from such drive, but many other areas of science, from Production Engineering (e.g., Graph theory) to all areas affected by Modern Control Theory, for instance.

A MBS is defined as a mechanical system with many degrees of freedom. For instance, as Kane once remarked, a system is a MBS if it has 2 or more bodies [154]. The motion of a MBS is governed by equations called dynamical equations of motion. These equations comprise a set of differential equations, together with some algebraic equations. The differential equations are an expression of the physical laws (Newton's Laws of Motion) which describe the motion of rigid bodies and the algebraic equations take into account desired restrictions on the geometry of the system or its motion, such as linkages connecting adjacent bodies, or particular characteristics of the contact of bodies.

In the past, the derivation of the dynamical equations for mechanical systems could be achieved by hand, through the application of first principles in their various forms. However, with the increased complexity which is desired to study these systems, this process has become cumbersome, tedious and error-prone. It can also be very difficult to accommodate design changes and even simple model variations. For these reasons computer programs for generating the equations of motion for MBS have been developed.

MBS modelling techniques can be used to perform the analysis and design studies of any mechanical system which can be modelled as a set of rigid bodies interconnected by joints, influenced by forces, driven by prescribed motion and restricted by constraints [254]. The equations of motion for these systems are very laborious and difficult to be generated by hand even for a system comprised of a small number of interconnected bodies. Therefore, it was a step forward that formalisms were developed through which the equations of motion for a MBS could be generated by computers based on simple data about the bodies (geometric and inertial) and their interconnections (kinematic constraints and force laws). Typical systems which can be studied by such techniques include articulated spacecrafts, ground vehicles, mechanisms and machines, manipulators, high speed electromechanical devices, etc. Some of the analysis which can be normally performed include assembly analysis, forward and inverse dynamics, static and steady motion, power flow, etc.

The development of MBS modelling by computers has two main areas of origin: The machine and mechanism community and the space research community. Only later on, as it always seems to be the case with ground transportation, the vehicle community, road and rail got involved.

MBS modelling techniques can be classified in many ways. One commonly adopted is to divide them according to the formalism which is used to describe the system and the nature of the computer implementation of the related algorithm. Other aspects which are normally considered include the class of systems which can be modelled, e.g., open or closed loop topology; the choice of dependent variables, whether automatic or done by the user; the number of equations used, whether minimal or redundant sets are employed; if the resulting set is the complete nonlinear set or if it is just the linearised terms; and so on.

3.3 Brief History of MBS Modelling

An extensive review on the subject of MBS history is presented by Schwertassek [253]. In his paper he also refers to specific reviews of MBS techniques based on Eulerian and Lagrangian approaches. A review of the application of MBS to ground vehicles is presented by Schiehlen and Kortum [243]. A recent book by Schiehlen [247] reviews a number of MBS programs performing a manipulator and a planar mechanism benchmarks. Also a forthcoming review paper by Kortum and Sharp is due in the winter of 1991 on MBS applied to vehicle dynamics, as a result of a workshop held last September in Czechoslovakia and benchmarks performed by MBS developers and users throughout the world, in which the author of this thesis was an active participant representing a set of computer packages. A preliminary report of these activities is presented in [129].

A brief historical overview of the development of MBS techniques is illustrated in the diagram of figure 3.1 [253, 243, 241, 230].

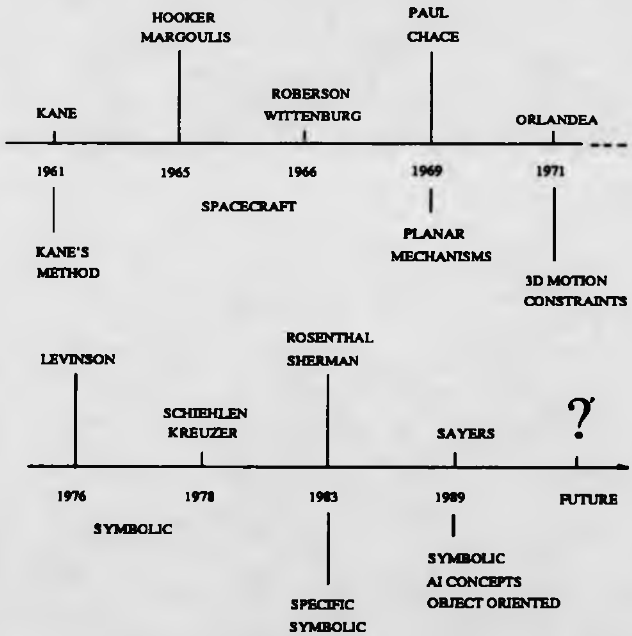

Figure 3.1: Historical evolution of MBS modelling techniques

Hooker and Roberson discussed together and presented their proposal for the development of MBS programs in California [253]. Afterwards they went separate ways and in 1965 Hooker and Margoulis, in California, and in 1966 Roberson and Wittenburg, in Germany at the DFVLR, now DLR, developed formalisms for the numerical derivation of the equations of motion for the 3d motion of spacecrafts with open loop topology.

When the MBS approach was introduced in the mechanical and ground transportation areas in the late 60's, early 70's, through the works of Paul and Chace [253], difficulties occurred with closed kinematical chains and nonholonomic constraints rarely found in space applications. Orlandea's work [209], which originated the program ADAMS, dealt with these kind of problems; however, because it had to be built around the most general form of the equations of motion it resulted in large systems which were then solved using sparse matrix techniques.

In order to overcome these problems, the use of symbolic programs were introduced by Levinson in 1976 and followed by Kreuzer and Schiehlen in 1978. However, with the use of general purpose symbol manipulation languages such as MACSYMA and REDUCE, special techniques were needed in order to generate efficient FORTRAN code and to avoid exceeding computer limits with intermediary expression swell.

Alternatively, simplified symbolic manipulation methods have been combined with MBS formalisms to create efficient MBS codes. Rosenthal and Sherman developed their program SD/EXACT [230] as part of a project for NASA in which the aim was to develop a MBS program which could be capable of running in real time in the computer $AD100$. The results were very impressive compared to general purpose multibody programs to date and a number of other programs have since appeared adopting a similar approach. They also used Kane's Method, and symbolic manipulation but they tried to incorporate facilities which were not so well catered for in the early version of SD/FAST. For this reason, the programs SYMBA [206] and AUTOSIM [237] have appeared. A recent version of SD/FAST [254] tries to cater for a larger group of users and presents a series of powerful facilities to deal with a variety of issues been currently addressed by the MBS community, such as closed loops and the handling of nonholonomic constraints.

3.4 Mathematical Formalisms

The dynamic behaviour of mechanical systems, with the relatively small order of magnitude of speeds involved, can be adequately described by Newton's Laws of Motion from Classical Mechanics and its numerous variations. Typical descriptions include Newton-Euler equations, Lagrange's equations, Jourdain's Principle,

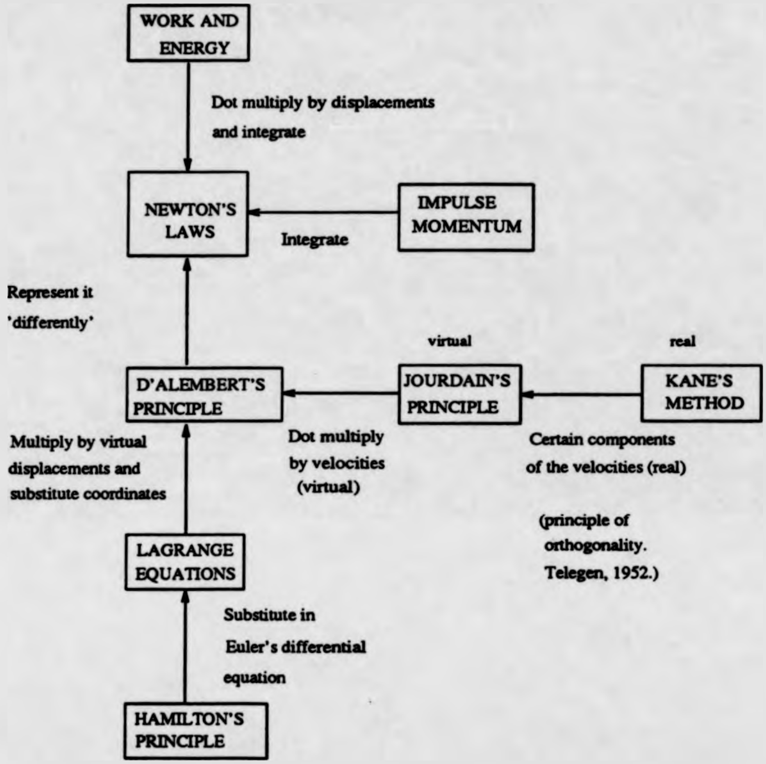

Figure 3.2: Newton's laws and their various related principles

D'Alembert's Principle and so on, as illustrated by the diagram of figure 3.2. This diagram shows in a simplistic way, the relationship between these various forms of expressing them and how they are obtained. Formalisms which are based on Newton-Euler equations are called *synthetic* and those based on Lagrange's are called *analytical.*

This present section on mathematical formalisms is divided in the following way: Initially some basic definitions are presented for the understanding of the forthcoming discussion, next Eulerian approaches are presented followed by Lagrangian ones. Finally a short discussion on Kane's Method which is the theoretical base for the MBS modelling approach used in this study is carried out.

3.4.1 Basic Definitions

The next sections are intended to give an overview of some basic definitions necessary to the development and understanding of the equations which are going to be discussed later on. It has no intention to be a complete and precise presentation, since it is trying to summarise topics which are covered by whole books of dynamics of MBS. The material presented hereafter is based on Kane, Likins and Levinson's book [117] and Kane and Levinson's book [116]. It also follows a similar nomenclature.

Scalar quantities are represented by plain typeface while dyadic or vectorial quantities are represented by boldface characters. Superscripts are used to identify the frame and body or point to which the vectorial quantity refers. When its meaning is clear in the context which it is used, or when it does not matter to which frame it refers, they are omitted. When present, the right superscript refers to a body or a point in a system and the left superscript to the reference frame to which it refers. Subscripts are used as identifiers for the quantities in which they appear, such as position in vector arrays, and their meaning should be clear as they appear in the text in each particular situation.

3.4.1.1 System's Configuration

The *configuration* of a set S of v particles, $P_1, \ldots, P_v$ in a reference frame A is known whenever the position vector of each particle relative to a point fixed in A is known. Thus v *vector quantities* or equivalently $3v$ *scalar quantities* are required for the specification of the configuration of S in A.

The scalar quantities are called Cartesian coordinates and they are defined as follows. If $\boldsymbol{a}_x, \boldsymbol{a}_y, \boldsymbol{a}_z$ are orthogonal unit vectors in A, then

$$\begin{aligned} x_i &= \boldsymbol{p}_i \cdot \boldsymbol{a}_x \\ y_i &= \boldsymbol{p}_i \cdot \boldsymbol{a}_y \\ z_i &= \boldsymbol{p}_i \cdot \boldsymbol{a}_z \end{aligned} \tag{3.1}$$

where $\boldsymbol{p}_i$ is the position vector from a point O fixed in A to the point P_i, and they represent the components of $\boldsymbol{p}_i$ in the directions of $\boldsymbol{a}_x$, $\boldsymbol{a}_y$ and $\boldsymbol{a}_z$. The operation denoted by '$\cdot$' is the dot or inner product. This situation is illustrated in 3.3.

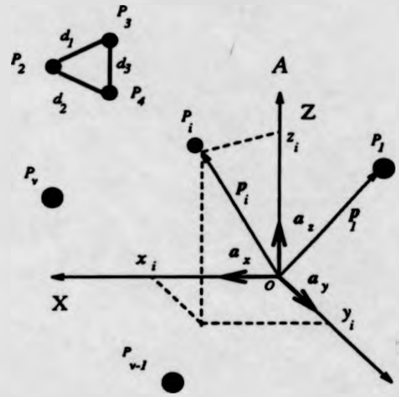

Figure 3.3: Configuration of a system of particles

3.4.1.2 Configuration Constraints

If the motion of S is affected by the presence of bodies that come into contact with one or more of the P_i, restrictions are imposed on the positions that the affected particle may occupy. S is then said to be subject to *configuration constraints* and an equation expressing such constraints is called a *holonomic constraint equation*. An example of configuration constraint is represented by the rigid links d_1, d_2, d_3 between particles P_2, P_3, P_4, of figure 3.3. It can be expressed as

$$f(x_1, y_1, z_1, \ldots, x_v, y_v, z_v, t) = 0 \tag{3.2}$$

Equations of the type 3.2 are classified as *rheonomic* or *scleronomic*, according to whether the function f does, or does not contain the quantity *time* explicitly.

3.4.1.3 Generalised Coordinates

When a set S of v particles $P_1, \ldots, P_v$ is subject to constraints represented by M holonomic constraint equations, only

$$n = 3v - M \tag{3.3}$$

of the $3v$ Cartesian coordinates x_i, y_i, z_i are independent of each other. Under these circumstances one can express each of them as a single valued function of time t and n functions of t, say $q_1(t), \ldots, q_n(t)$, in such a way that the constraint equations are satisfied. These quantities are called *generalised coordinates* for S in A.

The number n of generalised coordinates of a set S of v particles is the *smallest* number of independent scalar quantities such that to every assignment of values to these quantities and the time t, there corresponds a definite admissible configuration of S in A.

3.4.1.4 Generalised Speeds

Expressions for angular velocities of rigid bodies and velocities of points of a system S, whose configuration in a reference frame A is characterised by n generalised coordinates $q_1, \ldots, q_n$ can be brought into particular advantageous form by the introduction of n quantities $u_1, \ldots, u_n$ called *generalised speeds* for S in A, defined as

$$u_r = \sum_{s=1}^{n} A_{rs}\dot{q}_s + B_r \qquad (r = 1, \ldots, n) \tag{3.4}$$

where A_{rs} and B_r are functions of $q_1, \ldots, q_n$ and t. They must be chosen such that 3.4 can be solved uniquely for $\dot{q}_1, \ldots, \dot{q}_n$, that is

$$\dot{q}_r = \sum_{s=1}^{n} C_{rs}u_s + D_r \qquad (r = 1, \ldots, n) \tag{3.5}$$

where C_{rs} and D_r are known functions of $q_1, \ldots, q_n$ and t. Equations 3.5 are called *kinematical differential equations* for S in A, and they play an important role in the derivation of the equations of motion for the study of a mechanical system.

3.4.1.5 Motion Constraints

It can occur, for physical reasons, that the generalised speeds $u_1, \ldots, u_n$ for a system S in a reference frame A are not independent of each other. In that case S is said to be subject to *motion constraints*, and an equation that relates $u_1, \ldots, u_n$ to each other is called a *nonholonomic constraint equation.* In this case S is called a *nonholonomic system.*

When all the nonholonomic constraint equations can be expressed as the m relationships

$$u_i = \sum_{s=1}^{p} Y_{is}u_s + Z_i \qquad (i = p + 1, \ldots, n) \tag{3.6}$$

where

$$p = n - m \tag{3.7}$$

and Y_{is} and Z_i are given functions of $q_1, \ldots, q_n$ and t. S is referred to as a *simple nonholonomic system* possessing p degrees of freedom in A. Alternatively, the motion constraint equations can also be expressed as

$$\sum_{r=1}^{n} W_{ir}\dot{q}_r + T_i = 0 \qquad (i = p+1, \ldots, n) \tag{3.8}$$

where W_{ir} and T_i are also given functions of $q_1, \ldots, q_n$ and t, and they express equations 3.6 as a function of the derivatives of the generalised coordinates.

It is worth noting that equations 3.6 or 3.8 should be nonintegrable; that is, there should not be a function $f(q_1, \ldots, q_n)$ which is constant throughout every time interval in which 3.6 or 3.8 are satisfied. If such a function existed, then $q_1, \ldots, q_n$ would not be independent of each other, and thus they would not constitute a set of generalised coordinates. Or else, it can also be said that the motion constraints described by equation 3.6 or 3.8 cannot be obtained by differentiating a position constraint.

One classical example of nonholonomic constraint is a sharp edged discus rolling without slipping in a plane. There are restrictions concerning its motion (The direction of the velocity vector of its centre of mass should always coincide with its heading angle - no sideslip -), but the discus can be positioned anywhere in the plane, that is, there is no restriction concerning the configuration it can attain.

3.4.1.6 State Variables

The *state variables* of a system S in a reference frame A are a set of scalar quantities which allow the description of the *configuration* and *motion* of every particle P_i belonging to S.

The configuration of S in A in particular, for example, the location of every body's mass centre and the orientation of each body is determined by the generalised coordinates $q_1, \ldots, q_n$. Given the configuration of S in A, the motion of every P_i belonging to S is determined by the generalised speeds $u_1, \ldots, u_n$. In particular, again, for instance, the linear velocity of every body's mass centre and the angular velocity of each body.

Because of the difference in the treatment of constraints for the various mathematical formalisms, there are differences concerning the definition of the generalised coordinates and speeds and therefore the state variables. Normally, in each formalism, the state variables are defined to be those configuration and motion quantities which are obtained through *integration* of the equations of motion generated by the formalism, whether or not this is the only way to do it. Or else, it can be said that a certain quantity is a state variable in a certain formalism if it can only be obtained by integration in that formalism.

3.4.2 Eulerian Approaches

Synthetic formalisms which are also referred to as Eulerian approaches are based on equations which describe the equilibrium of forces in a system. They can be obtained through the momentum equations for the mechanical system.

The linear momentum for a rigid body B in an inertial frame N is given by

$$\boldsymbol{L}^B = m^B \boldsymbol{v}^{B*} \tag{3.9}$$

where m^B is the mass of B, and $\boldsymbol{v}^{B*}$ is the velocity of the mass centre, $B*$, of B in N.

The central moment of momentum or central angular momentum for the body B about its mass centre, $B*$, in an inertial frame N is described by

$$\boldsymbol{H}^{B*} = \boldsymbol{I}^{B*} \cdot \boldsymbol{\omega}^B \tag{3.10}$$

where $\boldsymbol{I}^{B*}$, called the central inertia dyadic of B, is the inertia dyadic of B relative to its mass centre, and $\boldsymbol{\omega}^B$ is the angular velocity of B in N. Note that the quantity $\boldsymbol{H}^{B*}$ is a vector.

Newton-Euler equations can be obtained from the previous equations 3.9 and 3.10, as follows. For the translational motion of the rigid body B, Newton's equations are

$$\boldsymbol{R}^B = \frac{d\boldsymbol{L}^B}{dt} = m^B \boldsymbol{a}^{B*} \tag{3.11}$$

where $\boldsymbol{R}^B$ is the resultant of all contact and distant forces acting on B, and $\boldsymbol{a}^{B*}$

is the acceleration of the mass centre of B in N. For the rotational motion of B, Euler's equations are given by

$$\boldsymbol{T}^B = \frac{d\boldsymbol{H}^{B*}}{dt} = \boldsymbol{I}^{B*} \cdot \boldsymbol{\alpha}^B + \boldsymbol{\omega}^B \times \boldsymbol{I}^{B*} \cdot \boldsymbol{\omega}^B \tag{3.12}$$

where $\boldsymbol{T}^B$ is the sum of all moments (torques of couples) applied to the body B about its mass centre and $\boldsymbol{\alpha}^B$ is the angular acceleration of B in N. The operator '$\times$' denotes the cross product of two vectors. This result derives from a well known fact relating the time derivative of a vector and angular velocity [116, 117, 105].

It is worth noting that scalar quantities can be obtained from these vector/matrix equations by projecting the vectors in some particular direction, using the dot product operation with vectors in the desired directions. For instance, if these vectors are the body fixed orthonormal frame aligned with the principal axes, the well known form of Newton-Euler equations results [116].

Equations 3.11 and 3.12 define a linear relationship between the derivatives of the velocities and the sum of external forces and torques applied to a body. For a system with n degrees of freedom, a set of equations of the form

$$\boldsymbol{M}\dot{\boldsymbol{u}} = \boldsymbol{f} \tag{3.13}$$

can be obtained from equations 3.11 and 3.12, in terms of the derivatives of the generalised speeds as defined in section 3.4.1.4. These equations are called *dynamical differential equations.*

The $\boldsymbol{M}_{n \times n}$ matrix consists of the coefficients of the derivatives of the generalised speeds and it is called *mass matrix* and its elements have units of mass, moments of inertia or expressions containing both. The n dimensional $\boldsymbol{f}$ vector is called the *force array* and its elements have units of force or moments. It includes all terms which appear in the right hand side of each equation of motion and all other remaining terms which are not multiplying a derivative of a generalised speed.

Equations 3.5 and 3.13 define a set of $2n$ differential equations which describe the motion of a MBS and which can be solved for $\dot{\boldsymbol{q}}$ and $\dot{\boldsymbol{u}}$ using linear algebra techniques. If the system was nonholonomic, with p degrees of freedom, still there would be n equations defined by equation 3.5, but instead of n, there would be p dynamical equations, 3.13, and m nonholonomic constraint equations, 3.6. The resulting vector of the derivatives of the generalised coordinates and speeds can then be integrated numerically in order to generate the response of the MBS.

Many approaches have been developed to solve these equations of which SD/FAST is one of them that uses Kane's Method to obtain equation 3.13. Its derivation is presented in a coming section when Kane's Method is discussed.

3.4.3 Lagrangian Approaches

Analytical formalisms are referred to as Lagrangian approaches and they are described by the balance of energy within the system in terms of the generalised coordinates and their derivatives. Lagrange's equations are derived from the application of the principle of virtual displacements to the equations of motion given by equations 3.11 and 3.12, represented in their D'Alembert form, and performing a substitution of variables.

Let $q_1, \ldots, q_n$ be the generalised coordinates which specify the configuration at the time t of a holonomic system S comprised of v particles $P_1, \ldots, P_v$. Let m^{P_i} specify the mass of a particle P_i of S. Let the cartesian coordinates of P_i in a Newtonian frame be expressed as a function of the generalised coordinates as

$$\begin{aligned} x_i &= \theta_i(q_1, \ldots, q_n, t) \\ y_i &= \phi_i(q_1, \ldots, q_n, t) \\ z_i &= \psi_i(q_1, \ldots, q_n, t) \end{aligned} \tag{3.14}$$

If R_{xi}, R_{yi} and R_{zi} are the components of the resultant of all forces acting on P_i, then Newton's equations for P_i are

$$\begin{aligned} m^{P_i}\ddot{x}_i &= R_{xi} \\ m^{P_i}\ddot{y}_i &= R_{yi} \\ m^{P_i}\ddot{z}_i &= R_{zi} \end{aligned} \tag{3.15}$$

Multiplying through the first equation by δ_{xi}, the second by δ_{yi} and the third by δ_{zi} and adding the resulting equations, yields

$$R_{xi}\delta_{xi} + R_{yi}\delta_{yi} + R_{zi}\delta_{zi} = m^{P_i}\ddot{x}_i\delta_{xi} + m^{P_i}\ddot{y}_i\delta_{yi} + m^{P_i}\ddot{z}_i\delta_{zi} \tag{3.16}$$

The quantities δ_{xi}, δ_{yi} and δ_{zi} are small displacements, called ***virtual displacements*** and they have been written as deltas to emphasise that they may have arbitrary values, consistent with the constraints and hence are ***not*** differentials.

Transforming the coordinates in equation 3.16, from the Cartesian coordinates (x_i, y_i, z_i) to the generalised coordinates $(q_1, \ldots, q_n)$ using the relations of equation 3.14, results for the components of the virtual displacements

$$\begin{aligned} \delta_{xi} &= \sum_{r=1}^{n} \frac{\partial x_i}{\partial q_r}\delta_{q_r} \\ \delta_{yi} &= \sum_{r=1}^{n} \frac{\partial y_i}{\partial q_r}\delta_{q_r} \\ \delta_{zi} &= \sum_{r=1}^{n} \frac{\partial z_i}{\partial q_r}\delta_{q_r} \end{aligned} \tag{3.17}$$

In order to introduce a simplification into the algebra of the derivation, suppose that $\delta_1 \neq 0$, while $\delta_2 = \cdots = \delta_n = 0$. This is permissible since the generalised coordinates have been defined as independent. In this case, equation 3.17 becomes

$$\delta_{xi} = \frac{\partial x_i}{\partial q_1}\delta q_1; \qquad \delta_{yi} = \frac{\partial y_i}{\partial q_1}\delta q_1; \qquad \delta_{zi} = \frac{\partial z_i}{\partial q_1}\delta q_1 \tag{3.18}$$

Substituting these values in equation 3.16 and summing over all v particles of S, results

$$\begin{gathered} \sum_{i=1}^{v}\left(R_{xi}\frac{\partial x_i}{\partial q_1} + R_{yi}\frac{\partial y_i}{\partial q_1} + R_{zi}\frac{\partial z_i}{\partial q_1}\right)\delta_{q_1} = \\ \sum_{i=1}^{v}\left(m^{P_i}\ddot{x}_i\frac{\partial x_i}{\partial q_1} + m^{P_i}\ddot{y}_i\frac{\partial y_i}{\partial q_1} + m^{P_i}\ddot{z}_i\frac{\partial z_i}{\partial q_1}\right)\delta_{q_1} \end{gathered} \tag{3.19}$$

The left hand side of 3.19 has a simple physical meaning: it is the work done by the external forces during the displacement δ_{q_1}. Equating this expression with

$F_1\delta_{q_1}$, then the *generalised force* is defined as

$$F_1 = \sum_{i=1}^{v} \left(R_{xi}\frac{\partial x_i}{\partial q_1} + R_{yi}\frac{\partial y_i}{\partial q_1} + R_{zi}\frac{\partial z_i}{\partial q_1} \right) \tag{3.20}$$

It should be noted that the generalised force does not necessarily have the dimensions of force. For example, if q_i is an angle, δ_{q_i} is dimensionless and F_i would be a moment.

Next, the quantities in the right hand side of equation 3.19 are substituted. Using the formula for the differentiation of a product, results for the x_i coordinate

$$\ddot{x}_i\frac{\partial x_i}{\partial q_1} = \frac{d}{dt}\left(\dot{x}_i\frac{\partial x_i}{\partial q_1}\right) - \dot{x}_i\frac{d}{dt}\left(\frac{\partial x_i}{\partial q_1}\right) \tag{3.21}$$

From equation 3.14, $\dot{x}_i$ can be expressed as

$$\dot{x}_i = \frac{dx_i}{dt} = \sum_{r=1}^{n}\frac{\partial x_i}{\partial q_r}\dot{q}_r \tag{3.22}$$

so that

$$\frac{\partial x_i}{\partial q_1} = \frac{\partial \dot{x}_i}{\partial \dot{q}_1} \tag{3.23}$$

It can also be used the fact that

$$\frac{d}{dt}\left(\frac{\partial x_i}{\partial q_1}\right) = \frac{\partial}{\partial q_1}\left(\frac{dx_i}{dt}\right) = \frac{\partial \dot{x}_i}{\partial q_1} \tag{3.24}$$

and substituting equations 3.23 and 3.24 into equation 3.21 results

$$\ddot{x}_i\frac{\partial x_i}{\partial q_1} = \frac{d}{dt}\left(\dot{x}_i\frac{\partial \dot{x}_i}{\partial \dot{q}_1}\right) - \dot{x}_i\frac{\partial \dot{x}_i}{\partial q_1} = \frac{d}{dt}\left[\frac{\partial}{\partial \dot{q}_1}\left(\frac{\dot{x}_i^2}{2}\right)\right] - \frac{\partial}{\partial q_1}\left(\frac{\dot{x}_i^2}{2}\right) \tag{3.25}$$

Developing similar expressions for the y_i and z_i components and substituting into equation 3.19 leads to

$$\begin{aligned} F_1\delta_{q_1} = \sum_{i=1}^{v} m^{P_i} \Bigg\{ & \frac{d}{dt}\left[\frac{\partial}{\partial \dot{q}_1}\left(\frac{\dot{x}_i^2}{2} + \frac{\dot{y}_i^2}{2} + \frac{\dot{z}_i^2}{2}\right)\right] - \\ & \frac{\partial}{\partial q_1}\left(\frac{\dot{x}_i^2}{2} + \frac{\dot{y}_i^2}{2} + \frac{\dot{z}_i^2}{2}\right)\Bigg\}\delta_{q_1} \end{aligned} \tag{3.26}$$

but the kinetic energy K of the system S is given by

$$K = \frac{1}{2}\sum_{i=1}^{v} m^{P_i}(\dot{x}_i^2 + \dot{y}_i^2 + \dot{z}_i^2) \tag{3.27}$$

which substituted in equation 3.26 results

$$F_1\delta_{q_1} = \left[\frac{d}{dt}\left(\frac{\partial K}{\partial \dot{q}_1}\right) - \frac{\partial K}{\partial q_1}\right]\delta_{q_1} \tag{3.28}$$

from which it can be written

$$F_1 = \frac{d}{dt}\left(\frac{\partial K}{\partial \dot{q}_1}\right) - \frac{\partial K}{\partial q_1} \tag{3.29}$$

This is Lagrange's equation in its most well known and useful form. If in the above derivation it is made $\delta_{q_2} \neq 0$ while $\delta_{q_1} = \delta_{q_3} = \cdots = \delta_{q_n} = 0$, an equivalent equation to 3.29 would be obtained for δ_{q_2}. Proceeding in a similar way for all n generalised coordinates, leads to n independent Lagrangian equations corresponding to the n degrees of freedom of the system S. That is

$$\frac{d}{dt}\left(\frac{\partial K}{\partial \dot{q}_r}\right) - \frac{\partial K}{\partial q_r} = F_r \qquad (r = 1,\ldots,n) \tag{3.30}$$

If all the external forces acting on the system, are *conservative*, so that they can be derived from a potential energy function V, it can be said by definition that

$$F_r = -\frac{\partial V}{\partial q_r} \tag{3.31}$$

and for a conservative system Lagrange's equations take the form

$$\frac{d}{dt}\left(\frac{\partial K}{\partial \dot{q}_r}\right) - \frac{\partial K}{\partial q_r} + \frac{\partial V}{\partial q_r} = 0 \tag{3.32}$$

If systems for which the principle of conservation of energy is not valid are considered, various forms of dissipative functions are possible. When a system is subject to resisting forces which are directly proportional to the velocities of their point of application, it is possible to express Lagrange's equations with just an extra term. Let the dissipative function be $\mathcal{F}$, Lagrange's equation is then described by

$$\frac{d}{dt}\left(\frac{\partial K}{\partial \dot{q}_r}\right) - \frac{\partial K}{\partial q_r} + \frac{\partial V}{\partial q_r} + \frac{\partial \mathcal{F}}{\partial \dot{q}_r} = F_r \qquad (r = 1, \ldots, n) \tag{3.33}$$

For a nonholonomic system, with p degrees of freedom, and m nonholonomic constraint equations of the form described by equation 3.8, Lagrange's equations can be obtained if one considers the system S to be subject to additional external forces; namely forces which have to be exerted by the constraints in order to compel the system to fulfil the motion constraints. In such case a set of m additional quantities called *Lagrangian multipliers* are introduced and the Lagrange's equation becomes

$$\frac{d}{dt}\left(\frac{\partial K}{\partial \dot{q}_r}\right) - \frac{\partial K}{\partial q_r} + \frac{\partial V}{\partial q_r} + \frac{\partial \mathcal{F}}{\partial \dot{q}_r} = F_r + \sum_{i=1}^{m} W_{ir}\lambda_i \qquad (r = 1, \ldots, n) \tag{3.34}$$

where λ_i are the Lagrangian multipliers and the coefficients W_{ir} are given by equation 3.8. Equation 3.34 together with equation 3.8 are then sufficient to determine the $(n + m)$ unknown quantities, $q_1, \ldots, q_n, \lambda_1, \ldots, \lambda_m$.

Some of the advantages of Lagrangian approaches are that the equations of motion are derived in the same way for any set of coordinates, accelerations do not have to be determined, the required number of equations are automatically obtained and some difficulties with algebraic signs are avoided. It is worth noting that some of these advantages are simply basic advantages of energy methods and Lagrange's equations may be thought of as the most general form of the energy principle.

3.4.4 Kane's Method

Kane's method can be classified as an Eulerian approach. It relies on some quantities to be defined shortly afterwards which are called *partial velocities*. It uses Newton's and Euler's equations, equations 3.11 and 3.12 in their D'Alembert form, and the principle of orthogonality of Telegen [11] to project the force vectors in some particular directions, via the dot product operation. It is similar to Jourdain's principle of virtual power, which states that the virtual power associated with constraint forces and torques must vanish. However, it does not use virtual, but specific components of the real velocities expressions for the bodies.

By proceeding in this way, as it has been mentioned before, Kane obtained scalar relationships between the projected inertia forces and torques, which he calls *generalised inertia forces* and the projected external forces and torques, which are defined by him as *generalised active forces.* Kane's dynamical equation in this form are valid to describe the motion of a system S of particles P_i in *any* reference frame. It is worth noting that the generalised inertia forces and generalised active forces should be defined in a Newtonian reference frame. Ultimately, the justification for regarding a particular reference frame as Newtonian can come only from experiments [116].

3.4.4.1 Partial Velocities

For a simple holonomic system S possessing n degrees of freedom in a reference frame A, then $\boldsymbol{\omega}^B$, the angular velocity in A of a rigid body B belonging to S and $\boldsymbol{v}^{P_i}$, the velocity in A of a particle P_i belonging to S, can be uniquely expressed as

$$\boldsymbol{\omega}^B = \sum_{r=1}^{n} \boldsymbol{\omega}_r{}^B u_r + \boldsymbol{\omega}_t{}^B \tag{3.35}$$

$$\boldsymbol{v}^{P_i} = \sum_{r=1}^{n} \boldsymbol{v}_r{}^{P_i} u_r + \boldsymbol{v}_t{}^{P_i} \tag{3.36}$$

where $\boldsymbol{\omega}_r{}^B$, $\boldsymbol{v}_r{}^{P_i}$ $(r = 1, \ldots, n)$, $\boldsymbol{\omega}_t{}^B$ and $\boldsymbol{v}_t{}^{P_i}$ are functions of $q_1, \ldots, q_n$ and t. The vector $\boldsymbol{\omega}_r{}^B$ is called the r^{th} *holonomic partial angular velocity* of B in A and $\boldsymbol{v}_r{}^{P_i}$ is referred to as the r^{th} *holonomic partial velocity* of P_i in A. If the system is holonomic scleronomic, $\boldsymbol{\omega}_t{}^B$ and $\boldsymbol{v}_t{}^{P_i}$ are zero, otherwise it is holonomic rheonomic.

If the system S is nonholonomic, the vectors $\boldsymbol{\omega}^B$ and $\boldsymbol{v}^{P_i}$ can also be expressed uniquely as

$$\boldsymbol{\omega}^B = \sum_{r=1}^{p} \tilde{\boldsymbol{\omega}}_r{}^B u_r + \tilde{\boldsymbol{\omega}}_t{}^B \tag{3.37}$$

$$\boldsymbol{v}^{P_i} = \sum_{r=1}^{p} \tilde{\boldsymbol{v}}_r{}^{P_i} u_r + \tilde{\boldsymbol{v}}_t{}^{P_i} \tag{3.38}$$

where the vectors $\tilde{\boldsymbol{\omega}}_r{}^B$, $\tilde{\boldsymbol{v}}_r{}^{P_i}$ $(r = 1, \ldots, p)$, $\tilde{\boldsymbol{\omega}}_t{}^B$ and $\tilde{\boldsymbol{v}}_t{}^{P_i}$ are functions of $q_1, \ldots, q_n$ and t. The vector $\tilde{\boldsymbol{\omega}}_r{}^B$ is called the r^{th} *nonholonomic partial angular velocity* of B in A and $\tilde{\boldsymbol{v}}_r{}^{P_i}$ is the r^{th} *nonholonomic partial velocity* of P_i in A. When speaking of partial angular velocities and/or partial velocities, one can generally omit the adjectives holonomic and nonholonomic, without any loss of clarity, but the tilde notation should be used to distinguish between the two. It is possible to establish a relationship between the holonomic and nonholonomic partial velocities using the motion constraint relations, equation 3.6 [116]. They are given for the nonholonomic partial angular velocities by

$$\tilde{\boldsymbol{\omega}}_r{}^B = \boldsymbol{\omega}_r{}^B + \sum_{i=p+1}^{n} \boldsymbol{\omega}_i^B Y_{ir} \qquad (r = 1, \ldots, p) \tag{3.39}$$

and

$$\tilde{\boldsymbol{\omega}}_t{}^B = \boldsymbol{\omega}_t{}^B + \sum_{i=p+1}^{n} \boldsymbol{\omega}_i^B Z_i \tag{3.40}$$

and similarly for $\tilde{\boldsymbol{v}}_r{}^{P_i}$ and $\tilde{\boldsymbol{v}}_t{}^{P_i}$ if they are expressed as

$$\tilde{\boldsymbol{v}}_r{}^{P_i} = \boldsymbol{v}_r{}^{P_i} + \sum_{i=p+1}^{n} \boldsymbol{v}_i^{P_i} Y_{ir} \qquad (r = 1, \ldots, p) \tag{3.41}$$

and

$$\tilde{\boldsymbol{v}}_t{}^{P_i} = \boldsymbol{v}_t{}^{P_i} + \sum_{i=p+1}^{n} \boldsymbol{v}_s^{P_i} Z_i \tag{3.42}$$

and therefore, one can use either notation in order to explain Kane's Method. One can say that in the case of a holonomic system, no motion constraints exist, therefore $m = 0$, $p = n$, $\tilde{\boldsymbol{\omega}}_r = \boldsymbol{\omega}_r$ and $\tilde{\boldsymbol{v}}_r = \boldsymbol{v}_r$.

Kane also keeps the terms $\tilde{\boldsymbol{v}}_t{}^{P_i}$ and $\tilde{\boldsymbol{\omega}}_t{}^B$ throughout his derivations. Sayers [241] states that that for a nonholonomic system, they can be accounted for in the Z_i terms of equation 3.6, and therefore can be ignored in further derivations.

The partial velocities can be seen to indicate the directions of the components of the body's angular and linear velocities. Given a body's angular velocity, $\boldsymbol{\omega}^B$, the r^{th} angular partial velocities of body B belonging to S can be expressed as

$$\boldsymbol{\omega}_r{}^B = \frac{\partial \boldsymbol{\omega}^B}{\partial u_r} \qquad (r = 1, \ldots, n) \tag{3.43}$$

and the r^{th} linear partial velocities of a particle P_i belonging to S when its linear velocity, $\boldsymbol{v}^{P_i}$, is given, is described by

$$\boldsymbol{v}_r{}^{P_i} = \frac{\partial \boldsymbol{v}^{P_i}}{\partial u_r} \qquad (r = 1, \ldots, n) \tag{3.44}$$

Since velocities are vectorial quantities and generalised speeds are scalar quantities, it can be seen that partial velocities are always vectorial quantities.

A physical interpretation of the partial velocities is also given by Sayers [241]. He states that they reflect the fact that forces and torques can only perform work if there is movement, and the partial velocities are the directions in which these movements take place. Later this fact will play an important role in the determination of the states of a MBS in formalisms which implement Kane's Method.

3.4.4.2 Kane's Dynamical Equations

In the present derivation of Kane's equations, the system S is assumed to be holonomic scleronomic for the sake of simplicity. This assumption does not affect their understanding as far as the explanation intended in this work is concerned; also they have been derived elsewhere for a generic nonholonomic system [116, 241, 231].

Kane's dynamical equations are obtained by writing Newton's and Euler's equations, 3.11 and 3.12, in their D'Alembert form, dot multiplying them by the partial velocities previously defined and adding them up, that is

$$\sum_{B}^{bodies} [(\boldsymbol{R}^B - m^B \boldsymbol{a}^{B*}) \cdot \boldsymbol{v}_r{}^{B*} +$$

$$(\boldsymbol{T}^B - \boldsymbol{I}^{B*} \cdot \boldsymbol{\alpha}^B - \boldsymbol{\omega}^B \times \boldsymbol{I}^{B*} \cdot \boldsymbol{\omega}^B) \cdot \boldsymbol{\omega}_r{}^B] = 0 \qquad (r = 1, \ldots, n) \tag{3.45}$$

where $\sum_B^{bodies}$ indicates summing over all the bodies in the system, $\boldsymbol{R}^B$ and $\boldsymbol{T}^B$ are the resultant of all the forces and torques for each body, as previously mentioned and $\boldsymbol{v}_r{}^{B*}$ is a special case of $\boldsymbol{v}_r{}^{P_i}$, by letting P_i be B^*.

Grouping the external forces and torques, and the inertial terms, results

$$\sum_{B}^{bodies} [(\boldsymbol{R}^B \cdot \boldsymbol{v}_r{}^{B*} + \boldsymbol{T}^B \cdot \boldsymbol{\omega}_r{}^B) - m^B \boldsymbol{a}^{B*} \cdot \boldsymbol{v}_r{}^{B*}$$

$$-(\boldsymbol{I}^{B*} \cdot \boldsymbol{\alpha}^B + \boldsymbol{\omega}^B \times \boldsymbol{I}^{B*} \cdot \boldsymbol{\omega}^B) \cdot \boldsymbol{\omega}_r{}^B] = 0 \qquad (r = 1, \ldots, n) \tag{3.46}$$

Next, *generalised active forces* are introduced as

$$F_r = \sum_{B}^{bodies} (\boldsymbol{R}^B \cdot \boldsymbol{v}_r{}^{B*} + \boldsymbol{T}^B \cdot \boldsymbol{\omega}_r{}^B) \qquad (r = 1, \dots, n) \tag{3.47}$$

and *generalised inertia forces* as

$$F_r^* = \sum_{B}^{bodies} [(-m^B \boldsymbol{a}^{B*} \cdot \boldsymbol{v}_r{}^{B*} - (\boldsymbol{I}^{B*} \cdot \boldsymbol{\alpha}^B + \boldsymbol{\omega}^B \times \boldsymbol{I}^{B*} \cdot \boldsymbol{\omega}^B) \cdot \boldsymbol{\omega}_r{}^B] \qquad (r = 1, \dots, n) \tag{3.48}$$

Then, *Kane's Dynamical Equations* are written as

$$F_r + F_r^* = 0 \qquad (r = 1, \dots, n) \tag{3.49}$$

and they describe the motion of a system S in *any* reference frame, when the forces F_r and F_r^* have been derived for S in an inertial reference frame [116].

It is interesting to observe that some forces that contribute to $\boldsymbol{R}^B$ and torques that contribute to $\boldsymbol{T}^B$ make no contributions to F_r. Actually, this is the main motivation for introducing generalised active forces. For instance, the total contribution to F_r of all reaction forces and torques between interacting bodies should vanish because they appear in pairs and with opposite signs in equation 3.47. In this case, the resultant set of equations for the MBS is minimal.

When Kane's Equation, 3.49, is cast in the form of equation 3.13, using equations 3.35 and 3.36, the coefficient of the mass matrix for the i^{th} row and the j^{th} column is given by

$$m_{ij} = \sum_{B}^{bodies} (\boldsymbol{\omega}_j^B \cdot \boldsymbol{I}^{B*} \cdot \boldsymbol{\omega}_i^B + m^B \boldsymbol{v}_j^{B*} \cdot \boldsymbol{v}_i^{B*}) \tag{3.50}$$

and the i^{th} element of the force vector is obtained by subtracting equations 3.50 from 3.46, to result in

$$f_i = \sum_{B}^{bodies} [(\boldsymbol{R}^B - m^B \boldsymbol{a}_{rem}^{B*}) \cdot \boldsymbol{v}_i^{B*} +$$

$$(\boldsymbol{T}^B - \boldsymbol{I}^{B*} \cdot \boldsymbol{\alpha}_{rem}^B - \boldsymbol{\omega}^B \times \boldsymbol{I}^{B*} \cdot \boldsymbol{\omega}^B) \cdot \boldsymbol{\omega}_i^B] \tag{3.51}$$

where $\boldsymbol{a}_{rem}^{B*}$ and $\boldsymbol{\alpha}_{rem}^{B}$ are called central acceleration remainder and angular acceleration remainder, respectively [231]. For a holonomic, scleronomic system they are given by

$$\boldsymbol{a}_{rem}^{B*} = \sum_{r=1}^{n} u_r \frac{d\boldsymbol{v}_r{}^{B*}}{dt} \tag{3.52}$$

and

$$\boldsymbol{\alpha}_{rem}^{B} = \sum_{r=1}^{n} u_r \frac{d\boldsymbol{\omega}_r{}^{B}}{dt} \tag{3.53}$$

Expressions can be obtained for the corresponding nonholonomic terms of 3.52 and 3.53 using equations 3.6 and proceeding in a similar fashion to that of the derivation of equations 3.39 and 3.41. Detailed derivations of these equations, as well as their proofs are presented in the literature [116, 117, 231, 241].

3.5 Computer Implementations

Regarding computer implementation, MBS programs can be classified as numerical, symbolic or a mixture of both. The term symbolic is a little misleading in the sense that the resultant code is not symbolic, but simply a FORTRAN, C, Pascal or ADSIM subroutine being the most common forms. The symbolic term applies to the generation phase of the equations of motion and it is usually transparent to the MBS program user.

Numerical programs can be either specialised code, which means it has been developed for a specific simulation or for a class of models. Some of the hand generated code still widely used in the industry and at universities and that incorporates some kind of flexibility for small model changes and can be used by various users, belong to this group. Normally these models are small and fast to run due to its specificity.

On the other hand, generalised codes are those developed around the most general form of the equations of motion because they must be capable of simulating systems with widely different topologies and must allow for bodies of arbitrary mass distribution, connected at arbitrary points by several types of joints. Typical codes of this type are normally large and slow to run, comparatively to the previous one.

Eulerian approaches were initially applied to spacecrafts problems in which the characteristics were 3d motion and systems with a tree topology without nonholonomic constraints. With the aid of Graph Theory, early numerical implementations of Newton-Euler equations were performed by Fleischer in the program MLTBDY, based on the Hooker/Margoulis method and by Roberson/Wittenburg with the program MULTIBODY. Later on, Hooker worked on a different way of describing how to isolate a single joint constraint torque in one dynamical equation. This method which seems to have been devised by Velman for a particular case of four bodies was given the name of 'Nested body method'. An implementation of such method is the program N.BOD2, developed by Frisch.

Formalisms based in Jourdain's Principle of virtual power were applied to systems with nonholonomic constraints by Kreuzer/Schiehlen in the program NEWEUL.

Kane's method, that can be viewed as a variation of Jourdain's principle in which the virtual velocities are substituted by some components of the actual velocities, as discussed beforehand, has proved to generate the shortest possible form of the equations of motion and therefore the code more likely to be more efficient. Implementations of algorithms based on this methodology include the programs SD/FAST [254], AUTOSIM [241], SYMBA [206], etc.

Other methods include the Vector Network Method developed by Andrews which uses concepts from electric network theory applied to mechanical systems and resulted in the program VECNET [11] and the Transformation Operator Method devised by Jerkovsky [253]. For vehicle dynamics, programs based on Eulerian approaches include SD/FAST [254], AUTOSIM [241], NUSTAR [129], SIMPACK [129], etc.

Authors who used Lagrangian formalisms were often from the mechanisms and machines community due to its ability to handle closed loops. Initial applications were mainly on systems with planar motion and the fact that Lagrangian multipliers gave the reactions forces between contiguous bodies also influenced such choice. Graph Theory was widely used to described system topology.

Paul developed the program DYMAC which was applied to planar motion of systems having general topology, and worked with a redundant set of coordinates. The solution, which includes Lagrange multipliers, is called the method of excess differential equations [253].

The program DAMN was developed in the University of Michigan and it is based on a work by Chace [253]. Later it was expanded by Smith and renamed

DRAM. It is also a program for systems with arbitrary structure, which undergo planar motion; however, it works with a minimal set of generalised coordinates.

In order to model systems of arbitrary configuration with three-dimensional motion, Orlandea developed the program ADAMS. The philosophy of programming is based on the tailoring of variables and equations to the numerical methods used. For this purpose redundant set of equations was adopted, with 15 first order equations for each body, corresponding to the kinematical relations, dynamical equations and the energy equations. The resultant system matrices are made very sparse by these choices. A Newton-Rapson algorithm is used to solve the algebraic part, while Gear's method is used to integrate the stiff differential equations [232].

Of the many programs developed in DFVLR for vehicle applications, MEDYNA illustrates the application of a Lagrangian approach to 3d motion using a kinematically moving reference frame, with the resulting equations of motion derived assuming small deviations from these frames, therefore a linear set of equations. MEDYNA was the result of a joint venture of the developers of LINDA, LINSYS and FADYNA [243]. For vehicle dynamics Lagrangian approaches are used by programs such as ADAMS [209, 143], DADS [91], MEDYNA [57], MADYMO [129], etc.

Programs which use symbol manipulation techniques have appeared as the answer to the question of how to generate efficient code which is specific to the problem at hand, but which can cater for a large class of systems, therefore avoiding the inefficiency of executing a fully general simulation for all system configurations [230, 247, 237, 298]. In the early days of symbolic programs, general purpose symbol manipulation programs such as MACSYMA and REDUCE were used. However these programs contain much more functions than is needed by the multibody formalism for generating of the equations of motion. Also, these programs require special techniques, like the creation of temporary variables and dynamic memory allocation, to generate efficient code and avoid intermediary expression swell. In order to overcome these problems specific symbol manipulation methods have been devised. An illustration of the role of MBS program generation by symbolic manipulation and a comparison with other methods is given in figure 3.4 [230]. Engineers work from a specification of the systems parameters to develop the explicit equations of motion. These equations are then coded and incorporated into a larger simulation framework. For a general purpose code, the specific system parameters are provided to the MBS code at runtime and are used in the fully general multibody model to produce the dynamic response of the system. One serious problem of general purpose multibody code is abysmal runtime performance, because it is unable to exploit the simplifications available in most physical systems to reduce the complexity of the equations. Symbolic codes, on the other hand, instead of being a replacement for the generated equations of motion in the development phase, they are a replacement for the *engineers*, in the sense that they are capable of taking the description of a system and produce specific code employing all the information about the system to produce the simplest possible set of equations of motion.

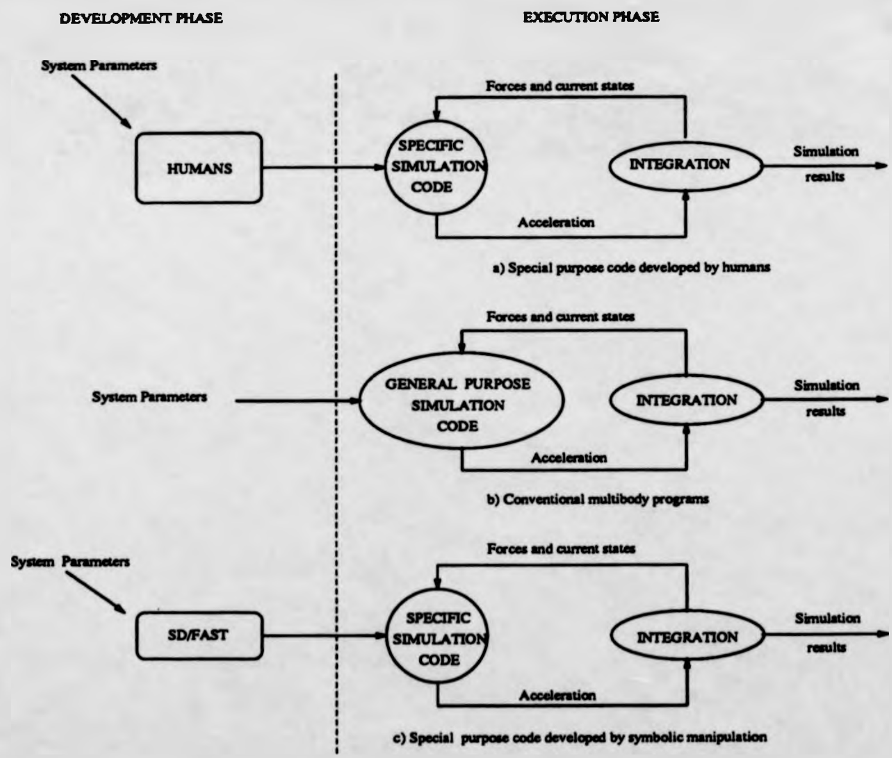

Figure 3.4: Three ways of generating a MBS simulation [254]

In SD/FAST, the computer code used in this work, a distinction is drawn between tree joints and loop joints. All the constraints associated with the tree joints are eliminated. That is, for the tree system, equations only appear for the degrees of freedom *granted* by the joint. No equations appear for the constraints *imposed* by the joint. Therefore, tree joints do not produce explicit constraints. This fact is due to the characteristics of Kane's Method in the use of partial velocities. The degrees of freedom impaired by the joint have zero partial velocity (no motion in those directions) and therefore the derivatives of possible generalised speeds do not appear. Loop joints on the other hand, are implemented by equations representing the constraints *imposed* by the joints, while none are needed for the degrees of freedom *granted* by the joint. That is so because holonomic constraints like loop joints are implemented via differentiation of the constraints equations, in order to have a set of algebraic-differential equations transformed into a set of ordinary differential equations only. This is a common approach in many computer implementations. However, the new equations will be subject to 'drift' during motion simulations caused by the imperfect nature of the numerical integration. This drift will cause the original set of algebraic constraints to be violated as the integration proceeds. In this case the numerical drift must be stabilised to prevent the constraints violations from becoming arbitrarily large. In SD/FAST, the Baumgarte method of constraint stabilisation is used. This involves feeding back a multiple of the velocity and position constraints violations into the calculations for the accelerations, so that subsequent integration tends to reduce these errors.

Other aspects of the computer implementations which should be considered are related to the user. As far as the user is concerned a MBS program should be easy to learn and easy to use while trying to prevent user's errors and helping her/him in the diagnosis of such errors. Another aspect which MBS programs should be concerned with is the danger of producing a functioning model which simulates the wrong system. For this matter graphic pre-processing and system's roadmaps are possible helps. Other forms of user-friendliness include free-format input file which is key-word oriented and/or graphical input.

If the program is not a stand alone simulation package, then it must be capable of efficiently integrating into a simulation/analysis/design environment. Even if this is not the case, it is still important to integrate in a more comprehensive CAD/CAM environment. In this respect, pre and post processing of data assumes an important role.

3.6 Conclusions

An overview of MBS modelling techniques has been presented with regard to its historical development. A discussion of the formalisms upon which the modelling is based, as well as the computer philosophies of implementing such formalisms has been carried out with aspects of program interfacing also being considered.

Regarding MBS programs currently available, it can be concluded that those which have been developed within a certain area tend to have advantages over others, although they all refer to themselves as general purpose and capable of modelling any system which can be represented within the classes of systems they claim to cater for. This is so because of the familiarity of those involved in the way of expressing quantities and terms in the model, as well as a better knowledge of the underlying assumptions for modelling those types of systems. Another possible bonus is normally the existence of libraries which might have been developed for their own models.

One important aspect to be considered when evaluating a MBS program it is its integrability in a CAD environment, since it is very difficult that the program itself will contain all necessary tools or facilities for the various task which might be undertaken in any study.

Actually, programs which concentrate in performing well the task of generating the equations of motion are more likely to result in a better code and the use of other state-of-the-art packages for the remaining parts of the study, such as the integration of the equations of motion and the calculation of other non-MBS quantities of the simulation, might allow for very powerful and comprehensive analysis/design studies to be performed.

Regarding trends in MBS programs development it can be stated that interface to flexible parts of the mechanical system via FEM, and the development of codes which are AI based to use more extensively symbol manipulation techniques in order to generate fast code for hardware-in-the-loop and human-in-the-loop based on Kane's Method and variations of it, are possibilities to be considered.

For the purpose of this work, it was evaluated that programs which implemented Kane's Method and used symbolic manipulation programs were highly recommended for the the derivation of the equations of motion of the mechanical parts of the automotive vehicle. The fact that it could be inserted into a more general simulation environment also played a decisive role in the choice of the program to be made. For these reasons the choice of SD/FAST seemed quite adequate. An important observation to be made is that initially the version of SD/FAST which was available was limited in the facilities it provided for the simulation of mechanical systems with the characteristics of automotive vehicles. The work developed by the author of this thesis pioneered the use of SD/FAST to automotive applications and at the time a number of facilities such as vector base transformations had to be implemented in the general simulation environment. Later, with the release of the present version, of which the author and other researchers at Warwick were part of the β-test phase, a number of these facilities in the form of subroutines have been added to the program and the initial program developed decreased significantly in size and complexity.

Chapter 4

Modelling of an Automotive Vehicle for Motion Control Studies

4.1 Introduction

The modelling to be performed in this chapter is motivated by the recognition of the need for a systematic approach to vehicle simulation which is capable of embracing all aspects of vehicle motion in order to allow the study of automotive vehicle control, which has been introduced under the motion management concept, whereby the automotive vehicle is viewed as a total system made up of a number of interacting subsystems.

Apart from aerodynamic resistances and gravitational effects, all external forces acting on a vehicle are applied through the wheels. Consequently, motion management will entail the application of control action at the wheels, and will be based on a combination of propulsion, steering and suspension control. Therefore it is necessary to develop vehicle models which are capable of representing all these actions simultaneously.

As it has been discussed in the previous chapter, the application of a MBS modelling approach to vehicle dynamics simulation involves two essential considerations, viz. the way in which the MBS model is formulated and the particular MBS program which is to be employed. The MBS model formulation determines how the equations of motion are derived. The MBS program determines the ultimate form of the dynamic equations, as implemented within the computer simulation program. Approaches to MBS modelling based on a combination of Kane's Method [116] and symbolic computation has been shown [243, 237, 230, 257] to generate the most efficient (run time) simulation code.

This chapter describes the application of a MBS technique to the modelling of an automotive vehicle chassis system for motion control studies. MBS simulation models of chassis systems usually include a complete representation of the suspension [209, 143] in which individual suspension elements are detailed. The modelling of this work is based on an alternative approach [41], in which suspension geometry effects are incorporated in a way which does not involve a detailed representation of the suspension system. The basis of the approach is an empirical black-box representation of the suspension kinematics derived from readily available experimental data describing the suspension trajectory at each wheel hub. From this data, an equivalent swing axle or trailing arm model is derived using techniques from differential geometry.

Other important kinematical quantities which are related to the suspension geometry and are considered in an approximate fashion in this work are camber and toe angles associated with suspension movements. If they were taken from a wheel rigidly attached to the swing axle, they would be overestimated as the suspension travelled. In this case, it was chosen to adopt their values at the vehicle's static position throughout the simulation. As our experience has shown [129] specially near the end of suspension stroke, these quantities can vary quite significantly. However, from a control point of view and for the initial purposes of this work, the present representation has proved adequate. An approach to correct this limitation of the swing axle for the toe-in angle has been recommended by prof. Hiller, and it has been implemented in a Ph.D. dissertation published in German [99]. An alternative approach is presented in section 4.7 and which have been partially implemented in a more advanced version of the model described in detail in this chapter.

The simulation model is constructed using the MBS package SD/FAST, and the ACSL continuous simulation language. SD/FAST combines Kane's formulation for a MBS model with an equation generator incorporating a transparent symbolic computation facility, while ACSL provides the simulation environment necessary to solve these equations.

4.2 Schematic Description of Vehicle Model

A schematic view of the nonlinear chassis model is presented in figure 4.1. The model represents the kinematics of the sprung mass and the four wheels and incorporates the effects of geometrical constraints associated with the suspension. At each wheel, the suspension geometry effects are represented as a swing axle [62] connecting the wheel hub H to the sprung mass via a single degree of freedom rotational joint (pin joint) at the point. The pin position and the arm length of the swing axle are derived empirically from data describing the suspension trajectory at each wheel hub. The suspension forces at each wheel are represented as a spring and damper which are active in a vertical direction only. In addition, the front suspension forces are supplemented by a further force to represent the effects of a single wheel bump anti-roll torsion bar. An equivalent lumped mass, concentrated at the point H, represents the combined wheel and suspension inertia effects for each wheel.

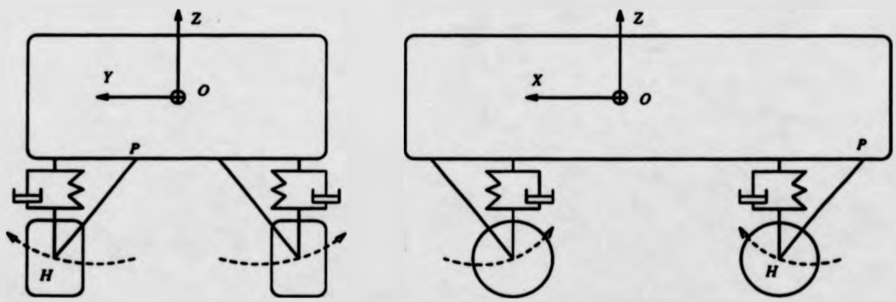

Figure 4.1: Nonlinear chassis model

Vertical tyre forces are represented at each wheel as a linear stiffness term. Longitudinal and lateral tyre forces at each wheel are also represented linearly, where it is assumed that the wheel does not spin or lock. The model provides a ten degree of freedom representation of the chassis, with six degrees of freedom resulting from the sprung mass and one degree of freedom from each swing axle. This results in a nonlinear twenty state model, in which the states correspond to ten generalised coordinates and ten generalised velocities.

This swing axle model provides the simplest possible representation of the suspension system which is consistent with a nonlinear chassis model intended for motion control studies. A less sophisticated approach would involve a representation of the chassis as a sprung mass with a spring and damper connected at each of its corners. This model is widely used for linear ride control studies. However, such a model does not represent steering or propulsion action at the wheels and, therefore, cannot be used to assess the dynamic effects of propulsion and steering manoeuvres on suspension control. More importantly, the model cannot be used for studies involving combinations of suspension, propulsion and steering control. A more sophisticated approach would involve a complex model involving a detailed representation of the suspension links and bushes. Complex models are widely used in design studies requiring the calculation of internal suspension loads and displacements, and in passive ride and handling investigations. However, complex models are not well suited to vehicle motion control studies since they give rise to simulation models with very large state dimensions.

The chassis model represents the first stage in the development of a vehicle dynamics model for use in motion control studies. The present model is limited, particularly in its representation of the tyre and wheel. However, the model can easily be extended to cater for complete motion control studies through inclusion of a more sophisticated representation of the tyre and wheel. The chassis model developed in this chapter is based on data corresponding to a luxury European saloon car. This data was selected for illustration purposes only. In particular, it should be noted that the chassis modelling approach presented here is not restricted to luxury saloon cars but is generic and applicable to a wide variety of automotive vehicle types.

4.3 Empirical Representation of Suspension Geometry Effects

The suspension geometry effects for each wheel are represented, in a black-box manner, as a swing axle derived empirically from data defining the wheelbase/bump and track/bump characteristics. This data, which describes the fore-aft and lateral wheel displacements as a function of vertical displacement, is presented as the stars in figure 4.2 for the front wheels, whilst the corresponding data for the rear wheels is presented in figure 4.3. The variation in fore-aft and lateral wheel displacements described in these figures appears small. However, the effect of these variations on the vehicle pitch and roll characteristics is highly significant, particularly during vehicle manoeuvres associated with large acceleration levels [19].

The wheelbase/bump and track/bump data defines relations in the form

$$x' = w(z') \tag{4.1}$$

for the wheelbase/bump characteristics and

$$y' = t(z') \tag{4.2}$$

for the track/bump characteristics, where, x',y' and z' represent the fore-aft, lateral and vertical wheel displacements, respectively, relative to the standard static condition. Note that, for the left side of the vehicle, the sense of $\boldsymbol{T}$ and $\boldsymbol{N}$ is defined by the right-handed axes set in which a positive value of z' corresponds to an upwards displacement, as described in figure 4.6. For the right side of the vehicle, the lateral displacement is the mirror image of the left side.

Relations 4.1 and 4.2 define a parametric equation, in 3-dimensional Euclidean space, E^3, in the form

$$\boldsymbol{r}(\xi) = (w(\xi), t(\xi), \xi) \tag{4.3}$$

where the position vector $\boldsymbol{r}(\xi)$ represents the trajectory, or curve, traced out by a wheel hub. Associated with this curve [208] is the Frenet frame field, which is specified by the unit tangent vector

$$\boldsymbol{T}(\xi) = \frac{\frac{d\boldsymbol{r}(\xi)}{d\xi}}{||\frac{d\boldsymbol{r}(\xi)}{d\xi}||} \tag{4.4}$$

the unit binormal vector

$$\boldsymbol{B}(\xi) = \frac{\frac{d\boldsymbol{r}(\xi)}{d\xi} \times \frac{d^2\boldsymbol{r}(\xi)}{d\xi^2}}{||\frac{d\boldsymbol{r}(\xi)}{d\xi} \times \frac{d^2\boldsymbol{r}(\xi)}{d\xi^2}||} \tag{4.5}$$

and the unit normal vector

$$\boldsymbol{N}(\xi) = \boldsymbol{B}(\xi) \times \boldsymbol{T}(\xi) \tag{4.6}$$

Also, the curvature at each point on the curve is given by

$$\kappa(\xi) = \frac{\|\frac{d\boldsymbol{r}(\xi)}{d\xi} \times \frac{d^2\boldsymbol{r}(\xi)}{d\xi^2}\|}{\|\frac{d\boldsymbol{r}(\xi)}{d\xi}\|^3} \tag{4.7}$$

and the torsion by

$$\tau(\xi) = \frac{\left(\frac{d\boldsymbol{r}(\xi)}{d\xi} \times \frac{d^2\boldsymbol{r}(\xi)}{d\xi^2}\right) \cdot \frac{d^3\boldsymbol{r}(\xi)}{d\xi^3}}{\|\frac{d\boldsymbol{r}(\xi)}{d\xi} \times \frac{d^2\boldsymbol{r}(\xi)}{d\xi^2}\|^2} \tag{4.8}$$

The Frenet apparatus $(\boldsymbol{T}(\xi), \boldsymbol{N}(\xi), \boldsymbol{B}(\xi), \kappa(\xi), \tau(\xi))$ provides a complete description of the curve $\boldsymbol{r}(\xi)$. For each value of ξ, the motion of a point H on this curve has instantaneous interpretation as an orbit, with radius (of curvature) $\kappa^{-1}(\xi)$, in the plane containing the vectors $\boldsymbol{T}(\xi)$ and $\boldsymbol{N}(\xi)$. This orbit is centred at the point P with position vector $\kappa^{-1}(\xi)\boldsymbol{N}(\xi)$ relative to H and describes the rotation of H about a vector parallel to $\boldsymbol{B}(\xi)$ and acting through P. The motion of the point H also has instantaneous interpretation as an orbit, with (torsion) radius $\tau^{-1}(\xi)$, in the plane containing the vectors $\boldsymbol{N}(\xi)$ and $\boldsymbol{B}(\xi)$ rotating about a vector parallel to $\boldsymbol{T}(\xi)$ and acting through the point with position vector $\tau^{-1}(\xi)\boldsymbol{N}(\xi)$ relative to H.

Polynomial representations of the functions $w(\xi)$ and $t(\xi)$, defining relations 4.1 and 4.2, were obtained for the front and rear wheels using least squares curve fitting techniques. For the front wheels, the wheelbase characteristics were represented as

$$w_f(\xi) = 0.283\xi^2 + 0.004\xi \tag{4.9}$$

and the track characteristics as

$$t_f(\xi) = -1.061\xi^2 - 0.129\xi \tag{4.10}$$

Similarly, for the rear wheels, the wheelbase characteristics were represented as

$$w_r(\xi) = 0.114\xi \tag{4.11}$$

and the track characteristics as

$$t_r(\xi) = -0.824\xi^2 - 0.225\xi \tag{4.12}$$

In all cases, the polynomial of lowest order which adequately fitted the data was chosen. A comparison of these polynomial representations and the corresponding wheelbase and track characteristics data is presented in figures 4.2 and 4.3.

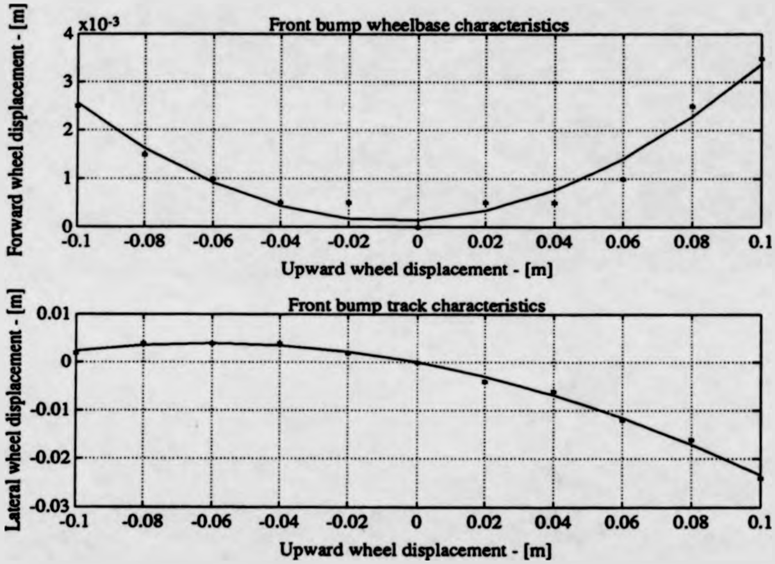

Figure 4.2: Front wheelbase and track characteristics

The polynomial representations 4.9 to 4.12 were used to define parametric equations

$$\boldsymbol{r}_f(\xi) = (w_f(\xi), t_f(\xi), \xi) \tag{4.13}$$

for the front wheel hub and

$$\boldsymbol{r}_r(\xi) = (w_r(\xi), t_r(\xi), \xi) \tag{4.14}$$

for the rear wheel hub.

The Frenet apparatus $(\boldsymbol{T}_f, \boldsymbol{N}_f, \boldsymbol{B}_f, \kappa_f, \tau_f)$ and $(\boldsymbol{T}_r, \boldsymbol{N}_r, \boldsymbol{B}_r, \kappa_r, \tau_r)$ corresponding to $\boldsymbol{r}_f$and $\boldsymbol{r}_r$, respectively, were calculated from the parametric equations

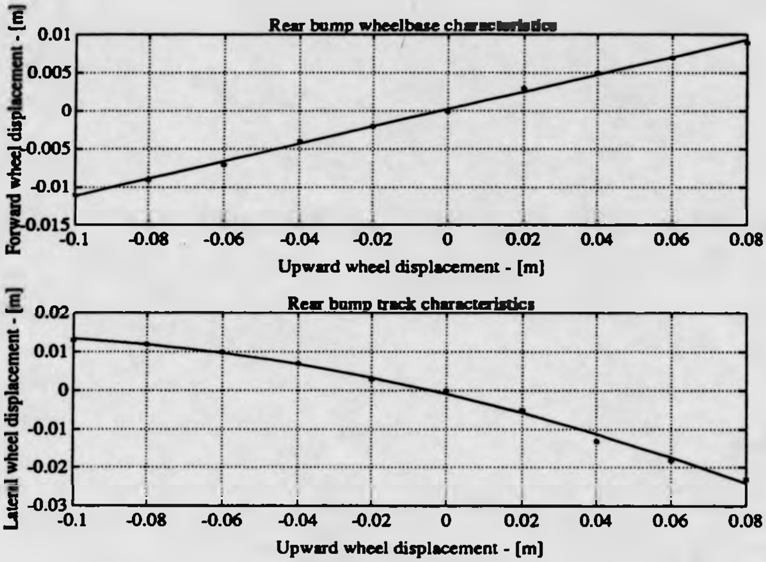

Figure 4.3: Rear wheelbase and track characteristics

4.13 and 4.14 using the formulae defined in expressions 4.4 to 4.8.

For the front wheel hub trajectory, the unit tangent vector field was obtained as

$$\boldsymbol{T}_f(\xi) = \frac{1}{d_1(\xi)}(0.004 + 0.566\xi, -0.129 - 2.122\xi, 1) \tag{4.15}$$

the unit binormal vector field as

$$\boldsymbol{B}_f(\xi) = (0.966, 0.258, -0.029) \tag{4.16}$$

and the unit normal vector field as

$$\boldsymbol{N}_f(\xi) = \frac{1}{d_1(\xi)}(0.261 + 0.062\xi, -0.966 + 0.017\xi, -0.126 - 2.195\xi) \tag{4.17}$$

where

$$d_1(\xi) = \sqrt{1.017 + 0.553\xi + 4.823\xi^2} \tag{4.18}$$

The corresponding curvature and torsion scalar fields were obtained as

$$\kappa_f(\xi) = \frac{2.197}{[d_1(\xi)]^3} \tag{4.19}$$

and

$$\tau_f(\xi) \equiv 0 \tag{4.20}$$

respectively.

Similarly, for the rear wheel hub trajectory, the unit tangent vector field was obtained as

$$\boldsymbol{T}_r(\xi) = \frac{1}{d_2(\xi)}(0.114, -0.225 - 1.648\xi, 1) \tag{4.21}$$

the unit binormal field as

$$\boldsymbol{B}_r(\xi) = (0.993, 0.0, 0.113) \tag{4.22}$$

and the unit normal vector field as

$$\boldsymbol{N}_r(\xi) = \frac{1}{d_2(\xi)}(-0.025 - 0.187\xi, -1.006, -0.223 - 1.637\xi) \tag{4.23}$$

where

$$d_2(\xi) = \sqrt{1.064 + 0.742\xi + 2.716\xi^2} \tag{4.24}$$

The corresponding curvature and torsion scalar fields were obtained as

$$\kappa_r(\xi) = \frac{1.659}{[d_2(\xi)]^3} \tag{4.25}$$

and

$$\tau_r(\xi) \equiv 0 \tag{4.26}$$

respectively.

For both front and rear wheel hub trajectories, the unit binormal vector fields are constant and the torsion scalar fields are zero. This implies that the wheel hub trajectories lie in a fixed (osculating) plane defined by their respective unit tangent and unit normal vector fields, as described in figure 4.6. The unit tangent and normal vector fields and curvature scalar fields vary with the parameter ξ for both front and rear wheel hub trajectories.

The variation in radii of curvature κ_f^{-1} and κ_r^{-1} is presented in figure 4.4. κ_f^{-1} varies gently between a maximum value of 0.539, a minimum value of 0.456, and takes the value 0.467 at the static condition $\xi \equiv 0$. At this static condition, κ_r^{-1} equals 0.661 and varies moderately between a maximum value of 0.758 and a

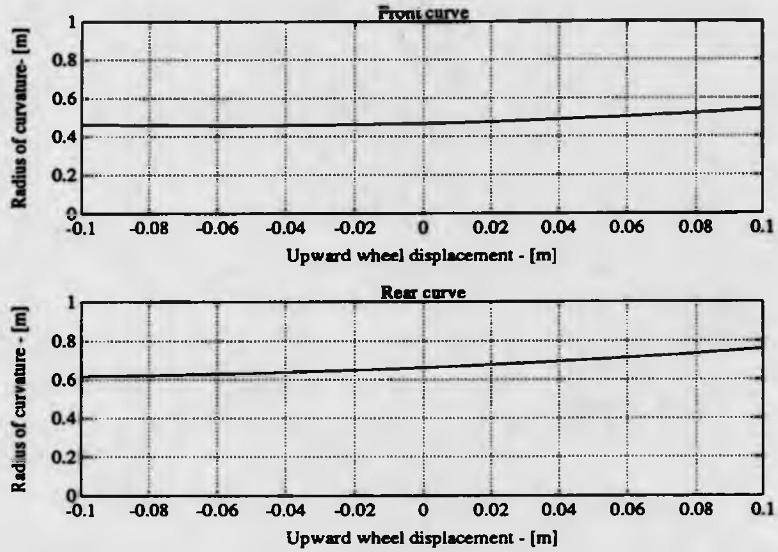

Figure 4.4: Front and rear radius of curvature

minimum value of 0.618.

A measure of the variation in the alignment of an arbitrary unit vector field $\boldsymbol{a}(\xi)$ is provided by the inner product

$$\boldsymbol{a}(\xi) \cdot \boldsymbol{a}(0) \tag{4.27}$$

which equals unity at values of ξ for which $\boldsymbol{a}(\xi)$ and $\boldsymbol{a}(0)$ coincide and equals zero for values of ξ where $\boldsymbol{a}(\xi)$ and $\boldsymbol{a}(0)$ are orthogonal.

Figure 4.5 present the variation in this alignment measure for the front tangent and normal vector fields, and rear tangent and normal vector fields, respectively. These figures indicate that for both the front and rear wheel hub trajectories the unit tangent and normal vector fields vary only slightly and, therefore, can be approximated by their corresponding values at the static condition $\xi = 0$.

For both the front and rear wheel hub trajectories, the equivalent swing axle representation, defined by its arm length and the position and orientation of its pin joint P, with respect to the wheel hub H, was obtained from the corresponding Frenet apparatus. The orientation of the front and rear pin joints were taken as the binormal vector fields $\boldsymbol{B}_f(0)$ and $\boldsymbol{B}_r(0)$. The front and rear pin joint positions

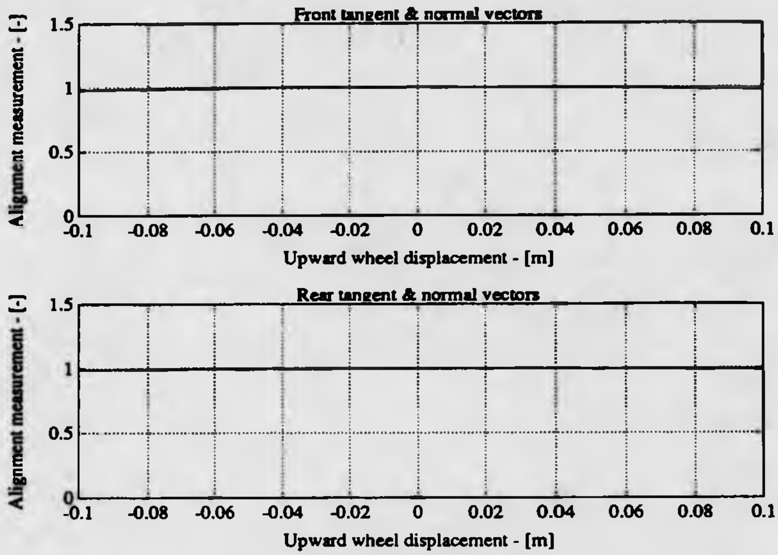

Figure 4.5: Variations of front and rear tangent and normal vector fields

were taken to be defined by the vectors $\kappa_f^{-1}(0)\boldsymbol{N}_f(0)$ and $\kappa_r^{-1}(0)\boldsymbol{N}_r(0)$ relative to the static wheel hub locations, and the corresponding arm lengths were $\kappa_f^{-1}(0)$ and $\kappa_r^{-1}(0)$. The situation is illustrated in figure 4.6.

4.4 Multibody System Model of Chassis

4.4.1 Topological Representation of MBS Model

The chassis model is considered as a multibody system made up from five hinge connected rigid bodies organised in a tree topology, as described in figure 4.7. The central, or base body corresponds to the sprung mass and the four branch bodies to each of the four wheels and associated suspension links. Each branch body is attached to the base body by a single degree of freedom rotational, or pin joint which permits rotation about a common hinge axis in the branch and base body. Associated with the base body and each branch body is a local right-handed coordinate frame with fixed orientation and location (at the centre of mass) in the respective body. Figure 4.8 describes a reference configuration such that all body based coordinate frames and an inertial frame are aligned.

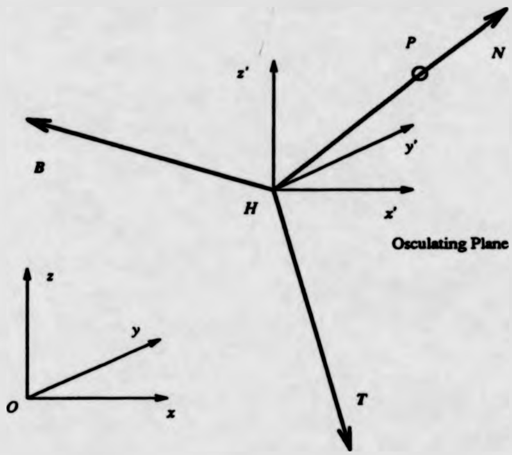

Figure 4.6: Frenet frame for swing axle

The geometrical description of the MBS model can be specified by a set of three vectors for each of the branch bodies. The first of the three vectors ***op*** describes the position of the pin joint relative to the centre of gravity of the base body. This vector is fixed in the base body. The second vector ***hp*** describes the position of the pin joint relative to the centre of gravity of the branch body and is fixed in the branch body. The third vector ***b*** describes the orientation of the pin joint axis.

		Left	Right
Front	***op***	(1.441, 0.303, -0.242)	(1.441, -0.303, -0.242)
	hp	(0.121, -0.447, 0.058)	(0.121, 0.447, 0.058)
	b	(0.966, 0.257, -0.029)	(0.966, -0.257, -0.029)
Rear	***op***	(-1.516, 0.105, -0.157)	(-1.516, -0.105, -0.157)
	hp	(-0.016, -0.645, 0.143)	(-0.016, -0.645, 0.143)
	b	(0.994, 0.0, 0.113)	(0.994, 0.0, 0.113)

Table 4.1: Geometrical description of MBS chassis model

A convenient choice of reference configuration for the body fixed axes is a right handed coordinate system as described in figure 4.1. For each branch body, specifications for the vectors ***op***, ***hp*** and ***b*** were obtained relative to this reference

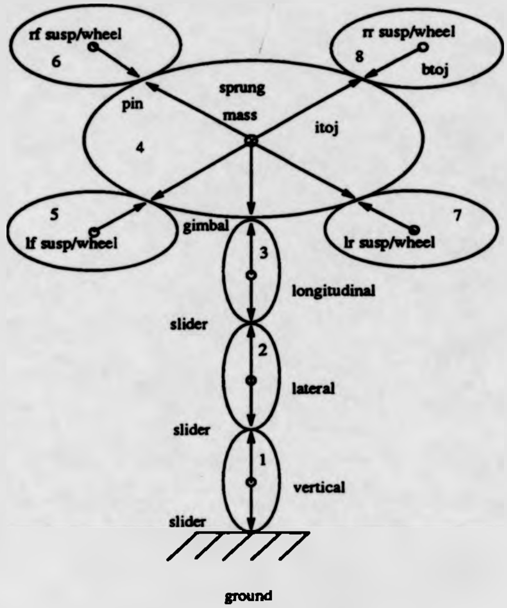

Figure 4.7: Topological representation of chassis

coordinate frame. The vectors ***hp*** and ***b*** were derived in the manner described in section 4.3 and the vector ***op*** was derived from the vehicle dimensional data presented in table 4.2 The resultant values of the vectors ***op***, ***hp*** and ***b*** are presented in table 4.1 and the general layout is described in figure 4.8.

To complete the physical description of this MBS model, the mass and inertia matrix for each body is required. This data is also presented in table 4.2.

4.4.2 External Forces Acting on MBS Model

4.4.2.1 Sprung Mass

The sprung mass consists of the base body. Apart from gravitational influences, the forces acting on the sprung mass are aerodynamic and suspension forces, which also includes the effect of an antiroll torsion bar at the front axle.

The air flowing around a vehicle produces forces and moments which impact on the vehicle's behaviour. Vehicle performance is effected by the aerodynamic longitudinal force. The aerodynamic vertical force and roll, and pitch, moments influence the cornering behaviour of a vehicle. Directional control of a vehicle is

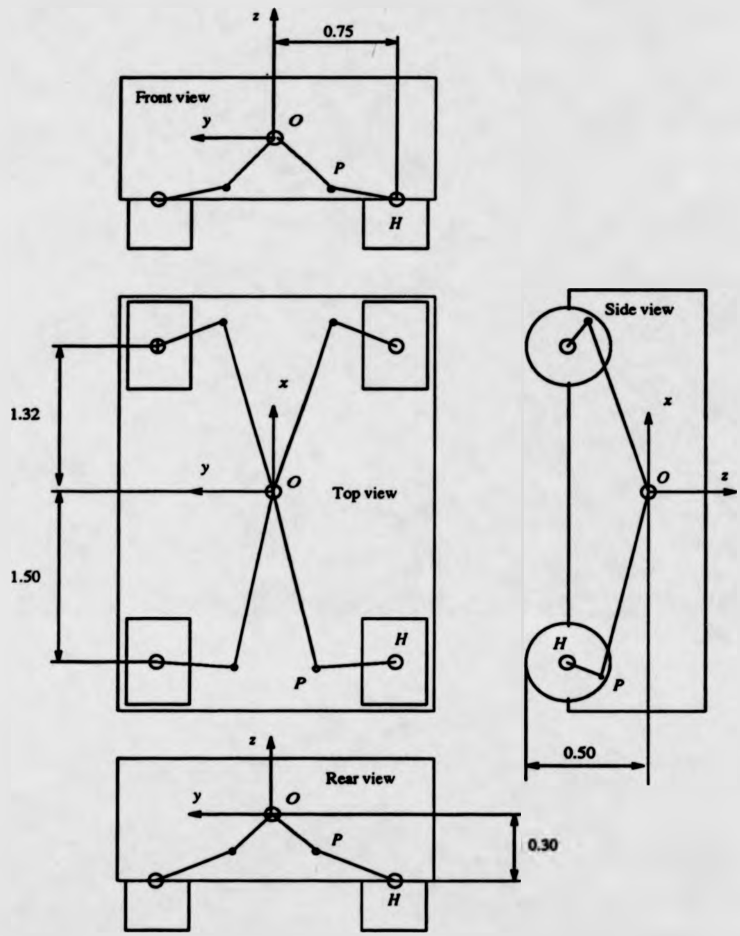

Figure 4.8: Layout of MBS model

effected by the aerodynamic lateral force and yaw movement.

The aerodynamic forces [109, 146] are considered to act through a point at ground level and at mid wheelbase and mid track. The longitudinal force is represented by

$$f_x = \frac{1}{2} c_x(\alpha_a) \rho A |v'|^2 \tag{4.28}$$

the lateral force by

$$f_y = \frac{1}{2} c_y(\alpha_a) \rho A |v'|^2 \tag{4.29}$$

Effective weight of wheel	front	50 kg
	rear	40 kg
Radius of wheel		0.2 m
Weight of sprung mass		1600.0 kg
Principle inertias of sprung mass	I_{xx}	500.0 $kg.m^2$
	I_{yy}	3000.0 $kg.m^2$
	I_{zz}	3000.0 $kg.m^2$
Centre of gravity of sprung mass	from front axle	1.32 m
	from rear axle	1.5 m
	above ground	0.5 m
	from right/left side	0.75 m

Table 4.2: Vehicle inertia and geometric data

and the vertical force by

$$f_z = \frac{1}{2} c_l(\alpha_a) \rho A |v'|^2 \tag{4.30}$$

with

$$c_l = c_{zf} + c_{zr} \tag{4.31}$$

and where A is the frontal area, ρ the air density, v' the relative velocity between the vehicle and head wind, and c_x, c_y and c_l are aerodynamic coefficients which are generally dependent on the aerodynamic slip angle α_a. The variation of the aerodynamic force coefficients with α_a are presented in figure 4.9. The stars represent experimental data and the solid lines fitted polynomials.

The aerodynamic moments [109, 146] are additional to the moments caused by the off-centre aerodynamic forces and are considered to act about the body coordinate axes. The roll moment is represented by

$$m_x = \frac{1}{2} c_{mx}(\alpha_a) \rho A |v'|^2 \tag{4.32}$$

the pitch moment by

$$m_y = \frac{1}{2} c_{my}(\alpha_a) \rho A |v'|^2 \tag{4.33}$$

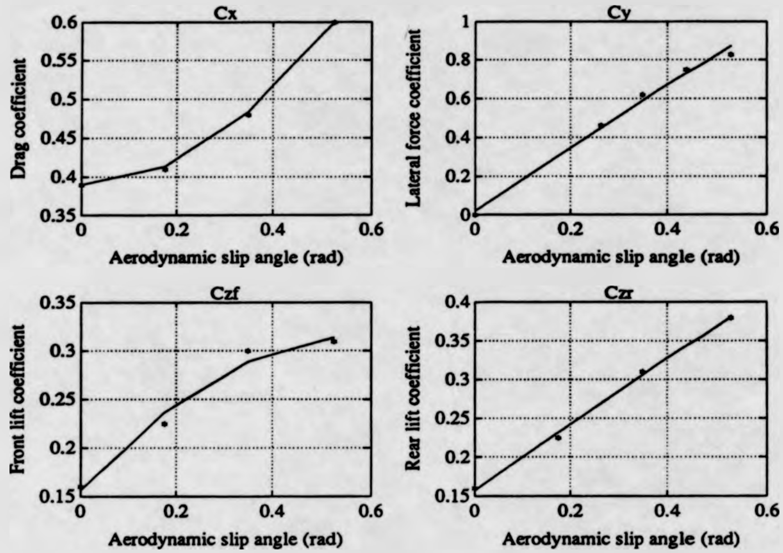

Figure 4.9: Aerodynamic coefficients

and the yaw moment by

$$m_z = \frac{1}{2}c_{mz}(\alpha_a)\rho A|v'|^2 \tag{4.34}$$

where c_{mx}, c_{my} and c_{mz} are aerodynamic torque coefficients which are also generally dependent on the aerodynamic slip angle α_a. They are presented on figure 4.10 in a similar manner to the force coefficients.

The six aerodynamic coefficients are based on empirical information. Over the range of aerodynamic slip angles investigated in this work, the coefficients c_y, c_{zr}, c_{mx} and c_{mz} are adopted to vary linearly with α_a and the coefficients c_x, c_{zf} and c_{my} are approximated by parabolas as a function of α_a. The vehicle aerodynamic data is presented in table 4.3.

The suspension forces generated at each wheel are considered to act in a vertical direction relative to the base body coordinate frame. Each force is applied through a nominal point located at the respective wheel hub positions given in figure 4.8. The wheelbase/bump and track/bump data presented in figures 4.2 and 4.3 are used to correct this point of application for vertical movement in the suspension.

Gravitational effects on the sprung and unsprung masses are represented as

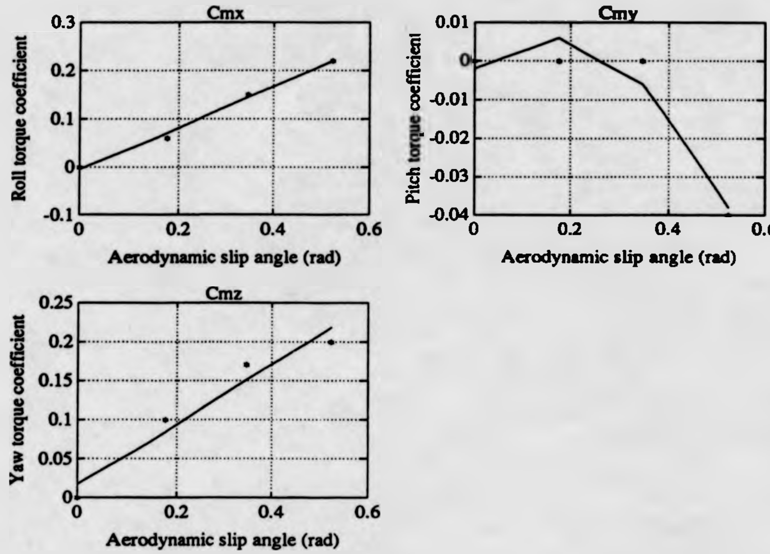

Figure 4.10: Aerodynamic torque coefficients

vertical forces, relative to the inertial coordinate frame, acting through each body centre of gravity, and are taken automatically into consideration by SD/FAST. These effects cannot be ignored since, under dynamic conditions, the sprung mass roll and pitch axes are not constant and do not necessarily pass through the centre of gravity.

4.4.2.2 Unsprung Masses

The unsprung masses consist of the four branch bodies. The forces acting on each unsprung mass are the longitudinal, lateral and vertical tyre forces, the suspension forces and gravitational forces.

Each suspension force is considered to react through the centre of gravity of the unsprung mass in a vertical direction relative to the base body coordinate frame. The tyre forces are represented relative to the branch body coordinate frame. The vertical tyre force acts through the branch body centre of gravity. The lateral and longitudinal tyre forces act through a point vertically displaced from the centre of gravity by the wheel radius. This displacement reflects the action of the longitudinal and lateral forces at the tyre/road contact patch, parallel to the ground plane.

A	$2.06\ m^2$
ρ	$1.202\ kg.m^{-3}$
c_x	$0.766\alpha_a^2 + 0.390$
c_y	$1.629\alpha_a$
c_{zf}	$-0.451\alpha_a^2 + 0.537\alpha_a + 0.156$
c_{zr}	$0.427\alpha_a + 0.1570$
c_{mx}	$0.430\alpha_a$
c_{my}	$-0.328\alpha_a^2 + 0.103\alpha_a$
c_{mz}	$0.384\alpha_a$

Table 4.3: Vehicle aerodynamic data

4.4.3 Representation of Suspension and Tyre Forces

4.4.3.1 Suspension Forces

Each suspension force is modelled as a spring in parallel with a damper. The spring and damper forces are taken as linear functions of the spring displacement, and displacement rate, respectively, relative to the base body. An additional roll torque represents the effect of an antiroll torsion bar acting at the front axle. This torque is modelled, relative to the base body, as a linear function of the left/right difference in spring displacements at the front axle. Static values for the vertical suspension and tyre deflections are computed to balance the weight of the sprung mass in the steady-state. The stiffness and damping coefficients for the suspension are presented in table 4.4

4.4.3.2 Steering System Compliance

At the front axle an additional steering stiffness is included, which acts in series with the cornering stiffness of the tyres. In this section, only a limited model of the steering system is considered in which the equivalent cornering stiffness for the combination tyre/steering system is derived. More comprehensive models are described in the literature and they include the effects of steering system dynamics [257, 36].

In this case, it is desired to obtain the tyre lateral force as a function of

		Front	Rear
Suspension	Spring stiffness	20.0 $kN.m^{-1}$	27.0 $kN.m^{-1}$
	Damper coefficient	1.4 $kN.s.m^{-1}$	2.0 $kN.s.m^{-1}$
	Antiroll bar stiffness	20.0 $kN.m^{-1}$	
Linear tyre	Vertical stiffness	250.0 $kN.m^{-1}$	250.0 $kN.m^{-1}$
	Cornering stiffness	66.0 $kN.m^{-1}$	70.0 $kN.m^{-1}$
Steering system compliance		0.0051 $rd.kN^{-1}$	

Table 4.4: Suspension and linear tyre data

the steering wheel angle and the equivalent cornering stiffness, when steering system compliance is considered. The steering mechanism, including its torsional stiffness, is depicted in the diagram of fig 4.11. In this diagram only one wheel is illustrated and the tyre/ground contact is represented by point 2. All the elements in the steering system are considered pure and ideal and inertia effects are neglected.

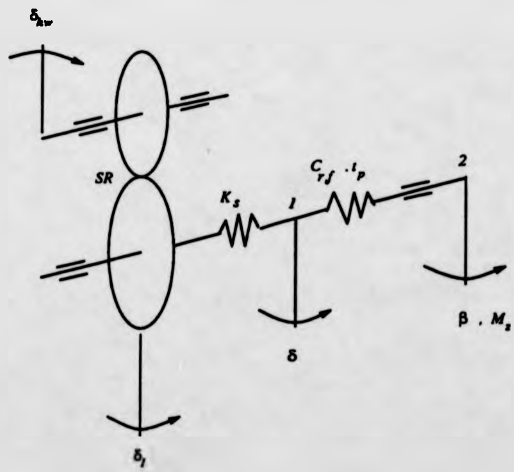

Figure 4.11: Steering system diagram

The following relationships can be obtained from figure 4.11. For the steering system gear mechanism results

$$\delta_1 = \frac{\delta_{hw}}{SR} \tag{4.35}$$

where δ_{hw} is the steering wheel angle, and SR is the steering system gear ratio.

Applying Newton's law to point 1 in the diagram, gives

$$-K_s(\delta - \delta_1) = C_{f,r}.t_p(\delta - \beta) \tag{4.36}$$

where $C_{f,r}$ is the cornering stiffness and the subscripts indicate whether it is a front or rear tyre, t_p is the pneumatic trail, K_s is the steering system torsional stiffness, β is the sideslip angle of the wheel and δ is the actual steering angle of the wheel. The steering system torsional stiffness is described by

$$K_s = \frac{t_p}{\left(\left.\frac{\partial\delta}{\partial F_{ty}}\right|_{F_{tyo}}\right)} \tag{4.37}$$

where $\left.\frac{\partial\delta}{\partial F_{ty}}\right|_{F_{tyo}}$ is the steering system compliance and it is presented in table 4.4.

Applying Newton's law to point 2, which represents the tyre/ground contact, yields

$$-C_{f,r}.t_p(\beta - \delta) = M_z \tag{4.38}$$

where M_z is the self-aligning torque.

The *slip angle* is defined as the angle between the wheel's x-axis and the direction of the wheel travel given by the wheel's velocity vector [233]. It can be expressed as

$$\alpha = \beta - \delta \tag{4.39}$$

It's worth noting that the slip angle could also be described using the wheel's x, y-axes directly, but using relation 4.39 provides a better insight into its physical meaning, as it will be seen in the next chapter.

Figure 4.12 illustrates these quantities. It can be seen from this figure that the steering angle, δ, is the angle between the *wheel* heading (wheel's x-axis) and *vehicle* heading (vehicle's X-axis), projected in the ground plane; the sideslip angle, β, is a measure of the angular displacement of the wheel's velocity vector in relation to *vehicle* heading, and the slip angle, α, can be seen either as the difference of the two previously mentioned quantities or as a measure of the angular displacement of the *wheel* heading angle in the relation to the wheel's velocity vector, projected in the ground plane (Note: In figure 4.12, δ and β are depicted negative whilst α is positive).

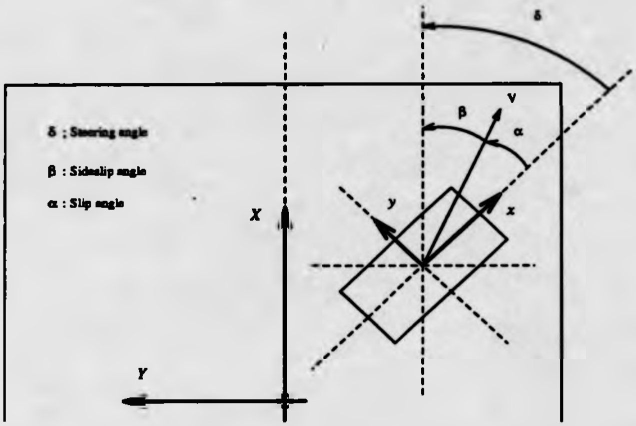

Figure 4.12: Illustration of steering quantities

Considering the *cornering stiffness* [1] for each wheel to be given by:

$$C_{f,r} = \left.\frac{\partial F_{ty}}{\partial \alpha}\right|_{\alpha_o} \tag{4.40}$$

where F_{ty} is the tyre lateral force, with all other quantities held constant; then the lateral force produced by a tyre running at a slip angle is described by

$$F_{ty} = -C_{f,r}.\alpha \tag{4.41}$$

similarly to a spring. However, because of the deformation of the contact patch between the tyre and the ground during a cornering manoeuvre, the lateral force does not act at the centre of the contact patch, but at a point behind it. This distance is the pneumatic trail, t_p. The side force and the self-aligning torque are thus related

$$F_{ty}.t_p = M_z \tag{4.42}$$

Performing the necessary substitutions, results for the lateral force

$$F_{ty} = -C_{f,r_e}\left(\beta - \frac{\delta_{hw}}{SR}\right) \tag{4.43}$$

where C_{f,r_e} is the equivalent cornering stiffness for the tyre/steering system com-

[1] This quantity is sometimes defined as negative in the literature.

bination and it is given by

$$C_{f,r_e} = \frac{C_{f,r}}{\left(1 + \frac{C_{f,r}}{K_s/t_p}\right)} \tag{4.44}$$

4.4.3.3 Tyre Forces

For the linear tyre model, the longitudinal tyre forces represent propulsion and braking action. These forces are modelled as an externally defined forcing term, where it is assumed that wheel spin and wheel lock do not occur. The vertical and lateral tyre forces are represented as linear functions of the vertical wheel displacement and wheel's sideslip angle, respectively. The linear tyre stiffnesses are presented in table 4.4

Lateral force coefficients			
b_{11}=15.014522	b_{12}=0.417075^{-2}	b_{13}=-0.132836^{-5}	b_{14}=0.914832^{-10}
b_{21}=503.258991	b_{22}=-0.339215	b_{23}=0.672331^{-4}	b_{24}=-0.421518^{-8}
b_{31}=-19291.713	b_{32}=9.901508	b_{33}=-0.171675^{-2}	b_{34}=0.994122^{-7}
b_{41}=219991.02	b_{42}=-113.39167	b_{43}=0.193546^{-1}	b_{44}=-0.109341^{-5}
b_{51}=-835912.2	b_{52}=443.517648	b_{53}=-0.765168^{-1}	b_{54}=0.43227^{-5}
Pneumatic trail coefficients			
c_1=0.194631^{-3}	c_2=-0.105903	c_3=0.800973^{-5}	c_4=0.169201^{-10}
c_5=-0.432221^{-4}		c_6=0.121753^{-3}	

Table 4.5: Nonlinear tyre model coefficients

For the nonlinear tyre model, the longitudinal and vertical forces are the same as in the linear tyre; however, the lateral force is described as a nonlinear function of the slip angle and vertical load. Also the pneumatic trail is calculated as a function of these quantities. The model used in this work was provided by the vehicle's manufacturer and it is similar to one described by Chiesa [32]. It consists of polynomial interpolations on experimental data generated by the tyre's manufacturer.

The first polynomial gives the lateral force, F_{ty}, as a function of slip angle, α, and tyre normal load, F_{tz}; whilst the second one gives the pneumatic trail, t_p, as a function of the same quantities.

These polynomials are described in equations 4.45 and 4.46, for the lateral force and in equation 4.47 for the pneumatic trail. The numerical values of the coefficients of these equations are listed in table 4.5.

For the tyre lateral force it was adopted

$$F_{ty} = a_1\alpha + a_2\alpha^2 + a_3\alpha^3 + a_4\alpha^4 + a_5\alpha^5 \tag{4.45}$$

where the a_is are described as a function of F_{tz} by

$$a_i = b_{i1}F_{tz} + b_{i2}F_{tz}^2 + b_{i3}F_{tz}^3 + b_{i4}F_{tz}^4 \qquad (i = 1,5) \tag{4.46}$$

For the pneumatic trail the interpolated polynomial is given by

$$t_p = c_1 + c_2\alpha + c_3F_{tz} + c_4F_{tz}^2 + c_5\alpha F_{tz} + c_6\alpha^2 F_{tz} \tag{4.47}$$

Three-dimensional plots have been generated of these functions and they are illustrated in figure 4.13 for the lateral force and in figure 4.14 for the pneumatic trail.

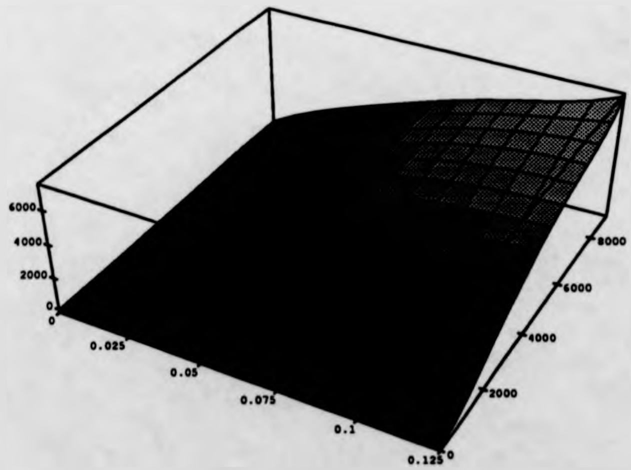

Figure 4.13: Lateral force versus slip angle and normal load

Actually, there is a dynamic delay in the tyre's lateral force generation process. Basically, the tyre must roll through some distance before the contact patch fully develops a commanded slip angle. These delays are normally small and a first order time lag is considered adequate in the majority of the publications [7, 146, 212]. It is assumed that the time constant is a function of the characteristic length, c_l, and vehicle's speed, V_x. That is

$$\tau_l = \frac{c_l}{V_x} \tag{4.48}$$

Then, the lagged lateral tyre force is given as the solution to the first order differential equation

$$\frac{dF_{tyl}}{dt} = \frac{1}{\tau_l}(F_{ty} - F_{tyl}) \tag{4.49}$$

For the vehicle modelled in this work, the characteristic length adopted was $0.3m$, according to manufacturer's data and the vehicle speed varied according to the simulation been run. It is assumed that τ_l varies slowly due to V_x, so that it can be assumed constant during an integration interval.

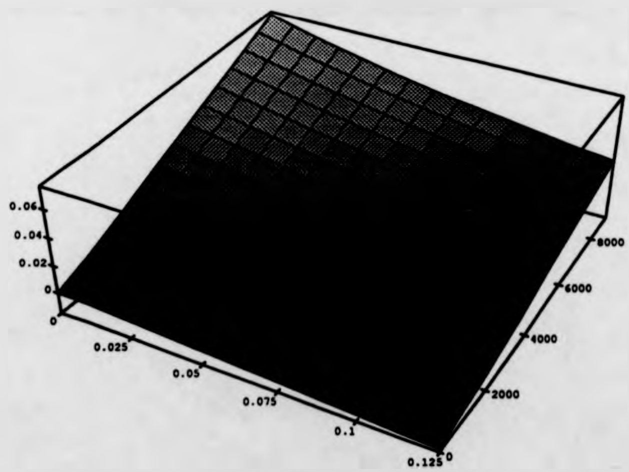

Figure 4.14: Pneumatic trail versus slip angle and normal load

Therefore, in this simulation four combinations of tyre models are possible: linear and nonlinear tyre models, with and without time lag in the generation process of the lateral force. These possibilities are discussed and analysed in chapter 5.

4.5 Simulation Model of Chassis

The computer simulation model was built using the MBS modelling package SD/FAST and the ACSL continuous simulation language. The SD/FAST program provides a means of automatically generating FORTRAN code describing the equations of motion for MBS modelling applications via a combination of Kane's formulation and symbolic computation techniques. This approach to MBS model generation results in computer simulation models with far higher runtime performance than is normally associated with general purpose MBS modelling packages and which are generally far more reliable than simulation models which are derived and coded by hand. The runtime performance gains result from the ability to exploit all the simplifications associated with a particular MBS application which is facilitated by the symbolic computation facilities in SD/FAST. The reliability improvements result from by-passing the need for hand derivation of the equations of motion for a MBS application and their subsequent coding in a programming language such as FORTRAN, a process which is both error prone and time consuming.

The SD/FAST program processes an ASCII source code file containing a user-supplied description of a MBS model. The resulting object code, which is generated autonomously by SD/FAST, consists of two main FORTRAN subroutines and several secondary subroutines. The first of the main subroutines defines the state derivatives for the equations of motion of the MBS model and the second subroutine serves to initialise the parameters and states of the MBS model. These states correspond to the generalised coordinates and velocities of each body in the MBS model. The secondary subroutines provide utilities for coordinate frame transformations and other housekeeping functions. These SD/FAST generated FORTRAN subroutines can be easily called from a variety of simulation packages. In particular, the two main subroutines can be incorporated directly into the `DERIVATE` and `INITIAL` sections, respectively, of an ACSL program.

The equations of motion for the MBS chassis model discussed in section 4.4 were generated by SD/FAST from the input file presented in appendix A. This input file corresponds closely with the topological description of the MBS model given in section 4.4.1. Approximately 10 s of CPU time on a SUN 44/330 computer was used to generate the resulting output file. This output file consists of around 1500 lines of FORTRAN code, about 70% of which is contained in the main subroutine defining the state derivatives for the equations of motion of the MBS model. This subroutine accepts the values of the states and external torques and forces acting on the MBS model and returns the derivatives of these states. The subroutine was incorporated into an ACSL program to form a complete simulation model. The external forces and torques acting on the MBS model are computed externally to the SD/FAST generated subroutines. These forces, which are detailed in sections 4.4.2, and 4.4.3 are calculated by user-supplied code written in ACSL.

The approach to simulation model development for an automotive vehicle chassis system, described in this section, has the advantage that the model can be easily extended to include nonlinear tyre and suspension force characteristics without the need to modify the kinematic description of the sprung and unsprung masses. More importantly, the approach facilitates the development of vehicle models which combine a MBS description of the chassis with other vehicle subsystems which are not amenable to MBS modelling techniques, such as the powertrain or a digital control system, but which can be easily represented in a simulation language such as ACSL.

4.6 Simulation Results

The simulations to be performed in this chapter have a twofold purpose. Firstly, It is intended for the validation of the developed model and secondly, for a preliminary assessment of the capability of the model to cater for all aspects of vehicle motion, in an isolated way and with all aspects considered simultaneously.

Unfortunately, the amount of experimental data available is very limited, and therefore extensive validation is not possible to be performed, as would be desirable. The only experimental results available consist of the static deflections of the suspension, which was used to derive the swing axle model. Other results available were an eigenmode analysis resulting from simpler models in use by the vehicle's manufacturer and which had been previously validated by them.

The linear eigenmode analysis was performed with the computer simulation model as a prelude to nonlinear simulation experiments. The natural modes of the MBS model and their corresponding frequencies and damping ratios were computed using the linear analysis facilities of ACSL. The linear analysis was carried out with the MBS model in a steady state condition corresponding to a constant forward speed of 20 m/s. The eigenmode analysis resulted in four modes corresponding to the sprung mass and four further modes, two for each of the front and rear wheel pairs. The results were in close agreement with experimental observation and are presented in table 4.6.

Modes	Frequency		Damping
	Hz	rd/s	Factor
Front-end bounce	1.1	6.88	0.206
Rear-end bounce	1.35	8.46	0.266
Roll on springs	1.81	11.38	0.267
Yaw rate/lateral velocity	1.29	8.13	0.834
In-phase front wheel hop	12.02	75.46	0.200
Out-of-phase front wheel hop	12.50	78.52	0.195
In-phase rear wheel hop	13.00	81.69	0.304
Out-of-phase rear wheel hop	13.05	81.95	0.303

Table 4.6: Eigenmode analysis

The four sprung mass modes are yaw/sideslip, front end bounce, rear end bounce and roll on springs. Motion consisting of a combination of a lateral velocity and a yaw rate corresponds to the yaw/sideslip mode. The front end bounce mode describes the situation in which the front end undergoes a vertical displacement and the rear end remains stationary. The reverse situation in which the rear end undergoes a vertical displacement and the front end remains stationary coincides with the rear end bounce mode. The roll on springs mode describes motion consisting of rotation about a roll axis. Note that this roll axis is not necessarily the fore/aft axis of the sprung mass coordinate frame.

The four unsprung mass modes are in phase front wheel hop, out of phase front wheel hop, in phase rear wheel hop and out of phase rear wheel hop. These modes describe the angular displacement of the swing-arms in their respective osculating planes. These angular displacements each have fore/aft, lateral and vertical components, with the vertical components dominant. The in phase front wheel hop mode describes the situation where the angular displacements of the front swing axles have vertical components in the same direction. The reverse situation in which the angular displacements of the front swing arms have vertical components in opposite directions coincides with the out of phase front wheel hop mode. The in phase and out of phase rear wheel hop modes are defined in a similar manner.

Several simulation experiments were performed with the computer simulation model in order to confirm its utility for motion control studies. For this purpose, the computer simulation model was exercised with a variety of external disturbances and control inputs. The external disturbances considered were a vertical road disturbance and a lateral wind gust. The control inputs consisted of acceleration and deceleration forces, and a steering angle. The simulation experiments were restricted to ranges over which the suspension and tyre characteristics are valid. The results of these simulation experiments are presented in figures 4.15 to 4.20. These results confirm that the MBS model realistically represents the response of an automotive vehicle to a variety of external disturbances and control inputs.

The road disturbance consisted of a vertical displacement with a triangular profile of height 0.1 m and width of 1.0 m, and which was applied to both the left and right wheels. This input was applied such that the front left wheel encountered the input 0.5 m in advance of the front right wheel and with a constant vehicle velocity of 5 m/s. The equivalent road inputs in the time domain and the resulting responses of each of the four wheels are presented in figure 4.15.

These figures show a 0.1 s time shift in the road input between the left and right wheels, and a 0.56 s time shift in the road input between the front and back wheels. The resulting wheel transient responses include a frequency component at approximately 12 Hz which is associated with the wheel motion and a frequency component at about 1.3 Hz which is connected with the motion of the sprung mass. Figure 4.16 presents the response of the sprung mass to this road input. As would be expected, the lateral, longitudinal and yaw motion responses were negligible.

The wind disturbance consisted of a lateral velocity pulse of magnitude 7.08 m/s and duration 2 s. With a constant vehicle velocity assumed as 40 m/s

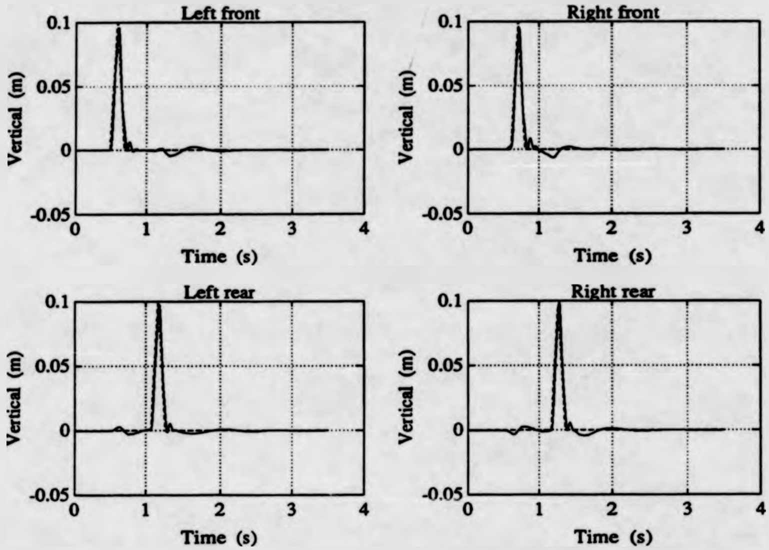

Figure 4.15: Response of the wheels to a road input

the corresponding aerodynamic side slip angle was 10°. Figure 4.17 present the response of the sprung mass to this wind input. Again, as would be expected, the longitudinal, vertical, and pitch motion responses were negligible.

The acceleration input consisted of a ramp in the total acceleration force at the rear wheels from 0 to 7.846 kN over 0.5 s. This force was maintained at this level until the vehicle speed reached 25 m/s at which point the force was stepped down to 0 kN. The vehicle was assumed to have an initial velocity of 2 m/s. Figure 4.18 present the vehicle response to this acceleration force. The tractive effort described is for a single wheel. Again, as would be expected, the lateral, vertical, roll and yaw motion responses were negligible.

The deceleration input consisted of a ramp in the total acceleration force at the four wheels from 0 to -0.8 g (kN) over 0.5 s. This force was maintained at -0.8 g until the vehicle speed reached 1 m/s at which point the force was stepped down to 0 g. The vehicle was assumed to have an initial velocity of 20 m/s . The braking effort was distributed such that 70% was applied at the front wheels and 30% at the rear wheels. Figure 4.19 presents the response of the vehicle to this deceleration force. The braking effort described is for a single wheel. Also shown are the effects on the forward velocity and pitch angle. Again, as would

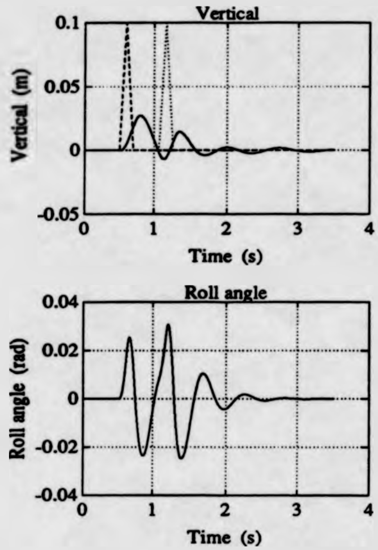

Figure 4.16: Response of the sprung mass to a road input

be expected, the lateral, vertical, roll and yaw motion responses were negligible.

The steering input consisted of a parabola to step input in the hand wheel angle of 45°. The vehicle was assumed to have a constant velocity of 20 m/s. Figure 4.20 presents the response of the sprung mass to this steering input. Again, as would be expected, the longitudinal, vertical and pitch motion responses were negligible.

4.7 Further Model Developments

In order to be able to include the rotational degrees of freedom of the wheels and eventually the steering (toe) and camber effects due to the suspension movements, a separate wheel hub and wheel have been added to the end of each suspension arm of the topological model described in figure 4.7. The resulting topological model is illustrated in figure 4.21. In order to include these additional bodies, each unsprung mass have been divided in three; a suspension arm, the wheel hub and the wheel itself. The front arms were assumed to weight 10 kg and the rear ones 5 kg. The hubs were assumed to be massless and the front wheels weight is 40 kg whilst the rear ones is 35 kg. However, the moments of inertia adopted

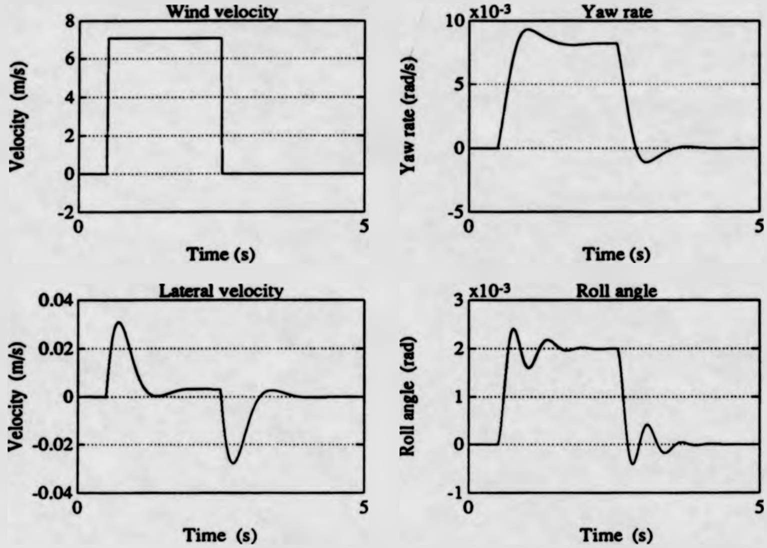

Figure 4.17: Response of the sprung mass to a wind input

for the wheels are the same. They are I_{xx}=0.75, I_{yy}=1.50 and I_{zz}= 0.75 $kg.m^2$. The location of the *cg* of the suspension arms are assumed to be at midpoint of the arm length and the resulting vectors are described in the SD/FAST input file given in appendix A. The ACSL program listings are presented in appendix B.

This is the model used for the simulation studies of chapter 5. Because of the modular nature of the simulation program, the addition of these extra bodies were performed with minimum effort, highlighting the modular characteristics of the simulation approach adopted. A comparison between the previous program and the present, illustrates this point. Because the present tyre model does not include a longitudinal slip description, the wheels' rotational degrees of freedom are disabled through the prescribed motion facility of SD/FAST, with their motion set to zero. Wheel toe-in due to suspension travel can be taken into consideration by adding it to the desired steering angle in case of the steered wheels or directly at the nonsteered wheels by using the prescribed motion facility and setting them to be equal to the equivalent rotation around the king pin. Because wheel camber is ignored presently, the wheel hub is assumed to be rigid. However it can be easily added on by proceeding in similar fashion. One might then wonder since this is such a powerful package why not include a proper description of the whole

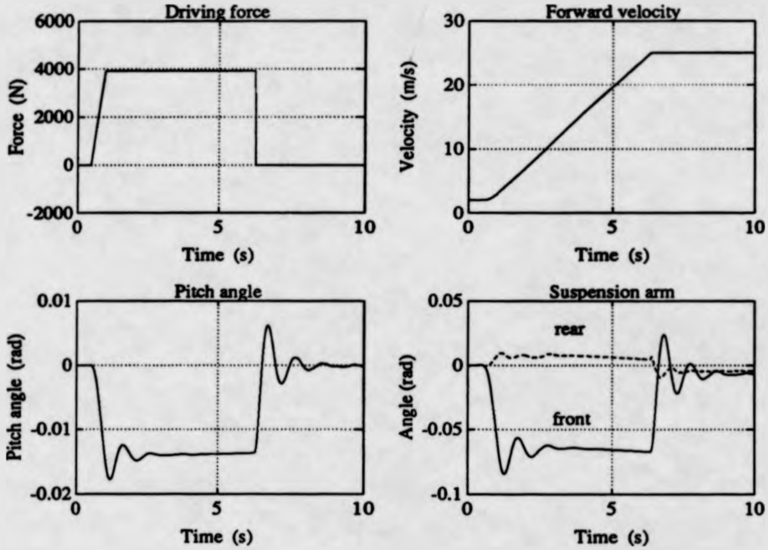

Figure 4.18: Response of the sprung mass to an acceleration force

suspension? The answer is that a proper suspension would include various closed loops in each suspension and therefore a significant amount of complexity to the model from a simulation point of view. Just as a simple example, the model as it stands, with rotating wheels, and wheel hubs has a complexity of one order of magnitude *smaller* than a similar suspension model as described in one of the IAVSD benchmarks performed by the author and a colleague [129]. This fact fully justifies the use of these simpler models, because they are much smaller, therefore quicker, and they are accurate enough for the purposes of the study of motion management.

4.8 Conclusions

This chapter has described work concerning the application of MBS modelling techniques to automotive vehicle chassis dynamics simulation. The work was motivated by the need for nonlinear simulation models capable of supporting vehicle motion control studies. The nonlinear MBS model developed in this chapter represents the chassis in terms of a sprung mass and four unsprung masses, and incorporates a simple geometrical description of the moving parts of the suspen-

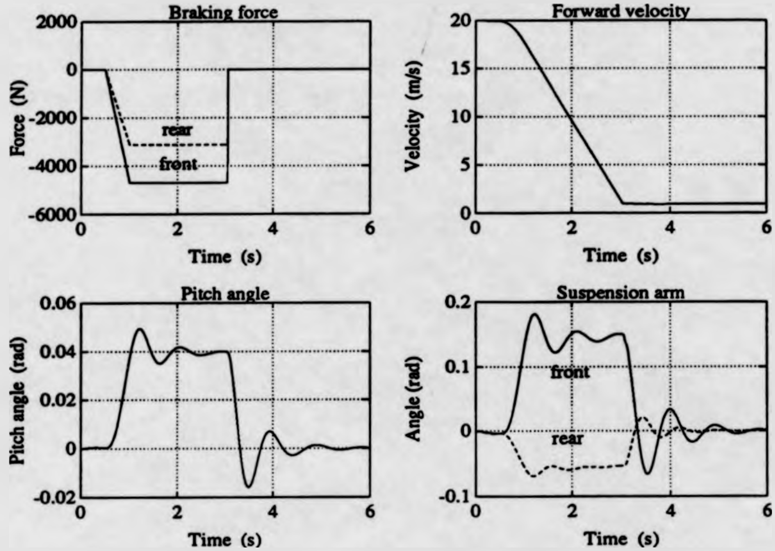

Figure 4.19: Response of the sprung mass to a deceleration force

sion. The simpler MBS model consists of five bodies with a total of ten degrees of freedom or, correspondingly, twenty states. The model incorporates a simple representation of the tyre and suspension based on linear force characteristics, and assumes no spin or lock in the rotational wheel motion.

The modelling of the suspension geometry was based on experimental data describing fore-aft and lateral wheel displacement as a function of the vertical displacement of each wheel. From this data, which represents the suspension trajectory at each wheel hub, an equivalent swing axle model was derived using techniques from differential geometry. This resulted in a pin direction vector and the arm length of the swing axle for each wheel.

The nonlinear computer simulation model was built using the MBS modelling package SD/FAST and the ACSL continuous simulation language. SD/FAST automatically translates a user-supplied description of a MBS model into FORTRAN code representing the equations of motion for the MBS model via a combination of Kane's formulation and symbolic computation techniques. This approach to MBS model generation results in computer simulation models with far higher runtime performance than is normally associated with general purpose MBS modelling packages, and which are generally far more reliable than

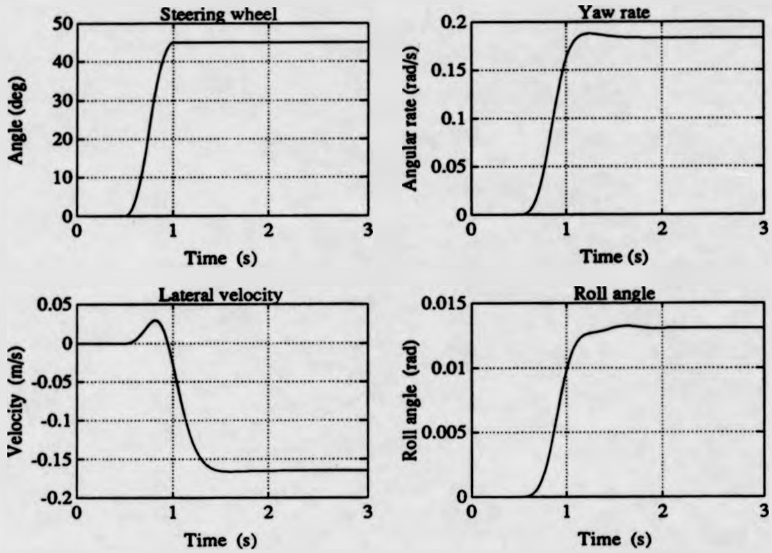

Figure 4.20: Response of the sprung mass to a steering input

simulation models which are derived and coded by hand.

The computer simulation model has been exercised with a variety of external disturbances and control inputs within ranges where the suspension and tyre characteristics are valid. The external disturbances consisted of a vertical road disturbance and a lateral wind gust, and the control inputs included acceleration and deceleration forces, and a steering angle. The results confirm that the MBS model realistically represents the response of an automotive vehicle to a variety of external disturbances and control inputs.

The modelling work described in this chapter represents the first stage in the development of a vehicle dynamics model for use in motion control studies. The present model is limited, particularly in its representation of the tyre and suspension forces. Nevertheless, the model is applicable to chassis control studies, including investigations of nonlinear problems associated with vehicle handling. Moreover, the model can easily be extended to include nonlinear tyre and suspension force characteristics without the need to modify the kinematic description of the sprung and unsprung masses.

This chapter has presented a systematic and straightforward approach to the modelling of automotive vehicle chassis systems for vehicle motion control stud-

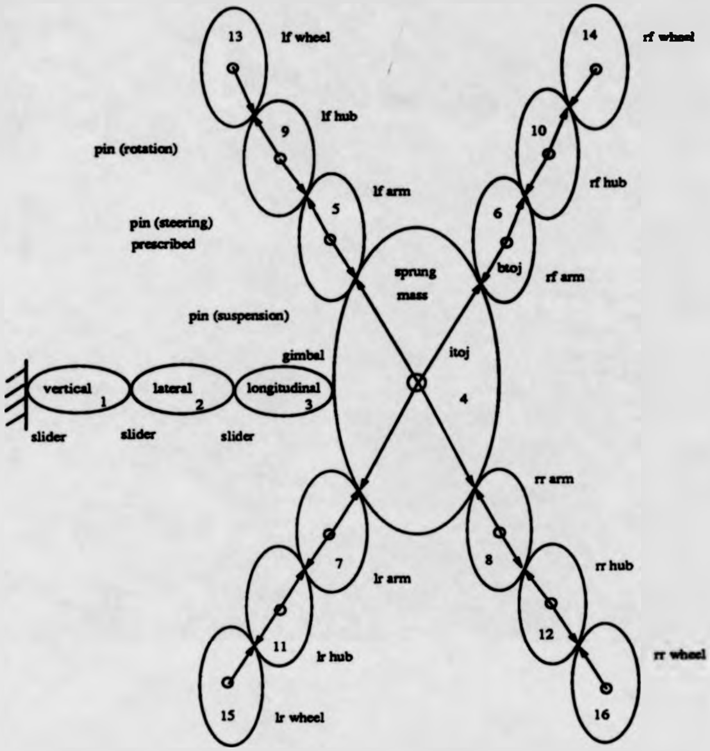

Figure 4.21: Extended topological representation of chassis

ies. The methodology is generic and applicable to a wide variety of automotive vehicles types, and it can be easily extended in order to consider aspects initially ignored. Furthermore, the approach facilitates the development of vehicle models which combine a MBS description of the chassis with other vehicle subsystems which are not amenable to MBS modelling techniques, such as the powertrain or a digital control system, but which can be easily represented in a simulation language such as ACSL.

In the next chapter extensive simulations are carried out in order to determine an operating range and conditions for the use of a linearised model in control system analysis and design. These simulations cover all aspects of vehicle motion under realistic operating conditions considering driver's inputs as well as external disturbances. For that purpose transient and steady-state analysis are performed.

Chapter 5

Simulation and Analysis of Vehicle Motion

5.1 Introduction

In order to confirm the utility of the previously developed model, as well as characterising its behaviour within an operating range which allows the definition of appropriate control strategies, for example, sets of operating conditions in which linear approximations would still be valid, a large number of tests which exercise the model within the boundaries of its simplifying assumptions will be carried out. These tests or experiments, are defined to be either driver inputs or external disturbances and they will be looking at both transient or steady-state behaviour of the simulation model. The analyses to be performed will consist of a detailed study of these simulations and will include eigenvalue analysis, intended at verifying the validity of transient behaviour, and gain analysis, aimed at determining the characteristics of the model's steady-state characteristics.

The intention of these experiments is to allow the definition of a driving envelope in which the interactions present, their nature and magnitude are discussed and a valid range of operating conditions for control studies can be defined. The definition and properties of the resulting linear models are then analysed in their state space and transfer function matrix representations and a number of issues, relevant to control studies and their computational implementation, are discussed in the next chapter.

The simulation and analysis of model behaviour will encompass the following quantities which are related to vehicle motion: six degrees of freedom related to sprung mass movement, four degrees of freedom associated with suspension deflections and two or four associated with the steering system. The four degrees of freedom related to the rotational motion of the wheels are not considered in this analysis, although they have been included in the model.

The analysis of the simulation results will be looking at the transient and steady-state characteristics of the behaviour of these quantities. For example, in transient manoeuvres, the vibrational aspects of the various modes associated with vehicle motion, such as front-end-bounce, rear-end bounce, roll on springs or yaw and sideslip for the sprung mass, or wheel hop modes for the unsprung masses. For the steady-state manoeuvres, the gain relationships will be analysed, for example, steady-state ratios between pitch and roll angles, yaw rate, lateral velocity or acceleration as functions of different cornering speeds or steering angles or height of sinusoidal road profiles.

The different tyre models for the lateral dynamics will be used in circumstances in which the effect that they incorporate becomes more apparent. For this purpose, for instance, at high speeds or/and large steering angles the nonlinear tyre model will show the difference in tyre lateral forces, when load transfer effects are considered.

5.2 Vehicle Simulation

The simulation runs to be executed comprise two types of manoeuvre; one intended to analyse the vehicle's transient behaviour and the another aimed at obtaining the vehicle steady-state response characteristics. Simulation results are presented in appendix C.

5.2.1 Transient Manoeuvres

The simulation runs which will be used to analyse the transient behaviour of the model can be divided in two types; driver related inputs and external disturbances inputs. Driver's inputs will be in the propulsion system, either in acceleration or deceleration manoeuvres, and in the steering system, in cornering manoeuvre. External disturbances will consist of road vertical input and/or aerodynamic disturbances. Results for the transient manoeuvres are presented in section C.1 of appendix C.

5.2.1.1 Steering Input

The simulation runs which will exercise the vehicle's handling or stability and steering control properties will consist of manoeuvres of the so called 'free-control' type, in which steering wheel angle is considered as the input. In this sense, effects of steering system compliance, as described in chapter 4 are taken into consideration. However, it does not include the dynamics of the steering system as reported by Segel and McAdam [256], for example.

The input will consist of a terminated parabola steering wheel angle input, which approximates a step input. The reason for choosing such an approximation to a step input is related to the fact that these quantities are defined as prescribed motion in SD/FAST, and in this form it presents better numerical behaviour in the enforcement of the constraints. It's worth noting that the parabola duration is negligible compared to the response times of the model quantities which are influenced by it. In this sense, it is a good approximation to a step input.

The steady-state value of the steering angle input will be $10^o, 45^o$ and 90^o, in most of the simulations. Intermediate values of the steering angle will be used to illustrated the iminence and/or occurrence of either vehicle skidding or instability. Turns will be negotiated at speeds of 10, 20 and 30 m/s. A rear wheel driving force, proportional to speed error is introduced in order to maintain constant forward velocity during cornering.

Because of the importance of tyre lateral forces in vehicle handling, the simulation runs were executed for the different possible combinations of tyre models, as described in the previous chapter. For this purpose, the linear and the nonlinear tyre models with and without time lag were used. The differences in vehicle behaviour become more apparent as the manoeuvres approach limit conditions.

The utility of the 4 different descriptions of the tyre lateral force models available can be classified according to the following operating conditions:

- linear, without lag: slow manoeuvres, small amplitudes;
- linear, with lag: fast manoeuvres, small amplitudes;

- nonlinear, without lag: slow manoeuvres, large amplitudes;
- nonlinear, with lag: fast manoeuvres, large amplitudes.

In the case of the steering input manoeuvres, the results are shown for only two lateral force tyre models; the linear one without time lag in the lateral force build-up process, and the nonlinear model, incorporating a first order model description for the tyre lateral dynamics. The occurrence of vehicle skid and loss of stability, is also verified by the model, through the comparison of the maximum available adhesion for each wheel with the demanded force to perform the manoeuvre, and illustrated in the results. The longitudinal forces due to rolling resistance are ignored.

Quantities depicted and analysed include yaw rate, lateral velocity and acceleration, roll angle, wheel slip angle, lateral and vertical tyre forces. Figures C.1 to C.32 of section C.1.1 of appendix C illustrate these results.

As it would be expected, the variation of other quantities of motion associated with vehicle's forward and vertical motion are negligible.

5.2.1.2 Traction/Braking Input

For the acceleration and deceleration simulations, the powertrain and/or braking system dynamics are considered in a very approximate manner. Deterministic functions of time are adopted which account for typical time delays involved in these system's responses and the available level of performance for the type of vehicle being modelled. The input quantities will be either driving or braking forces for the linear tyre which assume no slip in the longitudinal motion of the wheels. The results for these simulations are presented in section C.1.2, of appendix C.

For the acceleration simulation, the vehicle is assumed to be rear drive in accordance with the vehicle being modelled, the wheel force or torque is assumed to be a ramp to step input which lasts for a time corresponding to the response time of the powertrain and provides an acceleration compatible with the level of performance of the present vehicle. The model checks for the occurrence of wheel spin. Two different levels of acceleration are exercised with values of 0.3 g and 0.5 g corresponding to driving forces of approximately 2300 and 4000 N, for each wheel respectively. The ramp to step time adopted was 0.5 s, considering the time lags involved in the powertrain response. The vehicle initial velocity was 2.0 m/s and final velocities were 17.5 and 25 m/s, respectively, when the driving force was then made constant, just in order to overcome aerodynamic losses.

The quantities illustrated in the figures C.33 and C.34 are tractive effort for a single wheel, forward velocity, sprung mass pitch angle and front and rear suspension arm angular displacement.

The deceleration manoeuvres consisted of a ramp to step inputs in the total braking force which caused decelerations of 0.4g and 0.8 g, over a time of 0.5 s which accounted for the delays in the braking system. This force was maintained until the vehicle reached a speed of 1.0 m/s at which instant the braking force was stepped down to zero. The vehicle was assumed to have initial velocities of 10 and 20 m/s. The braking effort was distributed such that it followed a linear relation between front and rear, based on the braking diagram derived for this vehicle and illustrated in figure 5.1.

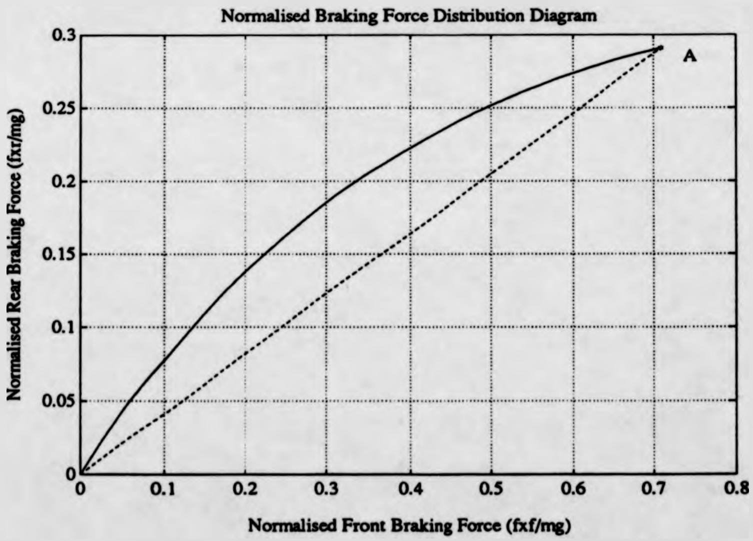

Figure 5.1: Normalised braking diagram

The choice of the inclination of the straight line was such that it was the one closest to the optimum braking effort distribution and which always locked the front wheels under any braking condition. This situation is normally adopted by vehicle manufacturers. In this case, the braking force distribution was 70/30 front/rear. The same quantities mentioned in the previous manoeuvres are illustrated in figures C.35 and C.36. Just as an exercise, the effects of locking the front wheels for a 1.0 g deceleration and locking the rear wheels if a 60/40 distribution is chosen, are shown in figures C.37 and C.38.

5.2.1.3 Road Vertical Disturbances

The road vertical inputs consist of crossing obstacles of different types such that all relevant aspects of vehicle motion are exercised. For this purpose, out-of-phase triangular bumps are transversed, together with terminated ramp and sinusoidal road profile inputs. The simulation model accepts a description of these input profiles in the space domain and calculates the equivalent time domain input function to the vehicle model. Input parameters are road height and obstacle length. The model also checks for the loss of contact between the tyre and the ground, since the road cannot apply any force to the wheels without contact. If such a wheel is a driven one, it also checks for the occurrence of wheel spin and halts the simulation. The choice of the magnitude of the input parameters is based on realistic circumstances which a vehicle of the type modelled may encounter in use and also close to values presented in the literature [129, 150].

The quantities of motion which undergo significant variation and are illustrated in the results include sprung mass vertical displacement and acceleration and pitch and roll angles. For the unsprung masses vertical displacement of the wheel's center of gravity, tyre force and suspension travel are included. As an aid to the analysis, the road profile is superposed in some of the simulation results. The results of these simulation are presented in appendix C, section C.1.3. The value of the other quantities of motion are negligible and therefore, omitted from the results.

For the out-of-phase triangular bump, the forward velocities adopted for the vehicle are 5, 10 and 15 m/s, transversing bumps 0.01, 0.05 and 0.1 m high and 1.0 m long. The difference between left and right hand sides was 0.5 m, with the left bump occurring first. The results are presented in figures C.39 to C.56.

For the terminated ramp, the same values of speed and length are adopted but only heights of 0.05 and 0.1 m are considered. The results are depicted in figures C.57 to C.62 for the terminated ramp input.

For the sinusoidal road profile, transient manoeuvres are illustrated for forward velocities of 5 and 15 m/s, with road profile 0.05 m high, 5 m long and with a left/right difference of 0.5 m, corresponding to frequencies of 1 and 3 Hz. Figures C.63 and C.64 illustrate the results for this simulation. Also illustrated in figures C.65 and C.66 are simulations for a speed of 5 m/s and bump 0.005 m high and 1.0 m long; and a speed of 12.5 m/s and height 0.007 m, respectively.

5.2.1.4 Aerodynamic Disturbances

The vehicle response to aerodynamic inputs is analysed through the response of the model to a lateral wind gust, which represents real life situations such as driving over bridges in open gaps, performing an overtaking manoeuvre, passing the ends of wind protection barriers or an opening in roadside vegetation. The wind model is similar to a suggested one in the literature [109] and it represents equivalent effects and the choice of the numerical values for the simulation parameters were based on engineering judgement. The side wind gust model is represented by a square pulse of amplitudes of approximately 3.5, 7 and 10.5 m/s which correspond to initial aerodynamic sideslip angles of 5^o, 10^o and 15^o for a vehicle speed of 40 m/s. The duration of the pulse is 2 s, which corresponds to an opening of 80 m, for the adopted vehicle speed.

The quantities analysed include sprung mass yaw rate, lateral velocity and roll angle. These results are presented in appendix C, section C.1.4. The variation of the other quantities of motion were found to be negligible. It is interesting to notice that because aerodynamic lift is included, the loss of available adhesion due to decrease of the tyre normal load is automatically considered.

5.2.2 Steady-State Manoeuvres

The primary task of the steady-state manoeuvre simulations is to obtain static and/or steady-state response information in order to assess the linearity of the model for the various quantities of motion involved in response to the already mentioned inputs and disturbances. Another important purpose was to determine directional stability parameters, related to the specific vehicle being modelled. In this case, both the linear and nonlinear tyre models, with and without time lag are used. Simulation runs which are executed include cornering with constant speed and different steering angles, cornering with constant steering angle and various forward velocities and transversing sinusoidal road profiles of different heights and lengths. In all cases the model was exercised for a long enough period of time which enabled the variables of motion to reach steady-state conditions, when their values were then stored. Steady-state simulation results are presented in appendix C, section C.2.

5.2.2.1 Cornering at Different Speeds

In this simulation the vehicle is considered to be driving at various constant speeds when the vehicle is steered by a parabola to step steering input with a parabola time of 0.1 s and a step value of 20^o angle of the front wheels. The speeds considered are 5, 10, 15, 20, 30 and 40 m/s and are intended to cover a realistic range of operating conditions of the vehicle. Results are presented in section C.2.1 of appendix C.

Due to losses in cornering, a driving force necessary to maintain constant speed is applied similar to the one previously described in the transient cornering manoeuvre. Three tyre models will be used, the linear one without the first order time lag in the lateral force build-up and the nonlinear one with and also without the first order time lag.

Besides the steady-state characteristics of the variables of motion and the relationship amongst them, these simulations also provide information about the vehicle directional control characteristics and handling stability, such as static margin, understeer or oversteer, etc. It's worth noting that with the linear tyre model, no load transfer effect is taken into consideration, and therefore the differences in lateral force during cornering is solely due to the differences in slip angle. This effect becomes quite accentuated in hard cornering, and it is better illustrated with the use of the nonlinear tyre model.

The variables which are analysed as functions of forward velocity include pitch and roll angles, lateral velocity, yaw rate, sideslip angle and lateral acceleration of the sprung mass and slip angle of the wheels. The results in figures C.71 and C.72 illustrate the model's behaviour for the linear tyre, whilst figures C.73 and C.74 represent the model's response with the nonlinear tyre without time lag and figures C.75 and C.76 with time lag.

5.2.2.2 Cornering with Different Steering Angles

This simulation is similar to the one previously described, but in this case the forward velocity is kept constant and larger values of lateral acceleration are obtained by performing the cornering manoeuvres at different values of steering angles. The input function is also a parabola to step with values of $10°, 20°, 30°, 40°, 50°$, and $60°$. The forward velocity is kept constant at a value of 20 m/s. Section C.2.2 of appendix C illustrates these results. Plots for pitch and roll angles, yaw rate, lateral and forward velocities, lateral acceleration, and vehicle and wheel sideslip angles are obtained as a function of the steering angle and lateral acceleration. These results are presented as a function of the steering wheel angle in figures C.77 to C.82 and of lateral acceleration in figures C.83 to C.88. Regarding vehicle directional characteristics, information about understeer or oversteer is also obtained, with the additional information about the onset of instability for the case of a vehicle with oversteer, if this is the case.

In both simulation cases and for all tyre models the iminence of sliding or loss of lateral stability in any of the wheels is checked by the comparison of the demanded tyre force in relation to the maximum available adhesion for the road condition assumed in any simulation. The simulation in these cases is then halted. In all simulations a dry road condition is considered, with a friction coefficient of 1.0.

Another experiment concerning steady-state manoeuvres which is widely used in practice, is the constant radius of turn manoeuvre, because of the ease of carrying out such road tests. In these tests the vehicle is driven at various constant forward velocities around a path with a constant radius of turn. Normal practice is to use a radius of turn of 42.5 m amongst many mentioned in the literature [146]. Because the driver is included in real tests, a path following control such as this is feasible. For the purposes of a simulation which does not include the driver nor can be solved for the steady-state conditions analytically, it is not possible to know what is the corresponding steering angle necessary to result this radius of turn, at a given velocity.

5.2.2.3 Crossing of Obstacles of Different Heights

In order to investigate the nonlinearities of the quantities of motion in relation to road disturbances, their behaviour are analysed for the vehicle driving over a sinusoidal road profile with amplitudes of 0.005, 0,01 and 0,025 m and length of 1 and 10 m. The vehicle forward velocities are 5, 10, 12.5 and 15 m/s. The resulting frequencies are 0.5, 1.0, 1.25, 1.5, 5.0, 10, 12.5, and 15 Hz. In-phase and out-of-phase right and left road inputs are considered. For the out-of-phase obstacles, their distance is such that a time difference of 0.1 s resulted for all speeds (frequencies). These quantities are chosen so that the entire range of relevant frequencies for the variables of motion of interest are covered by the vehicle driving in these conditions. The frequency response results for these simulations are depicted in appendix C, section C.2.3.

A visual analysis of the steady state response of the quantities of interest, to assess the nonlinearities, and a numerical analysis, to determine the influence of the amplitude of excitation, is carried out and represented in a 'pseudo' frequency response form. The amplitude ratio and phase angle in relation to front road input is calculated for each variable at each amplitude of excitation. In multi-axe vehicle vibration studies it is common practice to adopt one single time function as reference input and generate the other inputs to the wheels by applying different time delays [197, 95].

Although the values of the output quantities in the simulation were produced by the road input to the four wheels, the use of a frequency response between these quantities and the front wheel road input only, is justified on the basis that this is done for comparative purposes of the effect of the amplitude of excitation and all the results were generated on the same basis. The variables chosen as output were such that they included the significant modes and frequencies of interest for the so called ride motion. For this purpose, vehicle vertical displacement and acceleration and pitch and roll angles were the variables chosen, related to the sprung mass movement, and front and rear suspension forces and tyre deflections were chosen as those related to the unsprung mass movement.

5.3 Analysis of Simulation Results

It has been a traditional practice in vehicle dynamics studies [62, 146, 255, 301] to divide the quantities of motion in groups according to which aspects of vehicle motion they are related to, in order to allow the use of more simplified models which could take the most relevant aspects of these motions into account. As a result of this, the areas of performance, handling and ride studies have been established. Although the purpose of the present work is to look at the automotive vehicle as a total system, these divisions are well consolidated by their use in practice and also they can be very helpful in understanding overall vehicle behaviour and the relationship amongst the innumerable variables of motion.

The area of performance studies or longitudinal dynamics, is concerned with the behaviour of the vehicle when driving straight ahead or at very small lateral accelerations. Quantities of motion related to this area are vehicle forward velocity and acceleration, pitch angle, suspension deflections and wheel rotational dynamics. Some related quantities include drivetrain and braking system characteristics. Typical parameters used to specify vehicle's performance are maximum longitudinal acceleration, climbing ability, top speed, braking distance, etc.

Vehicle handling or stability refers to a vehicle's behaviour in response to the application of lateral forces and yawing moments caused by steering inputs and vehicle motion. It is also concerned with vehicle lateral stability when driving straight ahead. The degrees of freedom normally associated with this area are lateral motion and yaw and roll rotations. In the majority of the cases, forward velocity is adopted as a major stability parameter, but it is not considered a variable of motion. In this simulation model, forward velocity is actually a variable of motion, but in some manoeuvres it is assumed to be constant. This assumption of constant speed is a reasonable one in many cases, since normal steering manoeuvres do take place at nearly constant speed. Parameters used to specify a vehicle's handling characteristics include static margin, under/oversteer gradient, Ackermann angle, etc.

Ride studies are concerned with a vehicle's behaviour as it transverses uneven road surfaces. The degrees of freedom most relevant in this area are vertical displacement and roll and pitch angles of the sprung mass and displacements of the unsprung masses. Parameters used in evaluating a vehicle's ride performance embraces weighted vertical acceleration, suspension travel space, tyre deflections, etc. Suspension design concepts and its parameters are some of the related topics.

Under certain operating conditions, these areas can be studied separately, as has often been the case. However, the coupling amongst these variables must be taken into consideration under other conditions, especially limit manoeuvres. The objectives of this simulation analysis is to look at these couplings, their magnitude and nature. For didactic purposes, the analyses will be divided into performance, handling and ride characteristics of the vehicle's behaviour. It attempts to identify the couplings which are apparent in the model and the effects and implications they have for overall vehicle behaviour.

5.3.1 Definition of a Driving Envelope

The concept of a driving envelope has been originated in aeronautical research and it refers to the operating range of an airplane in terms of its speed, normally expressed in mach number, and the altitude range which it covers. A similar concept called driving range has also been used in automotive engineering [146] and it usually refers to the range of longitudinal and lateral accelerations and forward velocity in which a car can operate, given a certain road condition.

In the present case, the concept of driving envelope are used in a different manner. The idea is to define an operating range, or set of operating ranges, in which a linearised, or set of interrelated linearised conditions, would still be valid, and therefore amenable for control design and analysis purposes. This analysis is based on the previous simulation results.

The quantities which are used to define the envelope are longitudinal, lateral and vertical quantities. Its concept is diagrammatically illustrated in figure 5.2. The figure represents the limits of linear validity of the models in the X, Y and Z directions.

The variation in a vehicle's behaviour could be analysed in two different ways. One is related to the influence of the system's *parameters* and the other is related to the system's *variables of motion*. In this section the analysis is based on the influence of the *variables of motion*. The implication of some system parameters in vehicle behaviour are also included, as an aid to understanding the analysis, but it is not the main objective of this analysis, since the numbers of parameters

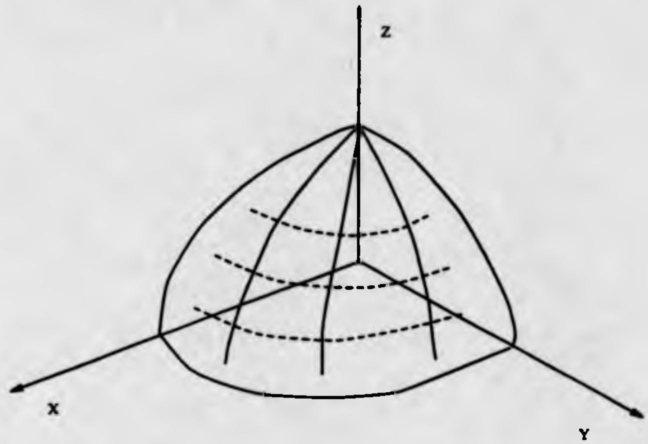

Figure 5.2: Concept of driving envelope

which influence vehicle motion are far too large, and should be done with a specific purpose in mind, not in a study which aims at studying the generic characteristics of vehicle motion.

In our case, because of the extensive simulation runs, and also based on previous experience and published work in vehicle system dynamics, the variables intended to be more closely analysed could be chosen beforehand. This is contrary to 'normal' ways of discussing system models because, usually, steady-state relationships are seen as a particular case of the generic dynamic relations. In such case, it is assumed that the derivatives are zero and static values are analysed. Because in the model of this thesis, explicit nonlinear equations of motion are not available, steady-state values cannot be directly obtained from them but from lengthy simulation runs. Therefore, it was more natural and easier to perform the transient analysis first.

The transient manoeuvre simulations are analysed in comparison to the expected behaviour of a linear system resulting from an eigenvalue-eigenvector analysis around the corresponding operating points. Also, in order to allow a comparison between the various linear system representation, that is, eigenvalues, eigenvectors and system matrices, around different operating points, a certain driving condition will be taken as a *reference* configuration.

The steady-state manoeuvres will be used to obtain the static and/or gain relationship between the various variables of motion and the inputs or disturbances. In this case it will be possible to analyse the strength of the coupling between these variables, and how linear this coupling is. This analysis, together with the previous one, allow the decision of a range of operation in which a linear representation of the system is still valid, and further studies, such as model order reduction, controller analysis and design for active systems, etc. is possible.

5.3.2 Eigenvalue Analysis (transient)

The analysis of the transient motion of the previous simulation runs is performed based on an eigenvalue/eigenvector analysis of the matrices of the linearised system, determined at certain operating conditions. As described earlier, motion quantities are divided according to which modes they are associated with and to which they contribute more significantly. In this sense, some quantities of motion are either taken into consideration or neglected, according to the circumstances of the simulation run.

A brief discussion about obtaining the linearised model is carried out in order to provide a minimum background to the forthcoming analysis. Later in chapter 6, it will be carried out in more detail.

The nonlinear vehicle model can be represented by

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}, t) \tag{5.1}$$

where $\boldsymbol{x}$ is the state vector and $\boldsymbol{u}$ the input vector, which includes driver's inputs and external disturbances. ACSL through numerical perturbation of the states provides an approximation of the system's Jacobian at an operating point, $\boldsymbol{x}_o$ of the state vector. In this case, it can be written for the autonomous linearised system that,

$$\dot{\boldsymbol{x}} = \boldsymbol{J}\boldsymbol{x} \tag{5.2}$$

where the ij^{th} element of $\boldsymbol{J}$ is given by

$$(J)_{ij} = \left.\frac{\partial f_i}{\partial x_j}\right|_{\boldsymbol{x}_o} \tag{5.3}$$

ACSL can also determine the linear representation for the nonautonomous system. With the definition of the control variables, $\boldsymbol{u}$, and the output quantities,

$\boldsymbol{y}$, a perturbation analysis is performed as before, on an augmented 'state' vector returning the system's matrices in the form

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{u} \tag{5.4}$$

$$\boldsymbol{y} = \boldsymbol{C}\boldsymbol{x} + \boldsymbol{D}\boldsymbol{u} \tag{5.5}$$

where $\boldsymbol{A}$,$\boldsymbol{B}$,$\boldsymbol{C}$,$\boldsymbol{D}$ are the matrices of the state space representation of the nonlinear system described by equation 5.1. These matrices are discussed in the next chapter.

For the present simulation analysis, the autonomous system representation is used and an eigenvalue/eigenvector analysis is performed on $\boldsymbol{J}$ at various operating points, according to the manoeuvre which has been executed. As has been mentioned before, for comparative reasons and to facilitate the analysis, an operating condition in which the vehicle is driving in a straight line, over a flat road surface, at a speed of 20 m/s and with a linear tyre model which does not include the first order time lag in the lateral force build-up process, is taken as the reference configuration. Under these circumstances, the eigenanalysis of the linearised representation of the system results in the values of natural frequencies, damping factors and modes presented in table 5.1

Modes	Frequency		Damping
	Hz	rd/s	Factor
Front-end bounce	1.1	6.88	0.206
Rear-end bounce	1.35	8.46	0.266
Roll on springs	1.81	11.38	0.267
Yaw rate/lateral velocity	1.29	8.13	0.834
In-phase front wheel hop	12.02	75.46	0.200
Out-of-phase front wheel hop	12.50	78.52	0.195
In-phase rear wheel hop	13.00	81.69	0.304
Out-of-phase rear wheel hop	13.05	81.95	0.303

Table 5.1: Eigenanalysis for reference configuration

The numerical values of the system's Jacobian is given in the appendix D, section D.4. Originally $\boldsymbol{J}$ is of dimension 36x36, but because some quantities are defined to be either prescribed motion in SD/FAST or result in zero eigenvalue in $\boldsymbol{J}$, they have been eliminated from the state variable vector. The resulting matrix in this case is then 18x18.

Because this is still a large matrix and a significant number of them are generated for all operating conditions, 3d representations can be obtained using MATLAB, where rows are represented in one axis, columns in another and the value stored in each element in a third one. In order to look more closely at a parameters' influence on the elements of $\boldsymbol{J}$, specific sections of these matrices are taken separately, when necessary. These 3d plots allow a qualitative analysis of such matrices and their sections before proceeding to a more detailed analysis based on the numerical values of the elements themselves. For illustration purposes, the 3d view of the full Jacobian of the reference configuration, and sections with the acceleration, spring and damping coefficients of the derivatives of the generalised speeds are depicted in figure 5.3.

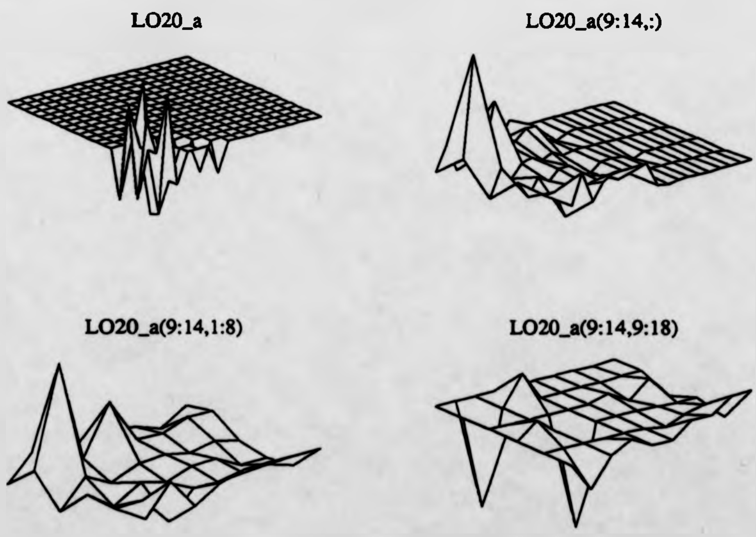

Figure 5.3: Qualitative 3D plots of Jacobian

The physical meaning of the generalised coordinates and velocities as given in the notation section, together with the eigenanalysis results of table 5.1, play an important role in the analysis which follows. Its division is similar to that of the previous section.

5.3.2.1 Steering Input

Steering inputs are related to a vehicle's handling behaviour which is concerned with the ability of the vehicle to change directions at the driver's request and to maintain directional stability at various operating conditions.

The simulation results for the transient behaviour to steering inputs are analysed in comparison to the expected behaviour of a linearised model. The characteristics of such linear system have been obtained from an eigenanalysis. Also a qualitative physical discussion about the behaviour of the most important variables of motion related to this mode is carried out. The vehicle's behaviour is discussed as a function of the three steering angles, $10^o, 45^o$, and 90^o, and the three forward velocities, 10, 20 and 30 m/s, as mentioned before in section 5.2.1.1.

The variables of motion which seem to undergo significant variation during cornering manoeuvres include sprung mass yaw rate, lateral velocity or sideslip angle, lateral acceleration and roll angle, and unsprung mass slip angle. Related quantities which are important for the motion, and/or stability purposes, are the tyre forces. These quantities are illustrated in figures C.1 to C.32 of section C.1.1 of appendix C, for the above operating conditions and for both types of tyre models simulated. Because the eigenanalysis at these operating points results in the full set of frequencies, damping factors and modes, the influence of these quantities in the ride and performance modes is also discussed.

With the knowledge of the position of the variables of motion related to the handling modes in the state vector, and using the 3d plots of the system's matrices, certain regions of the system's matrices are identified as being related to these modes. A physical interpretation is given to them and the effects of steering angle and forward velocity are qualitatively analysed.

Vehicle directional stability characteristics are related to the eigenvalues of the characteristic equation which is associated with the handling modes. However, a number of parameters which specify a vehicle's handling behaviour, and therefore stability, have been derived from simple models and are well established and understood in the vehicle dynamics community [62, 146, 255, 301]. For handling studies, three *parameters* play a central role in defining vehicle's behaviour:

- front cornering stiffness (gradient);
- rear cornering stiffness (gradient) and
- location of the vehicle's centre of gravity in relation to the wheelbase.

The value of these parameters determine a vehicle's transient and steady-state characteristics, as well as directional stability properties. The handling mode characteristics of a vehicle are shown to be those most susceptible to variations in operating conditions. The *quantity of motion* which seems to affect the handling characteristics most significantly is the vehicle's *forward velocity.*

The eigenvalue analysis of the Jacobian of the nonlinear vehicle model shows that for the vehicle driving in a straight line the handling modes, yaw rate and lateral velocity (or sideslip angle, because forward velocity is assumed to be constant) show the following behaviour with increased speed:

- decrease in natural frequency;
- decrease in damping factor.

The presence of the first order time lag in the lateral force also affects significantly the frequencies and damping factors. For values of forward velocities of 5 to 40 m/s the natural frequencies and damping factors, of the present vehicle model, are presented in table 5.2, for the linear tyre model, without time lag, (Linear 1), with time lag (Linear 2), nonlinear without time lag (Nonlinear 1), and nonlinear with time lag (Nonlinear 2). The presence of the time lag in the tyre model adds four extra states, two of which couple with the 'normal' handling modes. For the tyre models which include the time lag, the eigenvalues of these states are also presented in the table. The system matrices for the speed range of 5 to 40 m/s and the full set of eigenvalues for all tyre models are given in appendix E.

Values of speed up to 100 m/s have been used for the linear tyre model without time lag and the nonlinear one, with time lag. As can be seen from the above table, as the speed increases the effect of time lag decreases, because it is inversely

	Linear 1		Linear 2		Nonlinear 1		Nonlinear 2	
Speed	ω_n	ξ	ω_n	ξ	ω_n	ξ	ω_n	ξ
m/s	rd/s	–	rd/s	–	rd/s	–	rd/s	–
5.0	31.11	1.0	23.38	0.31	28.96	1.0	22.73	0.32
	22.57	1.0	19.39	0.48	23.62	1.0	19.80	0.46
			16.67	1.0 (2)			16.67	1.0 (2)
10.0	13.99	0.97	22.30	0.87	13.72	0.96	22.47	0.88
			22.31	0.65			20.36	0.65
			33.33	1.0 (2)			33.33	1.0 (2)
15.0	9.99	0.90	13.11	0.86	9.72	0.91	12.56	0.83
			38.76	0.99			39.57	0.99
			50.00	1.0 (2)			50.00	1.0 (2)
20.0	8.13	0.83	9.32	0.81	7.85	0.85	9.18	0.75
			58.88	1.0			59.71	1.0
			59.40	1.0			59.71	1.0
			66.67	1.0 (2)			66.67	1.0 (2)
40.0	5.80	0.59	6.10	0.57	5.47	0.61	6.30	0.49
			129.62	1.0			129.88	1.0
			130.16	1.0			130.56	1.0
			133.33	1.0 (2)			133.33	1.0 (2)
100.0	5.01	0.29					4.92	0.28
							333.33	1.0 (4)

Table 5.2: Handling mode frequencies and damping factors as function of forward velocity for 4 tyre models

proportional to it, and at high speeds both models show similar behaviour. The effect of forward velocity on the remaining modes are negligible in the operating range of speeds for a vehicle of this type, when the vehicle is driving in a straight line.

The behaviour of the vehicle with the linear tyre without time lag is analysed first. From the transient response of the low speed manoeuvres (10 m/s), figures C.1 to C.5, it is possible to observe that both lateral velocity and and yaw rate present similar behaviour, with equivalent response times. However, they are slightly different, and as forward velocity is increased to medium (20 m/s), figures C.7 to C.12, and higher values (30 m/s), figures C.13 to C.16, the lateral velocity response, although still similar in shape, presents a change of sign in steady-state gain and a initial transient behaviour typical of nonminimum phase systems. This fact is confirmed later on, as the various transfer functions for the transfer function matrix (TFM) of the system are determined, and the steady-state manoeuvres are carried out. The physical interpretation of this fact is as follows.

In steady-state turning for a given steering angle

$$V_x = u_4.R_t \tag{5.6}$$

where V_x is the vehicle's forward velocity, u_4 is the yaw rate and R_t is the radius of turn. If the vehicle has constant speed, the product $u_4.R_t$ is also constant. For small values of V_x this product should also be small. The value of the radius of turn is inversely related to the value of the steering angle input. The larger the steering angle, the smaller the radius of turn. The yaw motion is a consequence, mostly, of the yaw torque resultant from the lateral tyre forces applied to the wheels, at the contact point between the wheels and the road. The lateral tyre forces are mainly a function of the slip angle of the wheel, which is a result of steering angle and vehicle 'positioning'. The slip angle of the wheels can be described either by the wheel's lateral and forward velocities or it can be more easily understood through the sideslip angle, β, as described in chapter 4 and illustrated in figure 4.12.

In the case of a nonrotating wheel, it can be written in a simplistic way that

$$F_{ty} = f(\alpha_i, F_{tz}) \tag{5.7}$$

where F_{ty} and F_{tz} are the lateral and vertical tyre forces, respectively. Because of vehicle inertia, initially almost all the lateral force generated at the tyres is due to the steering angle input. If the yaw torque produced by this force is larger than that necessary to produce the required yaw rate in steady-state to satisfy equation 5.6, then the vehicle should be positioned in a way to counteract this torque. For these reasons, at low speeds a condition in which the vehicle is sideslipping positively, therefore decreasing the front wheel's slip angle and, consequently, the lateral force at the steered wheels and the yaw torque which they cause, results. This situation is illustrated in figures C.2 to C.6 for all three steering angle inputs.

For higher values of V_x, the lateral force produced by the steering angle is not sufficient to attain the equilibrium condition required by the steering input. Therefore, an additional torque is necessary and it is obtained by the vehicle being positioned with a negative sideslip, consequently, increasing the wheel's slip angle and the yaw torque, balancing the yaw motion at the constant yaw rate necessary to satisfy equation 5.6. This situation for medium and high V_x is illustrated in figures C.7 to C.16.

From these figures it can also be seen that the rear slip angles are a result of vehicle motion only, thus it does not present the 'spiky' behaviour of the front ones at low speed, due to steering inputs. It's interesting to note that the 'shape' of the transient response is the same at each forward velocity, irrespective of the value of the steering angle. The steady-state value of the output quantities are obviously different, because different steering angles represent different path demands.

From the simulations, it can be observed that the lateral tyre forces, for the linear tyres, are nearly identical for the inside and outside wheels, for low speed and small steering angle manoeuvre for the whole duration of the simulation, because they are a function of the slip angle only. As the steering angle is increased, the vehicle's sideslipping causes the outside and inside slip angles to differ, but still only by a small amount. As the velocity is increased, for the same steering angle, the difference in lateral force remain unchanged.

An interesting point to be observed is the 'shape' of the transient behaviour of the slip angle and lateral force responses, which confirms the previous explanation about the vehicle's behaviour in cornering. Initially, the steering angle input predominates, which is confirmed by their shapes. Figures C.2, C.4 and C.6 illustrate this point. Later, because steering angle is then constant, their variation is mainly due to the sideslip angle. Because the wheel's sideslip angle is close to the vehicle's sideslip angle for slow yaw rates manoeuvres, the 'inverted' shape of the lateral velocity response can be seen for the later part of the manoeuvre in both slip angle and lateral force functions.

The characteristics of the roll motion can also be analysed at these cornering manoeuvres, and the results compared with those of the eigenanalysis of the linearised system. The eigenanalysis shows that as forward velocity is increased the roll mode frequency and damping factor changes are negligible. However, if one looks at the transient behaviour of the roll motion for various speeds, it seems that it undergoes quite a large change. This is not the case, though. These differences in transient roll behaviour are explained by the way the tyre lateral forces are generated during each manoeuvre.

For low values of V_x, the sudden peak in lateral force induces the roll mode vibration much more strongly and the behaviour seen in figures C.2, C.4 and C.6 confirms this point. On the other hand, for higher values of V_x the greater smoothness of the transient behaviour of the lateral force induces much less roll vibration, and it behaves as if it is slowly approaching the steady-state value it should present at that cornering manoeuvre, although a small superposed transient roll vibration is still perceptible, figures C.7 to C.16.

As forward velocity is increased, because the linear model does not take into account the decrease of cornering stiffness with decreasing normal load, the required tyre force, which is the lateral component for the front tyres, and a combination of longitudinal and lateral forces for the rear tyres, overcomes the total available adhesion. In the case of a 45° steering angle input at 30 m/s, the inside rear wheel looses contact, as illustrated in figure C.15 and C.16, and for a 75° steering input at 20 m/s, the inside front does so, as can be seen in figures C.11 and C.12.

The limit values of steering angle for the medium and high speeds are 70° and 42.5°, respectively, for the linear tyre model, and this is due to the fact of at least one wheel loosing contact. Because the description of a skidding vehicle is not easily dealt with, and in the present model it has not been fully implemented, the simulation is halted when it occurs for at least one wheel. An attempt has been made at implementing it using the RSW command in ACSL, but because of its on-off nature, it tends to present oscillatory behaviour around the point of occurrence of skidding. This situation can be more appropriately dealt with using the SCHEDULE facility in ACSL.

The behaviour of the transition from the situation of a wheel in 'full contact' with the road, to one in total sliding, is not a trivial problem and it deserves attention in future developments of the present research. It's worth noting that it is a complex phenomena, in the sense that mathematically a rolling wheel in a plane is a system with 3 degrees of freedom and one nonholonomic (motion) constraint equation. When the wheel starts sliding, it becomes a 4 degrees of freedom system, with no constraint equation, therefore becoming a system with a different mathematical structure.

The discussion of the vehicle's behaviour with the nonlinear tyre model follows the previous one, highlighting its effects when it becomes more apparent. The influence of the nonlinear tyre model is stronger at medium and larger steering angles, for any speed, and at medium and larger speeds, for all steering angles. The most notable effect being the right/left load transfer. The basic interpretation of the vehicle's behaviour remains unchanged, but features related to the tyre model become apparent.

The influence of speed on the variables of motion remains the same, but the tyre forces undergo large changes. Although the shape of the lateral force remains the same for small steering angles (10^o) and low speeds ($10\ m/s$), it varies dramatically for medium and large steering angles (45^o and 90^o) and speeds (20 and 30 m/s). It can be noticed in figure C.22, that even at slow speeds ($10\ m/s$), for a steering angle of 90^o the value of the initial peak of the lateral force is higher for the outer front wheel (from 2500 N to 3000 N) and lower for the inner one (from 2500 N to 2000 N), when compared to the linear tyre model values, due only to load transfer effects.

An interesting aspect of the nonlinear tyre behaviour was an instability presented in the lateral force generation when the lateral acceleration achieved levels of 4.5 - 5.0 m/s In this situation, the front tyre forces largely oscillated the inner tyre force dropping to nearly zero and the outer front tyre force increasing significantly. Figures C.27 and C.28 illustrate this situation for the 20 m/s and 60^o steering angle manoeuvre and figures C.31 and C.32 depict this condition for the

30 m/s and 45^o manoeuvre.

The reasons for this behaviour are unclear (or unknown) at the moment and technical discussions have been carried out with prof. Lugner who claimed that interpolated tyre data tend to be numerically sensitive. Trying various steps sizes and integration algorithms have not solved the problem, though. Later talks with Dr. Williams from Jaguar Cars Ltd. who have provided the data, seems that he himself has adopted different functions for the description of the tyre data. One major drawback of polynomial interpolations is that they cannot normally be used outside the interpolated range because of abrupt variations, specially if they are of high order. However, in these cases the variables did not exceed the valid limits for the interpolation. Possible causes could be approximations made in the representation of the tyre forces, such as the neglecting of camber angle. They are assumed to act in the wheel frame (since camber is ignored), but because of suspension movement, the component of this force in the ground frame vertical direction could be such that it decreases the suspension vertical stiffnesses, causing the equivalent bounce and wheel hop modes to become unstables. Another possibility is that it could be a phenomenon similar to self-excited vibrations in which there is a feedback in the force generation process. It hasn't been possible to investigate these factors in depth, since the nonlinear tyre model was made available only recently. Also, these explanations were found only later on and they were not checked or implemented yet, therefore, they require further investigation.

However, apart from not allowing to explore the limits of the nonlinear model, the model is still adequate to show the effects of the nonlinearity of the tyre model in vehicle behaviour.

The behaviour of the vehicle before the onset of instability and with the nonlinear tyre, is qualitatively similar to that with the linear one and there are no significant differences in most variables of motion. Transient responses present similar behaviour, for most variables but yaw rate and lateral velocity present a faster response time and a less damped response. Steady-state values are also slightly different and this aspect is discussed in section 5.3.3. Although further investigation is still needed, the nonlinear tyre model provides a good insight into the amount of influence normal load and pneumatic trail considerations have on system behaviour. The most obvious one been the difference in tyre lateral force due to load transfer effects, as it is illustrated in figures C.17 to C.32.

In an attempt to find an operating range which would support the definition of a driving envelope, it was noted that the factor which influences the linearised model's behaviour more significantly is load transfer. The quantity of motion which is linked to lateral load transfer, and therefore makes the model with linear tyres loose contact and the model with the nonlinear one go unstable (in the case of the present model) is *lateral acceleration.*

From analysing all situations in which loss of adhesion for the linear tyre, or instability for the nonlinear one occurred, it can be stated that for values of lateral acceleration greater than 5.0 m/s^2, the model is inadequate in its present state. For lateral accelerations between 2.5 to 4.0 m/s^2, the linear tyre model is not totally adequate, due to load transfer effects. However, one must bear in mind that the concept and the amount of what is considered 'adequate' is related to each particular situation.

Two points to be emphasised, which were not possible to pursue further in this research, due to lack of experimental evidence. The first one is concerned with the model itself. It seems very unlikely, from the author's point of view, that a vehicle such as the one modelled in this study, would have a limit on lateral performance of only 5.0 m/s^2 of lateral acceleration. It's seems more logical to suppose that this limitation is due to model characteristics than to actual vehicle behaviour. Second, it is felt that a longitudinally slipping tyre model, in which there's a coupling between slip angle and longitudinal slip, and which affects cornering stiffnesses and therefore lateral stability characteristics, would allow the model to operate in a wider range of lateral acceleration, more in line with real vehicle behaviour.

5.3.2.2 Traction/Braking Input

Traction and braking inputs are associated with a vehicle's performance. In the case of the present model, studies are affected and limited due to the lack of engine, transmission and braking system models, since the dynamics of these subsystems determine vehicle motion. Nevertheless, a number of interesting features concerning overall vehicle behaviour and the amount of interaction between the performance 'mode' and the others can still be illustrated. Also, the use of very simplified models for the driving and braking forces has the main purpose of illustrating the model's capability to respond to these types of input. Obviously, it also illustrates the transient motion of performance quantities for a full vehicle model. In this case, for instance, the effect of an active suspension design on pitch angle, tyre normal load variation and suspension travel during either hard acceleration or braking, which is important for vehicle stability, can be analysed. Besides, future developments of the vehicle model can easily incorporate propulsion system models, because of its modular nature.

For all acceleration and deceleration manoeuvres a dry road condition with adhesion coefficient equal to 1.0 was considered. Although this value of μ is achieved only under special conditions (worn tyres, dry, good road surface) it was adopted for simplicity reasons. From the results of the acceleration simulation depicted in figures C.33 and C.34, it can be seen that with accelerations of up to 0.5 g, with no lateral manoeuvre being executed, the vehicle doesn't present any behaviour which could lead to the loss of lateral stability if driven in a dry tarmac surface. However, a number of important factors related to vehicle lateral stability can be observed in these simulations. For example, the decrease of vertical front tyre force indicates a decrease in the front wheel cornering coefficient and a tendency towards increased understeer in vehicle's behaviour.

For the 0.3 g acceleration, only hard cornering manoeuvres which generate large cornering forces, and therefore decreases the amount of adhesion available for traction (or vice-versa) would endanger vehicle stability. Obviously, this situation is deteriorated in case of a low adhesion surface condition. However, for the 0.5 g manoeuvre, small to medium steering angles could cause the car to loose lateral stability due to the decrease in cornering ability and use of available adhesion for traction, even on good road conditions. The specific situations and conditions in which these would occur precisely are numerous and therefore almost impossible to enumerate, but one important thing is that the present model does take them into account and is adequate to describe their occurrence. This is a simple example of the utility of a full vehicle model which illustrates the interactions that occur amongst the various modes in vehicle behaviour.

It can also be seen from the simulation results that the vehicle accelerated from 2 to 25 m/s in 6.3 s in the 0.5 g manoeuvre, and it pitched at an angle of -0.02 rd which corresponds to a 0.026 m front end lift at a wheelbase distance from the cg. The vertical front tyre force decreased from approximately 4700 N to 4000 N, with a loss of adhesion of 700 N for a dry road condition. The vertical rear force increased from 4000 to 4700 N, but it needed almost 4000 N for the driving force, leaving approximately 2500 N for a cornering manoeuvre, assuming a circle model for the interaction between longitudinal and lateral forces and adhesion, for a dry road condition. Also, the simulation illustrates the transient motion of the combined front-end and rear-end bounce, the velocities of 2 m/s at the beginning of the acceleration and 25 m/s at the end of it. It seems that there's very little influence on these modes of the forward velocity. This conclusion is supported by other simulation results and the eigenvalue analysis performed at different speeds.

One important observation that can be made concerning the deceleration manoeuvre is that the theoretical maximum deceleration calculated, based on the braking diagram of figure 5.1, cannot be realised due to the transient effect in front/rear load transfer. In this case it should be theoretically possible to brake the vehicle at 1.0 g for a road with friction coefficient equal to 1.0 and the braking force distribution adopted. This situation corresponds to point A in the braking diagram, if one uses the static relationships which originated figure 5.1 [7, 146, 182, 211]. However, due to vehicle transient pitching, front tyre normal load falls below the steady-state value assumed for the vehicle in a constant deceleration manoeuvre.

The deceleration manoeuvres illustrate similar behaviour regarding sprung mass vertical displacement and pitch angle as illustrated in figures C.35 to C.38. It is interesting to note that for a 60/40 front/rear braking force distribution in a 0.8 g braking manoeuvre, in a dry road condition, the rear wheels lock up being necessary to use higher values of front/rear force distribution in order to avoid it happening. Figure C.38 depicts this situation. As the vehicle is decelerated at 0.8 g the pitch angle is of the same order of magnitude as when it was accelerated at 0.5 g, showing the anti-diving effect of the suspension geometry. Also, the simulations illustrate the decrease in available cornering power due to the utilisation by the longitudinal braking forces, as in the acceleration manoeuvres, although the effects seem to be less marked since during braking there is an increase in the front tyre normal load and therefore available adhesion.

5.3.2.3 Road Vertical Disturbances

Road vertical disturbance simulations are aimed at testing vehicle ride modes. Muller [197] has classified the study of vehicle vibration and its consequences to passenger comfort, in three subproblems:

1. modelling and characterisation of road roughness;

2. prediction of vehicle motion for transversing a given road;

3. prediction and characterisation of passenger response to vibration exposure

Given the causality aspects of these subproblems, it can be said that the complete problem's analysis involves the solution of problems 2 and/or 3 for inputs from problems 1 and 2, respectively. If the main focus of the study is the design of suspension components, then the interest lies within the solution of problem 2. On the other hand, if it is passenger comfort evaluation, then problem 3 has to be solved. In which case, 1 makes use of road roughness models, 2 uses vehicle models, and 3 utilises experiments of human response to vibration exposure.

Typical road roughness models include deterministic and stochastic inputs. Deterministic functions can be either special 'shaped' functions for transient analysis or sinusoidal functions for frequency domain analysis. Stochastic functions are usually stationary gaussian coloured noise processes. Vehicle models can be linear or nonlinear, and known human response to vibration exposure is given in [112]. A summary of the possible approaches for carrying out the task for solving problems 2 and 3 is given in the diagram of figure 5.4.

In the present study, because the model is very complicated and not known analytically, it can only be represented by a so called 'nonlinear black box', therefore none of the analytical approaches of figure 5.4 can be used. In this case, what was chosen to be done was to define time functions, with certain characteristics such as to exercise vehicle transient behaviour, and sinusoidal functions, to verify steady-state behaviour. The present section is concerned with the analysis of the results for the transient behaviour of the model. The next section will be looking at the steady-state relationships. The aim of this analysis is to assess the level of nonlinearity of the model. Then, using linearisation techniques, a linearised

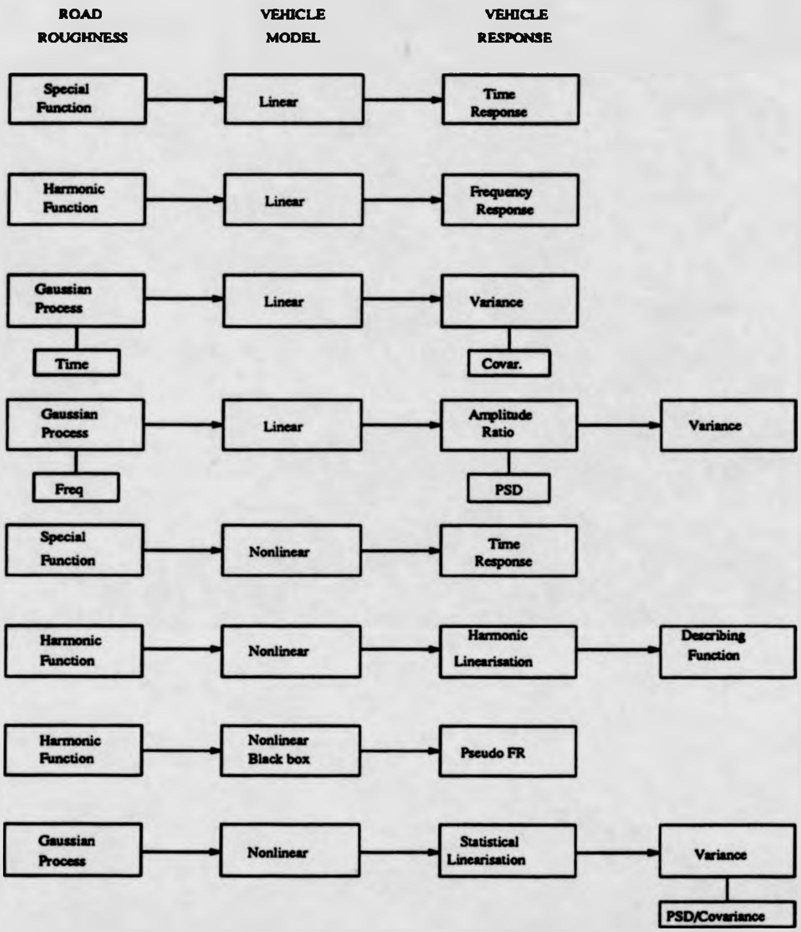

Figure 5.4: Summary of mathematical methods for ride studies

model can be obtained and any of the methods listed in figure 5.4 can be used for the description of the ride modes of the vehicle.

The study of vehicle ride modes assumes a contact point model for the interaction between tyre and road in the vertical direction. This situation is equivalent to a lumped parameter model for the tyre vertical force characteristics.

In order to generate the frequencies of excitation in the range of interest of the present work, of up to 15 Hz, velocities of 40 m/s have to be achieved, well within the capabilities of the vehicle. Sharp [264] discusses the filtering characteristics of pneumatic tyres due to enveloping effects of short wave length irregularities, of less than 0.75 m. He argues that the frequencies these irregularities generate in the normal speed range for ride studies is beyond the frequencies of interest (20 Hz for a speed of 15 m/s for irregularities of 0.75 m). Therefore, the assumption of a contact point is adequate.

Radial force relationships given by Clark [37] show that, for other than very small loads, force/deflection curves are quite linear and, although the damping coefficient of the tyre is said not to be insignificant in comparison with that of the suspension, it has been shown [264] not to influence suspension performance. Vehicle manufacturer data indicates that, for displacement of up to 0.04 m a linear relation can be adequately assumed. Because typical tyre static deflection varies between 0.015 and 0.020 m, the adoption of an operating range equal to $+/-$ tyre static deflection seemed adequate. Under some simulation conditions, this situation might not be satisfied, in which case the tyre loses contact with the ground for a negative value of deflection larger than that of the static value.

The deterministic road input functions were chosen such that all variables of motion related to the ride modes were excited adequately. The out-of-phase bump is intended to excite mainly roll motion and wheel hop. The ramp to step input is aimed at vertical and pitch angle motions, as well as wheel hop. Both tests allow tyre vertical deflection and force to be monitored. The effect of vehicle speed, bump height and lengths on the variables of motion are discussed.

For the out-of-phase triangular bump, it can be observed that the right wheel bump presents a constant spatial shift of 0.5 m in relation to the left one. Therefore the time between them varies with speed. In this case, values of 0.1, 0.05 and 0.033 s results for the speeds of 5, 10 and 15 m/s, respectively. The time shift between front and rear can also be seen to vary with speed because of the constant wheelbase of 2.82 m. For the previous values of speed, time shifts of 0.56, 0.28 and 0.14 s result.

Figures C.39 to C.56 illustrate vehicle response to bumps with heights of 0.01, 0.05 and 0.1 m, and length 1.0 m, for the above forward velocities. The variables of motion depicted in these figures are left and right front wheel displacements, vertical tyre forces and front and rear suspension arm angular displacements. For the sprung mass, the quantities illustrated include cg vertical displacement and acceleration and pitch and roll angles. Changes in forward, lateral and yaw motions are negligible, as expected.

Tyre jump-off occurred for the intermediary values of speed and bump height of the simulations. So, for road heights higher than 0.05 m, and speeds faster than 10 m/s, all four wheels lost contact when they reached the peak of the triangular bump. In these circumstances, the tyre forces became zero, figures C.47, C.49, C.53 and C.55. Because losses in forward velocity are negligible in the time span of the simulation, the driving forces were set to zero. Therefore the simulation was not halted when the wheels lost contact because no wheel overspin occurred.

Analysing the influence of bump height for the same speed, it can be observed that, for all variables of motion, the changes in transient behaviour is completely negligible. One cannot even distinguish to which height the simulation refers to, if the scale of the y-axis would not be shown! The only perceptible difference being the relative magnitude between the settling displacement the vehicle undergoes before the input and the steady-state value it achieves after the input, because it varies with input size. For example, the displacement of the vehicle's cg in this settling period is of the order of magnitude of 10^{-4} and it becomes more visible for the 0.01 m input than for the 0.1 m. But apart from these differences in the relative magnitude of transient and steady-state values, it can be said that the 'shape' of the response is identical, indicating good approximation of the nonlinear model to a linear one in the ride modes, as far as *amplitude of excitation* is concerned. The limitation in this case been wheel spin in the case of driven wheels loosing contact, or tyres undergoing larger vertical deflections than the valid range for linear behaviour.

The behaviour of the sprung mass for the same bump height and for the three different speeds shows that, irrespective of the value of the speed, their transient motion *after* the inputs have been applied, in the range in which they get excited by the bump input, are nearly the same, in terms of the frequency of oscillation and damping. In this way, vertical displacement and acceleration and pitch and roll angles show similar behaviour for all three speeds for each one of the bump heights. Besides this important conclusion concerning sprung mass response to triangular bump inputs, other observations related to its response can also be made. For instance, one aspect to be highlighted is the transmission of high frequency vibrations to the sprung mass, specially for the faster bumps. This can be easily seen in the vertical acceleration graphs, where large values of vertical acceleration are achieved at the cg (0.5 g) and of frequency content close to the wheel hop vibrations. See for example figure C.56.

For the wheels, it can be observed that both pairs, front and rear, present similar behaviour. There is a superposition of high frequencies, associated with wheel hop modes in the range of 75 to 80 rd/s, and low frequencies, associated with a combination of front and rear-end bounce and roll modes. The degree of influence of each aspect is dependent on the simulation conditions. The high frequency is more visible after the wheel has come down the bump, whilst the low frequency presents a more complicated behaviour. For the front wheels it is associated mainly with roll, after they have hit the bump. For the rear wheels, it is associated with roll, before they hit the bump and finally rear-end bounce combined with a little roll after they hit the bump. It can be clearly seen that the front wheel hop frequencies are lower than the rear ones, as well as more lightly damped, as results from the eigenanalysis. These results are illustrated in figures C.39 to C.56.

For small speeds, the transient behaviour of the vertical displacements shows a comparable level of contribution from the low and high frequency vibrations. However, as speed is increased the duration of the bump decreases, its bandwidth increases and excites more strongly the hop modes frequencies, whose contribution becomes dominant in the wheel's response. However, despite these changes in behaviour, the characteristics of the transient response in terms of frequency and damping factor have remained nearly constant with speed and this provides a good basis for using a linearised model as far as ride modes are concerned.

The behaviour of the suspension arm angular displacement closely resembles that of the wheels, even though they are defined as displacements relative to the sprung mass, in opposition to the wheels which are defined in relation to the inertial frame. This is so because the motion of the sprung mass (vertical, roll and pitch) can be considered to be 'slow' in relation to the wheel motion, due to the fact that the sprung mass equivalent inertia is about 8 times larger than those of the wheels. In which case, the wheel motion 'dominates' the displacement of the suspension arms. These quantities are also depicted in figures C.39 to C.56. Therefore, the same conclusion is applicable to the suspension arms, that their transient behaviour does not change with forward velocity.

These results are interesting in the sense that they confirm that the nonlinear vehicle model does not present significant differences in its transient behaviour when under ride mode types of excitation, in terms of *forward speed.* Together with the previous conclusion concerning the effect of bump height and constant speed, these results provide good insight into the use of the linear model approximation of the nonlinear vehicle model for the area of ride performance. It is worth noting that this area's main concern, from a control point of view, is analysis and design of controlled suspension.

The use of the ramp to step input serves as an aid and support to the triangular bump input for the analysis of the transient motion to road vertical disturbances. From a modelling point of view, they could even be related, if the triangular bump in this case was not out of phase and they were fast enough in relation to the system's dynamics. Under these conditions, the triangular bump would represent an impulse function and the ramp to step a step function.

If known concepts of system theory are applied [52], the step function can be seen as the integral of the impulse function, and if the system was linear the same relation would be true for the system's output. However under many practical situations, the impulse might not have enough energy to excite all the modes significantly, or it might cause some of the simplifying assumptions to be violated, e.g. concerning limit values of system variables. In the present work, for example, some of the modes were more distinguishable when the model was subject to a step to ramp input. In these tests, a better response results for the sprung mass vertical displacement and pitch angle, and also for the front and rear tyre deflections.

From the results presented in figures C.57 to C.62 it can be seen that both pitch angle and vertical displacement present a transient behaviour which is a combination of of front-end and rear-end bounces. These modes are better illustrated separately through the transient behaviour of the tyre deflections, also illustrated in the previously mentioned figures. Because of the negligible differences in vehicle behaviour to the triangular bump of different heights for the same speed, only two heights of 0.05 and 0.1 m have been exercised. The speeds remained the same as in previous simulations. The results, nevertheless, confirm more clearly, for the previously mentioned variables, the conclusions drawn from the triangular bump simulation.

Besides the triangular bump and the ramp to step inputs also a few simulation runs for a sinusoidal input have been performed, in order to evaluate the time domain characteristics of the system's response to this type of input, before performing a frequency domain analysis. Figures C.63 to C.66 present the results for these simulations.

The time responses for almost all the variables of interest and in almost all the simulation conditions, presented a behaviour nearly sinusoidal, when visually inspected. The only exception was vertical acceleration, at frequencies near the front and rear end bounce resonances. At these frequencies the pitch motion became quite large, and because the vertical acceleration that was chosen to be measured and expressed in the body frame, (It doesn't necessarily have to be so, but this was chosen as such because that's the value which is obtained from road tests where the accelerometers are bolted to vehicle body.) it caused the value of the acceleration to be distorted at the positive peaks because, at these points, vehicle pitch was maximum as well (approximately 0.08 rd or 0.145 m in the front end) therefore influencing the value of the vertical acceleration expressed in the sprung mass frame, since road input is sinusoidal in the inertial frame. These results are illustrated in figure C.63. Also a few other simulations have being included due to the fact that they present some peculiar behaviour worth noting about the model for the purpose of the present analysis.

In all sinusoidal simulations, a driving force to the rear wheels was necessary to be applied, since for a sinusoidal road profile in the space domain to remain sinusoidal, it has to be transversed with constant speed. In some cases, due to the large displacements of the tyres, normal forces became small enough to cause the driven wheels to spin, on which occasion the simulation was halted. These results can be used to define a boundary for a combination of speed and road amplitude for this type of input. Figure C.66 illustrates such situation in which the front tyre deflection approaches zero.

Other interesting situations include the filtering characteristics of the sprung mass displacement to high frequencies. Figure C.66 also illustrates this simulation for a vehicle driving over a road profile of amplitude 0.007 m and for a frequency of 12.5 Hz.

A final example of time response is from the left and right out-of-phase test. In this simulation there is a destructive interference amongst the suspension forces transmitted to the sprung mass, in such a way that at steady-state the wheels are hopping very strongly but the car body remains stationary !! This result is presented in figure C.65.

5.3.3 Gain Analysis (steady-state)

The gain, or steady-state analysis, is intended to assess the level of coupling amongst the variables of motion, the amount of nonlinearities in these couplings and also to determine vehicle directional stability parameters.

Coupling can be determined by the steady-state gain between input and output variables, which measures how much output is produced by the system per unit of input quantity. For instance, if there would be no coupling between an input and an output variable in a system, the steady-state gain for these variables would be zero and no such output would be produced by the system for that input quantity.

If the system is linear, the amount of output produced is then proportional to the amount of input given to it. This proportionality factor is denominated steady-state gain. For a generic system, assuming the system to be in an equilibrium operating point, the steady-state gain can be generically defined as:

$$k_{ij} = \left. \frac{\partial y_i}{\partial u_j} \right|_{\boldsymbol{x}_o} \tag{5.8}$$

for all the inputs $u_{k \neq j}$ held constant, and for the system's state equal to $\boldsymbol{x}_o$ at the operating point.

Vehicle directional properties can be measured by a large number of parameters, usually related to its steady-state response. Some of the most common ones include the static margin, understeer/oversteer gradient, Ackermann angle and so on.

For this analysis, three types of steady-state simulations have been performed:

- cornering at constant speed with various steering angles;
- cornering with constant steering angle at various speeds;
- crossing of obstacles with sinusoidal profile of different heights, and at different speeds.

For cornering at different speeds the results are presented in figures C.71 and C.72 for the linear tyre without time lag, in figures C.73 and C.74 for the nonlinear tyre without the time lag, and for the nonlinear tyre with time lag in figures C.75 and C.76. For the cornering with different steering angles results are given in figures C.77 to C.82 for the graphs as function of steering angle, and in figures C.83 to C.88 as function of lateral acceleration, for the three types of tyre used.

The results presented in these figures are pitch and roll angles, yaw rate, lateral velocity and acceleration, sideslip angle and slip angle of the wheels. They are presented as a function of the steering angle, as well as a function of lateral acceleration, for the constant speed manoeuvre. This is done for comparative purposes, since some vehicle directional characteristics are specified as functions of lateral acceleration in the literature [7, 62, 233]. For the constant steering manoeuvres the same quantities as above are depicted versus forward velocity.

For the sinusoidal obstacle crossing the simulation results are illustrated in figures C.89 to C.96 for the in-phase obstacles and in figures C.97 to C.104 for the out-of-phase ones. Variables depicted in these figures are amplitude ratio and phase angle of a pseudo-frequency response of vertical acceleration, displacement, pitch angle for the right/left in-phase road excitation, pitch and roll angles for the out-of-phase left/right road input, front and rear suspension forces, and front and rear tyre deflections. The input quantity is the front left road profile.

For multivariable systems, to which control is intended to be applied, the concept of coupling has two different meanings, although closely related. In the 'classical' control sense, coupling is expressed in terms of the input/output quantities. Because it deals with *input/output* relationships, it is desirable to characterise the amount of interaction present in the system to be controlled, by analysing the amount of each output produced by each input. If all the interactions are 'weak', single input, single output (SISO) control techniques can be applied to individual loops with good success. In this situation the controls will not 'fight' each other too much. If this is not possible, design techniques which decouple system inputs and outputs might be necessary. This is done through the use of 'compensators' introduced in the path of the input/output pair intended to be decoupled.

For the 'modern' control approach, coupling is related to model order reduction. Because the control action is based on *state variable* feedback, the concept of coupling describes the influence of the states variables on the input/output relationships. The effect an input has upon a state is measured by its controllability and the influence a state has on a certain output is measured by its observability. Therefore, states which are slightly controllable and observable and have little effect on the input/output characteristics of the system can be eliminated from the state vector. This is done by analysing controllability and observability gramians of balanced realisations of the system and eliminating those states which have 'small' coefficients.

The variables chosen for the two handling manoeuvres, cornering at constant steering angle and cornering at constant speed, were those found to be more affected by the mentioned manoeuvres and relevant for the handling modes. The effect of other variables were either neglected or their influence could be inferred from the behaviour of the chosen variables. In this respect, suspension displacements effects are assessed by their connection to sprung mass positioning, pitch and roll angles and tyre deflections.

The analysis is performed first for the cornering manoeuvres, before proceeding to the obstacle crossing. Some of the discussion included in the transient steering input analysis is used to clarify the explanation of vehicle handling behaviour.

In order to have an overview of the amount of interaction amongst these variables and the linearity of such interaction, linear regression has been applied to the data of figures C.71 to C.88, for both the constant speed and the constant steering manoeuvres, and for the three tyre models. These results are presented in table 5.3, for the model with linear tyre without time lag, in table 5.4, for the nonlinear tyre without time lag, and table 5.5, for the nonlinear one with time lag.

As it can be deduced from these tables, for all tyre models, all variables retained were largely affected by the steering input. The only exception being the pitch angle, which showed the smallest influence of all. This result confirms the widespread practice of neglecting vehicle pitch from the state variables in handling studies, and when it is considered, its effect is calculated in an approximated manner. It also confirms real vehicle behaviour, because a vehicle does not suffer large pitch angle variations in any normal operating condition. The other variables chosen for the sprung mass movement, yaw rate (u_4), and lateral velocity in vehicle body (V_y), and slip angle (α_i) for the wheels, are the main quantities in handling studies and their importance for handling studies is obvious.

Constant Speed Input: Steering (rd)			Constant Steering Input: Speed (m/s)		
Output	Gain	Corr.	Output	Gain	Corr.
a_y (m/s^2)	4.5921	1.0	a_y (m/s^2)	0.089	1.0
q_5 (rd)	0.0004	0.9792	q_5 (rd)	0.0020e-3	0.7921
q_6 (rd)	0.0165	1.0	q_6 (rd)	0.0003	0.9969
u_4 (rd/s)	0.2304	1.0	u_4 (rd/s)	0.0013	0.7928
V_y (m/s)	-0.2003	-1.0	V_y (m/s)	-0.0201	-0.9595
β (rd/s)	-0.0100	-1.0	β (rd/s)	-0.0007	-0.9962
α_f (rd/s)	-0.0447	-1.0	α_f (rd/s)	-0.0009	-0.9963
α_r (rd/s)	-0.0277	-1.0	α_r (rd/s)	-0.0005	-0.9960

Table 5.3: Steady-state gains: Linear tyre without lag

Constant Speed Input: Steering (rd)			Constant Steering Input: Speed (m/s)		
Output	Gain	Corr.	Output	Gain	Corr.
a_y (m/s^2)	4.4915	0.9955	a_y (m/s^2)	0.1059	0.9965
q_5 (rd)	0.0034	0.9720	q_5 (rd)	4.8200e-5	0.9714
q_6 (rd)	0.0184	0.9952	q_6 (rd)	0.0004	0.9969
u_4 (rd/s)	0.2275	0.9967	u_4 (rd/s)	0.0017	0.8490
V_y (m/s)	-0.3596	-0.9908	V_y (m/s)	-0.0268	-0.9519
β (rd/s)	-0.0180	-0.9907	β (rd/s)	-0.0009	-0.9985
α_f (rd/s)	-0.0530	-0.9976	α_f (rd/s)	-0.0010	-0.9986
α_r (rd/s)	-0.0355	-0.9994	α_r (rd/s)	-0.0007	-0.9984

Table 5.4: Steady-state gains: Nonlinear tyre without lag

Constant Speed Input: Steering (rd)			Constant Steering Input: Speed (m/s)		
Output	Gain	Corr.	Output	Gain	Corr.
a_y (m/s^2)	4.4971	0.9956	a_y (m/s^2)	0.1059	0.9965
q_5 (rd)	0.0034	0.9755	q_5 (rd)	4.8214e-5	0.9712
q_6 (rd)	0.0185	0.9953	q_6 (rd)	0.0004	0.9969
u_4 (rd/s)	0.2277	0.9967	u_4 (rd/s)	0.0017	0.8487
V_y (m/s)	-0.3625	-0.9899	V_y (m/s)	-0.0269	-0.9518
β (rd/s)	-0.0182	-0.9898	β (rd/s)	-0.0009	-0.9985
α_f (rd/s)	-0.0531	-0.9974	α_f (rd/s)	-0.0010	-0.9986
α_r (rd/s)	-0.0357	-0.9993	α_r (rd/s)	-0.0007	-0.9984

Table 5.5: Steady-state gains: Nonlinear tyre with lag

Interesting conclusions can be drawn concerning the type of tyre model adopted. For the linear model, in the constant speed manoeuvre, the correlation coefficient of the linear regression is 1.0 for every variable but pitch angle. This represents a highly linear behaviour in steady-state, of the simulation model, due to steering inputs. However, by the constant steering manoeuvre, it can be seen that a strong coupling between these variables and the *variable of motion*, forward velocity exists. Nevertheless, it can be seen that this interaction, for most of the variables in handling, is also quite linear, except for pitch angle and yaw rate. The pitch angle/forward velocity coupling during cornering is also weak, as it would be expected, and it can be seen by its gain. The effect of forward velocity on the yaw rate response which has been briefly discussed in the transient steering input manoeuvre, will be further pursued at the discussion of vehicle handling and directional behaviour parameters.

For the remaining variables of motion related to vehicle handling, it can be seen that the model coupling concerning forward velocity, can be adequately described by linear relations for vehicle steady-state behaviour. This can have interesting design implications from a control point of view, depending which strategy is adopted to implement the control. If forward velocity is adopted as a system *parameter*, the fact that model characteristics change in a linear way for the steady-state behaviour, together with the conclusions already made about the transient behaviour, may allow the implementation of some kind of adaptive control, for example, as a function of vehicle speed.

The two nonlinear tyre models show very similar behaviour, and this allows us to conclude that the time lag has no influence on vehicle steady-state behaviour, which is to be expected. Therefore the discussion and conclusions are applicable to both tyre models.

For the constant speed manoeuvre, the variables of motion showed less linear behaviour than the linear tyre model. However, the level of linearity is quite high, with correlation coefficients with values around 0.99 for all variables of motion with the exception of pitch angle, similar to the result obtained for the linear tyre model. Again, the most 'uncorrelated' result from both manoeuvres was yaw rate variation with forward velocity as a response to a constant steering input, which again is related to vehicle handling characteristics. However it can be seen that the nonlinear one is more correlated than the linear one and the implication of such fact is also discussed later.

In comparison to the linear model, it can be seen that for the steering angle input manoeuvre at constant speed, the vehicle with linear tyre model presents higher gain for the lateral acceleration, and marginally higher for the yaw rate. It presents an inverted pitch gain(!) and a smaller roll gain, but not significantly different. The variables lateral velocity, sideslip and slip angles of the wheels are closely related to each other for a constant speed manoeuvre, and they present similar behaviour amongst themselves for both tyre models. However, the model with nonlinear tyre model presents a much larger sideslip and lateral velocity gains, as well as wheel slip angle. This indicates that this vehicle model sideslips more (in the negative direction) for the same input, due to the nonlinear tyre characteristics and change in cornering coefficients caused by left/right load transfer effects. That means that the yaw torque generated by the steering angle is less than that for the model with linear tyre, indicating lower overall cornering stiffness.

Vehicle handling characteristics are discussed next. For this purpose, the yaw rate response to steering inputs for a constant speed, and to a constant steering input for various speeds are used. The constant speed manoeuvre is analysed first. Vehicle response to fixed steering input is called the ***static sensitivity*** and it embraces important aspects of overall vehicle behaviour.

During a turn, at constant steering, the vehicle presents a constant yaw rate, a fixed roll angle and a constant angle of sideslip. The yawing moment and lateral force which cause vehicle motion is due to the steering angle input and the sideslipping of the wheels. If the yawing moment due to sideslipping is assumed to be zero, either because it can be small [255], or because of vehicle handling characteristics [62], it then results for the steady-state yaw rate response

$$\left.\frac{u_4}{\delta}\right|_{ss} = \frac{V_x}{(a+b)} \tag{5.9}$$

where u_4 is the yaw rate, δ the road steering angle, V_x is the forward velocity and a and b are the front and rear wheelbases, respectively.

In steady-state turning,

$$V_x = u_4.R_t \tag{5.10}$$

where R_t is the radius of turn. It can be seen from equations 5.9 and 5.10 that the steering angle for this condition is equal to $(a+b)/R_t$. This angle is called the ***Ackermann steer angle*** and as such it is the basic handling response characteristic of an automotive vehicle. In this case the centre of turn lies in a line through the rear axle, perpendicular to the wheel base, and the front wheels are turned so as to be perpendicular to the line joining each wheel centre to the centre of turn, figure 5.5. This response is modified by the existence of vehicle directional properties and their relationships.

In the case where wheel sideslip is considered, the steady-state solution for the yaw rate to a road steering input is given by

$$\left.\frac{u_4}{\delta}\right|_{ss} = \frac{V_x}{(a+b).(1+K.{V_x}^2)} \tag{5.11}$$

with K expressed as

$$K = \frac{m}{(a+b)^2}\left[\frac{b}{C_{ft}} - \frac{a}{C_{rt}}\right] \tag{5.12}$$

where m is vehicle total mass, and $C_{f,rt}$ are the total front and rear cornering stiffnesses, respectively. The constant K defines fundamental vehicle properties as it has been noted in the literature [7, 255, 185] and it will be discussed later.

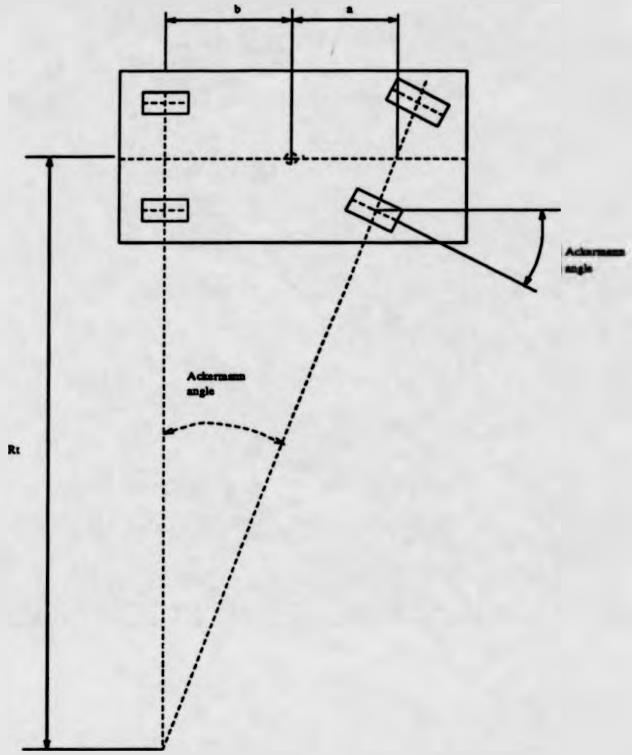

Figure 5.5: Ackermann steering angle

Another related quantity derived from these parameters, which also defines vehicle stability characteristics and handling behaviour, is the *static margin*. The static margin is a normalised measure of the distance between the center of reaction of the tyre forces, which has been named *neutral steer point*, and the *cg* of a nonrolling, nonpitching vehicle. As such, it represents the directional stiffness of the vehicle and it is a way of defining vehicle directional stability.

For a nonrolling, nonpitching vehicle, the static margin, *sm* is defined as

$$sm = \frac{C_{rt}.b - C_{ft}.a}{(C_{ft} + C_{rt})(a + b)} \tag{5.13}$$

As can be seen from equation 5.13, the static margin will be positive for values of $C_r.b > C_f.a$, negative for $C_r.b < C_f.a$ and zero for $C_r.b = C_f.a$. If the cornering stiffnesses are equal, it can be seen that the static margin is positive for the *cg* located forward of the wheelbase midpoint, zero if it is coincident and negative otherwise. In terms of the static margin, the constant K of equation 5.12 is expressed as

$$K = \frac{m}{(a+b)}\left[\frac{sm.(C_{rt}.C_{ft})}{(C_{rt}+C_{ft})}\right] \tag{5.14}$$

In order to characterise vehicle handling behaviour, the concepts of neutral steer, oversteer and understeer are adequate. These concepts have been defined by Olley [255] and they are connected to the path a vehicle, which is driving straight ahead, follows when acted upon by a *constant side force* at its centre of gravity. If the steady-state response is such that the new trajectory is another straight line, the vehicle is said to be *neutral steering*, if the rear end is projected outwards, the vehicle is said to be *oversteering* and if the front end is, the vehicle is *understeering*. In terms of the variables of motion, a neutral steering vehicle presents zero yaw rate in steady-state, for a positive side force, an oversteering vehicle presents negative yaw rate, and an understeering vehicle has positive yaw rate. In terms of the constant K, it can be said that a vehicle with $K < 0$ is oversteering, with $K = 0$ is neutral steering and with $K > 0$ is understeering. In terms of the static margin, a neutral steering vehicle has zero static margin, an oversteering one, negative and an understeering, positive.

When the vehicle is understeering, it is stable under all conditions of forward velocity, but, if it is oversteering, a value of a critical speed exists, above which the vehicle is unstable. Even for simple vehicle models [255], the expression for the critical speed is quite complex. However, Ellis [62] referring to an early work by Rocard, presents an expression for the critical speed in terms of vehicle characteristics, which gives a good insight into the influence of the main handling parameters in vehicle directional characteristics. The value of speed above which an oversteering vehicle is unstable is given by

$$V_{xc}^2 = \frac{C_{ft}.C_{rt}(a+b)^2}{m(a.C_{ft} - b.C_{rt})} \tag{5.15}$$

and in terms of the constant K, it is simply:

$$V_{xc}^2 = -\frac{1}{K} \tag{5.16}$$

It can be seen that, as perturbation values for $C_{f,r}$ change under high g manoeuvre operating conditions, it is possible for an understeering vehicle to become oversteering and, ultimately, unstable.

It should be emphasised that these relationships have been derived for very simple models and that a large number of other parameters, influence vehicle handling behaviour. Nevertheless, these relations just presented sum up a basic set of parameters for the understanding of directional behaviour of an automotive vehicle.

It can be seen from equations 5.12 and 5.13 that a neutral steering vehicle is one which has the constant K and the static margin sm equal to zero. For this type of vehicle the yaw rate/steering input gain as a function of forward velocity is a straight line with tangent $1/(a+b)$, according to equation 5.9 or 5.11. If the vehicle is understeering, that is, its static margin sm is positive, and therefore its cg is located forward of the neutral steer point, the constant K is positive and equation 5.11 shows a decreasing gradient for the yaw rate/steering input gain as a function of forward velocity. On the other hand, if the vehicle is oversteering, its static margin is negative, meaning that its cg is located after the neutral steer point, and K is negative. In this case the denominator of equation 5.11 decreases with forward velocity and the gradient of the yaw rate/steering gain versus speed increases. This situation is illustrated, for these three types of vehicles, in figure 5.6.

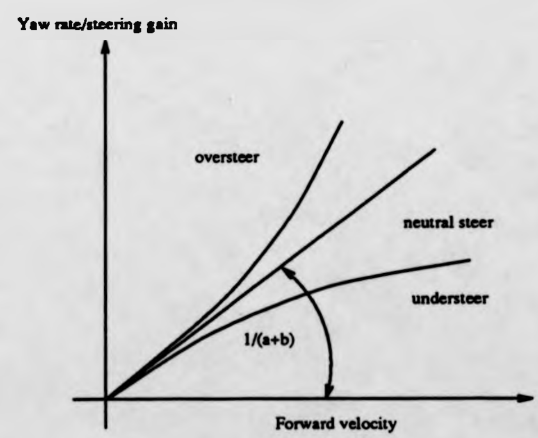

Figure 5.6: Handling diagram - constant steering

For the present vehicle model these results can be seen in figure C.71 for the linear tyre, in figure C.73 for the nonlinear tyre without time lag and figure C.75 for the nonlinear tyre with time lag. The vehicle is always understeering, therefore presents a positive static margin and it is always directionally stable. From the correlation coefficients given in tables 5.3, 5.4 and 5.5 and the results from the previous figures, it can also be seen that the model with linear tyre presents more understeer, that is, it has a greater static margin than the model with nonlinear tyre.

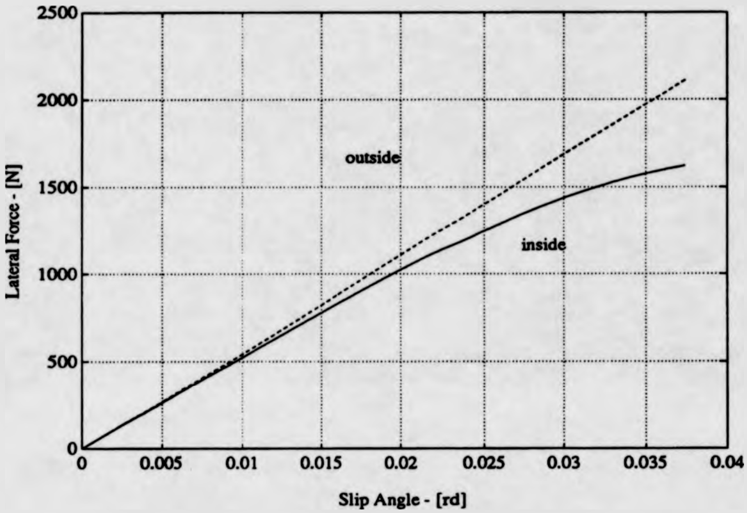

Figure 5.7: Variation of cornering coefficient - Front tyres

Two factors influence the static margin in the nonlinear tyre. The change in cornering stiffnesses due to normal load variation and change of wheelbase caused by pneumatic trial variation due to slip angle and normal force. There is an increase in the coefficients of the outer wheels and a decrease in the inner ones, figures 5.7 and 5.8. The rear inside cornering stiffness decreases more than the equivalent front one. Because both outside coefficients don't change much, the overall effect is a decrease in understeer. There is a decrease in front wheelbase and an increase in the rear one. The overall effect of this variation is to increase the understeer, see figure 5.9. However, the negative contribution to understeer from the cornering coefficient variation is larger than the positive contribution to it by the wheelbase variation. The net effect for the nonlinear tyre model is therefore a decrease in understeer.

Other steady-state simulation results which illustrate these points are those

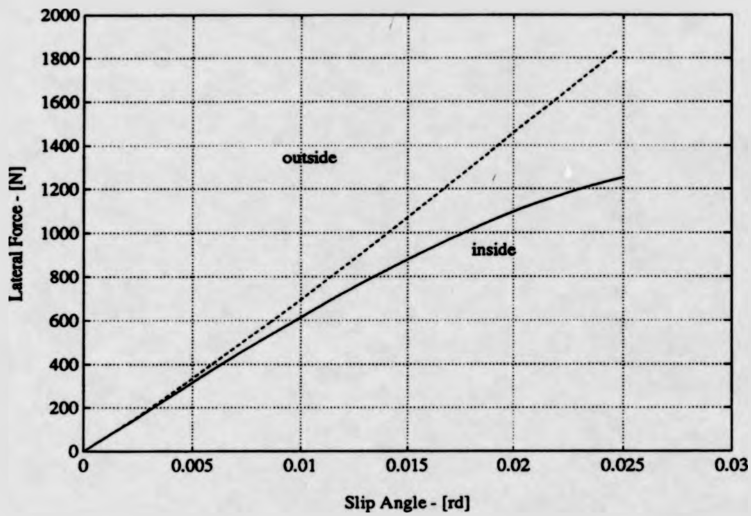

Figure 5.8: Variation of cornering coefficient - Rear tyres

from the constant speed simulation. The road wheel steering angle is plotted against either radius of turn, yaw rate or lateral acceleration, since they are equivalent in steady-state for a constant forward velocity. The neutral steer responses are now lines of constant tangent for each speed. When the gradient of the simulation result is greater than that from the neutral steer response, the vehicle is understeering, if it is smaller, the vehicle is oversteering and if it is negative the vehicle is unstable.

For the present simulation model with linear tyre the behaviour is understeer and for the nonlinear tyre, either with or without time lag, the behaviour is increasing understeer with increasing steering angle. Figure 5.11 illustrates this situation. It can be seen that for the linear tyre the understeer is constant, while it is increasing for the nonlinear tyres.

Another quantity called *stability factor* can be used to describe vehicle handling behaviour. It corresponds to the SAE oversteer/understeer gradient, and it is defined as the quantity obtained by subtracting the Ackermann steer angle gradient from the ratio of the steering wheel angle gradient to the overall steering ratio [233]. In terms of the constant K, the stability factor is given by

$$K_{sf} = (a + b).K \tag{5.17}$$

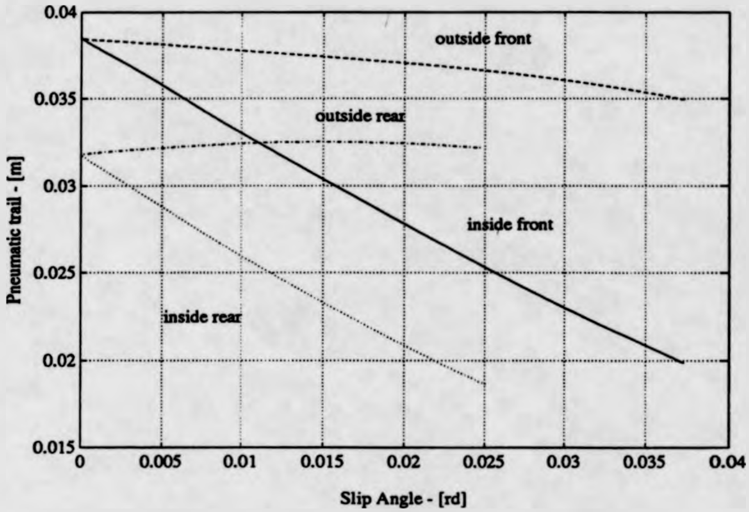

Figure 5.9: Variation of pneumatic trail

and similar results to the previously described ones can be obtained for the simulation model.

The sinusoidal profile obstacle crossing manoeuvres are discussed next. The objectives of these simulations are to verify the linearity of the ride modes of the vehicle model. Only one model for the vertical tyre force description is available.

The amplitudes of the road profile are 0.005, 0.01, and 0.025 m, the length are 1 and 10 m and the speeds with which the vehicle transverses them are 5, 10, 12.5, 15 and 30 m/s. The resulting frequencies are 0.5, 1.0 1.25, 1.5, 5, 10, 12.5, 15 and 30 Hz. In-phase and left/right out-of-phase obstacles are crossed. A constant time shift of 0.1 s was adopted for the out-of phase simulations.

Because it is necessary to apply a driving force to maintain sinusoidal behaviour in the time domain from a space domain road description, in some of the simulations the fall in wheel normal load caused the driven wheel to spin and the simulation was halted. When this occurred, no frequency response data was generated. It has happened for the 12.5, 15 and 30 Hz frequencies for the amplitude of 0.01 m and for the 10, 12.5, 15 and 30 Hz for the amplitude of 0.025 m. For this reason, two sets of frequency responses have been generated. One in the range of 0.5 to 5.0 Hz which include all three road amplitudes, and another

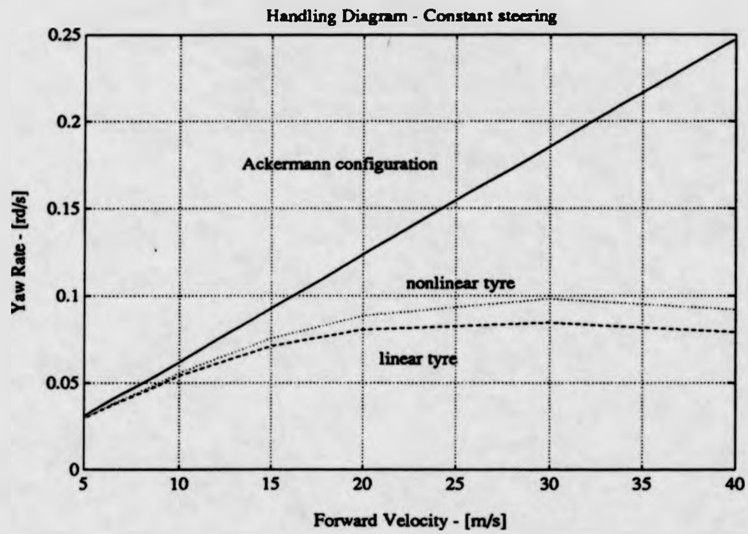

Figure 5.10: Handling diagram - Constant steering manoeuvre

one for the range 0.5-30 Hz which include only the 0.005 m amplitude. These two sets of results were obtained for the in-phase and the out-of phase bumps.

Figures C.89 to C.96 presents the in-phase frequency response for both the 0.5–5.0 Hz, for all three amplitudes and the 0.5–30.0 Hz for the 0.005 m amplitude. For the out-of-phase simulations results are presented in figures C.97 to C.104 for the two frequency ranges. The frequency response plots for the 0.5 to 30.0 Hz range is included in order to permit the analysis of a simulated frequency response of the full nonlinear vehicle model for the whole range of interest.

The procedure used to obtain this data was as follows. The concept of settling time [49] was used to define a minimum time after which the transient response due to the road profile input of all system variables of interest could be considered negligible. In this case, the front-end bounce mode, which has the slowest and least damped response, was used as the limiting mode. Adopting a five time constants specification results in a settling time of 3.64 s. With this value, the transient response is attenuated by 99.3%. A value of 4 s was then adopted for all simulations, after which approximately 4 periods of data were gathered. The data was visually examined for assessing nonlinearities, since direct Fourier transforming it was not enough to distinguish the amount of higher harmonic

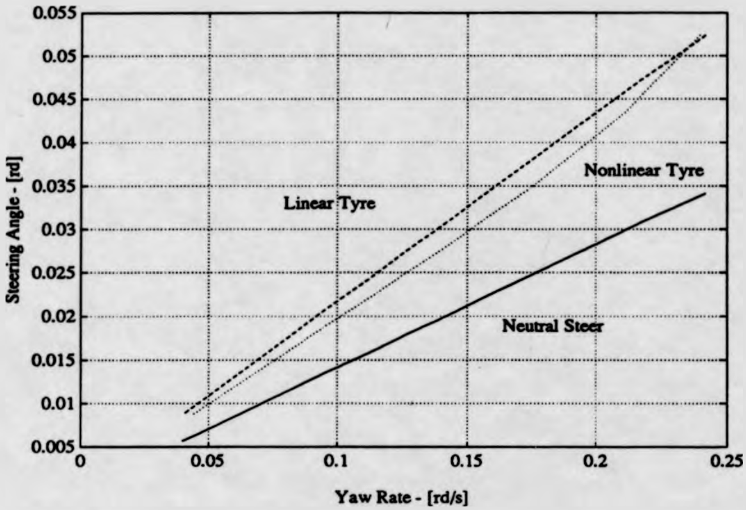

Figure 5.11: Handling diagram - Constant speed manoeuvre

content, unless the data was preprocessed beforehand. The only exception to nearly sinusoidal behaviour was the the vertical acceleration of the sprung mass near the bounce modes of the vehicle body, 1.0 to 1.5 Hz, as has been previously discussed in section 5.2.2.1.4. Figure C.63 of the transient response illustrates such response. However, because this nonlinearity is more related to the variable of motion itself, and therefore at these frequencies is present in all amplitudes of road crossing, the use of the pseudo frequency response is justifiable for comparative purposes.

The phase angle was calculated based on the time shift of the output and input variables, and the frequency, assuming the output to be always delayed in relation to the input, even though for the acceleration frequency response some textbooks consider it to approach 180^o phase advance as the frequency tends to zero. There is no need to be concerned with the phase when it went through a 2π change either, since it was obtained for comparative purposes and all the phases for all amplitudes were consistently calculated in the same manner. For this reason the abrupt phase changes can be noticed in the phase plots of the frequency response, for all variables which achieved that condition.

The in-phase 0.005 m amplitude excitation for the 0.5 - 30 Hz is discussed first.

Although no comparison can be made in terms of the amplitude of excitation, some interesting analysis can be performed, and conclusions drawn about vehicle behaviour, from these results.

When analysing frequency response plots, one should be looking at the response characteristics at the low and high frequency ends, both magnitude and phase, in order to get an initial view of the system dynamics in the frequency range being studied. If Bode plots are used, then phase angle plots together with initial and final values of asymptotes inclination are useful for giving an idea of system order. Transition characteristics in the frequency domain can also be observed in these plots by noting changes in direction of asymptotes and phase angle variation going through inflection points. In these simulations, however, due to the way the phase angle has been generated, and the type of input the vehicle is subject to, it can not provide this type of information. The frequency response is calculated only in relation to the left front wheel input, although all other road inputs are present. Because of the fact that they have been ignored, their effect on the frequency response is not taken into consideration. Therefore, unless it is for comparative reasons, these results have limited use. One use which can be made of them is to determine the presence and the values of resonances, and check them against those results obtained through eigenvalue analysis of the linearised model.

For the vertical displacement of the *cg* of the sprung mass, this resonance occurs near the natural frequency of the front-end bounce mode at approximately 6.9 rd/s. Another smaller resonance can be observed near the front wheel in-phase hop mode, around 75 rd/s. See figure C.90. The vertical acceleration of the *cg* presents a a smaller peak at the low frequency and a much larger one at the vicinity of the wheel hop frequency. This is so because acceleration amplitude ratio is equivalent to the dispacement response multiplied by ω^2. Therefore the effect of high resonances is amplified by this factor. Figure C.90 illustrate these results.

The pitch angle response presents similar behaviour to the vertical displacement, with a large resonance at 6.9 rd/s, and a small at 75 rd/s, as can be observed in figure C.92.

The front and rear suspension forces and tyre deflections, all present similar behaviour to each other, only the values of the resonances change for the front and rear quantities. Figures C.94 and C.96 illustrate these results. As the frequency tends to zero, their values tend to the static one. The suspension force is defined to include this term, just like the tyre forces. However, the tyre deflection is defined as the deformation from the static value, therefore it tends to zero. All four quantities present two resonances. A low one, connected to a body mode and a high one connected to a wheel hop mode.

For the front suspension force and tyre deflection, the low resonance is at the front end bounce mode, 6.9 rd/s, and the high one at the in-phase wheel hop mode, 75 rd/s. Similar results are presented by the rear suspension force and tyre deflection, where there is a resonance at the rear end bounce at around 8.46 rd/s and another one at 81 rd/s. It can be observed that the damping for the front modes is lower than that of the rear ones, in accordance with the eigenanalysis results previously obtained.

For the out-of-phase simulations, amplitude 0.005 m and 0.5 - 30 Hz range, a behaviour similar to the previous one is observed, and the responses are nearly identical, for both amplitude ratio and phase angle. An interesting behaviour arises for the sprung mass variables, though. The quantities discussed include vertical displacement and acceleration and pitch and roll angles.

For the vertical behaviour, the amplitude ratio curve for both displacement and acceleration, present two resonances, as before. However, they also have points of very high attenuation due to interferences of the suspension forces at 5 and 15 Hz. Figure C.98 depicts this situation.

For the pitch and roll angles the behaviour is slightly more complicated than before. They both have significant variations at the low frequency range. The pitch angle presents two close resonances at 6.9 rd/s and 8.5 rd/s corresponding to the tyre front and rear bounce modes frequencies. The roll angle has a resonance at about 11.5 rd/s corresponding to the roll-on-springs mode frequency. These plots are illustrated in figure C.100. However, at 31.4 rd/s (5 Hz) and 94.3 rd/s (15 Hz) the pitch angle is heavily attenuated similarly to the vertical quantities. The roll angle on the other hand varies slowly in the range 0.5 to 5 Hz, but suffers a heavy attenuation around 75 rd/s and peaks again at 81 rd/s. This behaviour can be explained by the wheel hop modes. At around 75 rd/s, the in-phase wheel hop modes induce large pitch angles, while around 81 rd/s the out-of-phase modes induce large roll motion.

The effect of the amplitude of excitation is discussed next, for the in-phase and out-of-phase simulations, in the frequency range of 3.14 (0.5 Hz) to 31.4 rd/s (5 Hz). This was the only range in which the amplitudes of 0.005, 0.01 and 0.025 m which were chosen for these simulations, did not result in loss of contact with the ground. In this range of frequencies, only the sprung mass modes could be checked. The wheels lost contact for excitations higher than 0.005 m and above 62.8 rd/s (10 Hz).

It can be seen from figures C.89 to C.95, for the in-phase obstacle crossing and in figures C.97 to C.103, for the out-of-phase ones, that there is no significant difference, neither in the amplitude ratio nor in the phase angle, for any of the amplitudes of road transversed. Also, because the 0.005 m amplitude is included in these simulations, the previous analysis is applicable to the present results, in terms of behaviour characteristics.

It can be concluded that the present vehicle model, with linear relations for the spring and damper of the suspension, and for a linear vertical tyre behaviour, as well, can be adequately represented by a linearised model, concerning ride modes behaviour. The limiting factor in this case being the loss of adhesion or compression of the tyre beyond the assumed linear range. These results show that the inertia couplings, in these models, for this mode are weak.

5.3.4 Determination of a Driving Envelope for Linear Control Analysis and Design

The discussion about a driving envelope for linear control analysis and design is centred at the previous simulation runs.

The discussion will be based on the type of vehicle model adopted and the area of vehicle behaviour intended to be addressed. Issues with regard to strong points as well as limitations of the present model are discussed and conjectures about the effects of possible variations of the model are elaborated.

The type of vehicle model available in the present study include a full vehicle capable of performing all manoeuvres, although at the moment, the rotational degrees of freedom of the wheels are not enabled. There are four types of models for the lateral force description and one for the vertical. The suspension forces at the moment are described by linear relationships for both the spring and damper, however, it poses no difficulty whatsoever to have nonlinear elements. It was chosen to be so in regard to the problems intended to be addressed by the present study.

The areas of vehicle behaviour which are discussed are the same according to the division established for the simulation runs. The effects and consequences of considering them together is also attempted to be discussed.

For the vehicle ride behaviour, with its present tyre and suspension models the range of validity of linear behaviour seems to be limited only by the loss of contact of the wheels with the road. These conclusions are based on the results of the transient and the steady-state analysis of the present simulations. For the transient results it can be observed that there is no significant changes in vehicle transient behaviour as the levels of model inputs are varied. All the main variables of motion presented similar transient behaviour for road inputs of different types and severity. Apart from a more realistic suspension model, the linear vertical tyre model is widely used in vehicle ride studies and seems adequate for the purpose of such studies. The limitation of suspension nonlinearities are not so bad for suspension control studies, since in these cases some form of actuator is involved in which the largest contribution to the nonlinearities in a normal range of operation are originated by the dampers and those normally are substituted in many controlled suspension design concepts, such as active or semi-active. These limitations do not influence so much the handling modes either, because during normal handling manoeuvres, the rate of variation of suspension quantities is small. A final point to be made is that as far as control design is concerned, the suspension behaviour has to be linearised anyway, at some stage of the process, since the existent nonlinear control theory is not capable of dealing with nonlinear problems of such complexity.

Concerning vehicle performance, not many conclusions can be drawn due to the limitations of the present model powertrain and braking system dynamics, as well as the longitudinal tyre model and which play an important role in vehicle performance mode and its coupling to other vehicle modes, most notoriously to vehicle handling. Nevertheless, the models of these subsystems can be incorporated into the whole simulation model with relative ease once they are available. Also the present vehicle model allows for the speculative studies of torque control in which the wheel torque generating process is immaterial. This approach together with judiciously applied control theory could lead to interesting results and should be considered in future studies. It can be stated that the lack of a model to describe wheel spin or wheel lock limits the present model.

Finally, with respect to the handling modes, it seems that for the linear tyre model, quite adequate linear approximations can be made especially for low value of lateral acceleration, based on the analysis of the present simulation results. In the transient manoeuvres it could not be observed significant changes in vehicle motion as the severity of the steering wheel input increased, unless, of course the vehicle slid as in the case of the linear tyre model or it became unstable as in the case of the nonlinear one. Based on the steady-state analysis it has not been observed any significant nonlinearities in the gain characteristics, neither for the linear nor for the nonlinear tyre, apart from those related to vehicle handling characteristics such as understeering or oversteering behaviour, although those are the most important and should be used as means of establishing performance criteria for vehicle behaviour. The results presented in the gain analysis section 5.3.3, indicates quite an interesting characteristics for the MBS part of the model, in which a high degree of linearity for many variables of motion was obtained for varying severity of steering wheel input. Even for the nonlinear tyre model, this linearity seems quite encouraging, if it was not for the instability problem that the model presented. The author believes that once this problem is solved similar conclusions might be possible for the nonlinear tyre in a broader range of vehicle behaviour, although this unstable behaviour has not impaired the fundamental conclusions of this thesis. The largest influence on the handling modes is forward velocity. This fact is well known and fairly obvious even from very simple models and has been confirmed in this study. Another influence is the existence of the time delay in the build-up process of the tyre lateral force. Without the time delay, the coupling of the forward velocity to the handling modes seem quite predictable and a situation in which vehicle speed is considered as an adaptation parameter in a linear vehicle model is quite possible. The only possible hindrance to this situation would be more complex tyre dynamics such as the time delay or the coupling between longitudinal and lateral tyre forces. The coupling with the vertical force does not seem so significant to the point in which a robust controller would not be capable of dealing with. Again if control studies are to be performed to vehicle motion linearisation at some stage might be necessary for the above mentioned reasons. Although the possibility of it not resulting in vehicle behaviour which meets the established objectives of enhanced vehicle performance should always be considered.

In any case, linear model which are adaptive with vehicle speed seem to be recommendable to successfully implement such controls. The coupling with the ride modes have been previously described and it does not seem likely that a steering control would affect suspension behaviour, however the contrary would be true.

5.4 Conclusions

This chapter has performed extensive experiments both in transient and steady-state circumstances in all aspects of vehicle motion, covering reasonable ranges of vehicle operation in order to enable the determination and the study of a valid range of model linearisation for the analysis and design of integrated vehicle motion control systems, the so called motion management. The analysis involved exercising the model in response to driver inputs such as accelerating, braking and steering as well as to external disturbances such as road and aerodynamics inputs. From these results, with the model with its present limitations, it can be concluded that for the ride aspects of vehicle behaviour the use of a linearised model seem adequate.

For the performance aspects of vehicle motion, not much could be concluded apart from the fact that the model is adequate and suitable to incorporate and encompass performance control studies due to its characteristics and modular nature. An interesting point to be noticed is that the model is not restricted to existing propulsion system concepts. It is felt that if a longitudinal tyre model was included, other conclusions could have been reached and this aspect denotes a present weakness of the model.

For the handling characteristics the existence of a linear and a nonlinear tyre models has shown that it is still feasible to consider the utilisation of integrated control to improve vehicle behaviour and the simulations have provided sufficient grounds for encouraging the design of decoupled controllers. The strongest coupling factor between the vehicle's variables of motion is forward velocity. However, in the simulation runs this coupling seemed predictable, in the sense that it affects one aspect of vehicle motion predominantly, the handling modes. This situation can be very positive from a control point of view because such coupling could be treated as an adaptation parameter which affects the other linearised model parameters in a predictable way, resulting in some kind of adaptive system.

In the next chapter, the resulting linearised models obtained from the vehicle operating in the range determined in this chapter are analysed from a numerical and control theoretical point of view. The variation in system properties such as conditioning, controllability, observability, principal gains, etc., are analysed for these models at some of these previous operating conditions.

Chapter 6

Properties of the Linearised Models

6.1 Introduction

In this chapter is discussed the control theoretic aspects of the linearised model of the vehicle developed in chapter 4 and analysed in chapter 5. Because it has been observed in chapter 5 that the model behaviour varies with forward velocity and tyre model, it was adopted to generate a set of linearised models for some of the operating conditions previously described. For this reason, linearised models have been obtained for the vehicle driving straight ahead and in steady-state cornering. According to SAE norms [233] these situations are referred to as *trim* conditions. For the analysis of the linear system, time and frequency domain representations of the model are discussed. Time domain representation is described by the state space model, in the form of the system matrices, $\boldsymbol{A}$,$\boldsymbol{B}$,$\boldsymbol{C}$,$\boldsymbol{D}$ and the frequency domain one is represented by the transfer function matrix, $\boldsymbol{G}(s)$. The discussion concerning the state space representation encompasses structural, numerical and control theoretical properties of the system matrices. Topics such as poles and zeros, frequency responses and principal gains are addressed for the transfer function representation.

6.2 State Space Model

The state space model is obtained by linearisation of the system equations through the `ANALYZ 'jacob'` facility in ACSL. Subcommands `'control'` and `'observ'` are used to define the lists of control $\boldsymbol{u}$, and observable $\boldsymbol{y}$ variables.

The nonlinear vehicle model can be represented by

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}, t) \tag{6.1}$$

where $\boldsymbol{x}$ the state vector is comprised of the generalised coordinates, the generalised speeds and the lateral tyre forces in case the time delay is included in the tyre model, and $\boldsymbol{u}$ is the input vector, which includes driver's inputs and external disturbances.

The Jacobian is the partial derivative of the state derivative vector with respect to the state vector and it is a square matrix of dimension equal to the number of state variables. When the `'control'` and `'observ'` subcommands are used, the addition of the input and output variables extends the state and the state derivative vectors, respectively. So the state vector is extended by the inputs and the derivative vector by the outputs.

ACSL through numerical perturbation of the states provides an approximation of the system's Jacobian at an operating point, $\boldsymbol{x}_o$ of the state vector. In this case, it can be written for the autonomous linearised system that

$$\dot{\boldsymbol{x}} = \boldsymbol{J}\boldsymbol{x} \tag{6.2}$$

where the ij^{th} element of $\mathbf{J}$ is given by

$$(J)_{ij} = \left.\frac{\partial f_i}{\partial x_j}\right|_{\boldsymbol{x}_o} \tag{6.3}$$

The Jacobians for the linearised systems are presented in appendix D. Section D.1 contains the 3d plots for the qualitative analysis and section D.2 their numerical values. ACSL can also determine the linear representation for the nonautonomous system. With the definition of the control variables, $\boldsymbol{u}$, and the output quantities, $\boldsymbol{y}$, a perturbation analysis is performed as before, on the augmented state vector returning the system's matrices in the form

$$\begin{aligned} \dot{\boldsymbol{x}} &= \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{u} \\ \boldsymbol{y} &= \boldsymbol{C}\boldsymbol{x} + \boldsymbol{D}\boldsymbol{u} \end{aligned} \tag{6.4}$$

where $\boldsymbol{A}$,$\boldsymbol{B}$,$\boldsymbol{C}$,$\boldsymbol{D}$ are the matrices of the state space representation of the nonlinear system described by equation 6.1. Because ACSL also includes an eigenvalue/eigenvector analysis facility, the Jacobian can be used to determine natural frequencies and modes in a modal analysis study or this same facility can be used for open loop stability analysis in a control system design. According to the way the state vector have been defined in this study, the Jacobian $\boldsymbol{J}$ and the system matrix $\boldsymbol{A}$ coincide. According to the way ACSL calculates the $\boldsymbol{A}$,$\boldsymbol{B}$,$\boldsymbol{C}$,$\boldsymbol{D}$ matrices there are normally no restrictions on the observable variables (outputs), but the control variables (inputs) specified must not be calculated in the program, that is, they must not be evaluated in the `DYNAMIC` section of the ACSL program. This is because the state and control variable lists are jointed together and then each is perturbed in the positive and negative direction calculating a column of the combined matrix:

$$\begin{bmatrix} \boldsymbol{A} & \boldsymbol{B} \\ \boldsymbol{C} & \boldsymbol{D} \end{bmatrix} \tag{6.5}$$

Since in most models, control variables are calculated quantities, extra variables are required to adjust the model, in which a perturbation term, declared as a constant, is added to the input variables and used as control variable. If an attempt is made to include a nonconstant variable to the control list, an error message (`INDEPENDENT VARIABLE CHANGED IN JACOBIAN EVALUATION`) is produced when the matrices are calculated. The remaining $\boldsymbol{B}$,$\boldsymbol{C}$,$\boldsymbol{D}$ matrices are discussed in a future section and their numerical values are presented in section D.2 of appendix D.

The calculation of the Jacobian in ACSL also verifies the occurrence of two situations, one related to function evaluation nonrepeatability and another to evaluation nonlinearity. Function nonrepeatability is verified by perturbing the state again in the positive direction, then checking to see if all values are identical

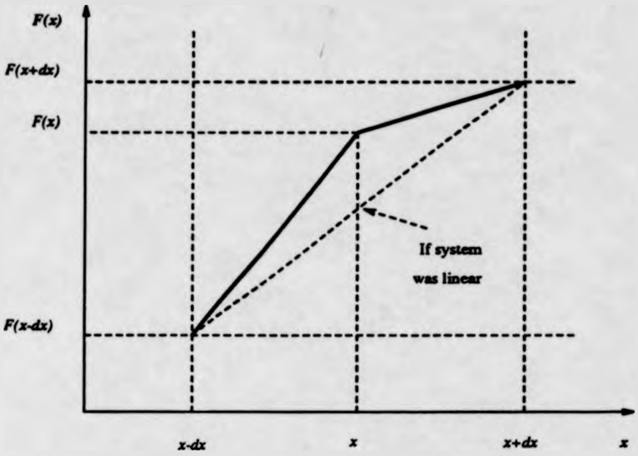

Figure 6.1: ACSL Jacobian evaluation

to the value obtained at the first perturbation. This guarantees that the state is a true function, with no side effects and will return the same value, no matter how often the function is evaluated, provided the arguments are identical. The nonlinearity is a measure of the difference between the slopes of the derivative function when perturbed positively and negatively. Figure 6.1 illustrates this situation. If the system is linear, the midpoint $F(x)$, lies on the average of the two outer points. Deviation from this midpoint implies nonlinearity. To avoid scaling problems during nonlinearity measurement calculations, the difference from midpoint is normalised by the total vertical dimension. This calculation does not work well if $F(x)$ is near a point of maximum or minimum. Therefore when $F(x)$ lies outside the rectangle of figure 6.1, no message is generated by ACSL. A number often seen in nonlinearity measures is +/-0.5, which means $F(x)$ lies at one of the corners of the rectangle of figure 6.1, indicating saturation points.

6.2.1 Properties of Resulting Matrices

Properties of the system matrices which are analysed include structural, symmetrical and block characteristics. Initially, a discussion of the A matrix and its elements is performed based on the theoretical background developed in chapter 3 and taking the considerations of chapter 5 into account. Afterwards, numerical properties such as conditioning and control related properties are discussed which include open loop stability, controllability, observability and model order reduction.

6.2.1.1 Elements of the A Matrix

The state vector x comprises the generalised coordinates, q, the generalised speeds u and the lateral tyre forces, if the first order time delay is included in the tyre model. The following quantities correspond to the state variables which are calculated in the subroutines generated by SD/FAST and have to be integrated in ACSL. The first 3 coordinates correspond to the translational degrees of freedom of the sprung mass, described in the inertial frame. They are

- q_1 = forward displacement.
- q_2 = lateral displacement.
- q_3 = vertical displacement.

The next three coordinates correspond to the rotational degrees of freedom of the sprung mass and they describe 3-2-1 body angles, according to the type of connection chosen (gimbal joint, fixed at the sprung mass centre of mass) and the order of orientation of the direction vectors of the joint. They correspond to

- q_4 = yaw angle, or rotation around the body fixed z-axis.
- q_5 = pitch angle, or rotation around the body fixed y-axis.
- q_6 = roll angle, or rotation around the body fixed x-axis.

The next four variables correspond to the angular displacement of the equivalent suspension, represented by the swing arm defined in chapter 4, and they are defined as rotations relatively to the sprung mass. The following order of arrangement was chosen

- q_7 = front left suspension angular displacement.
- q_8 = front right suspension angular dispalacement.
- q_9 = rear left suspension angular displacement.

- q_{10} = rear right suspension angular displacement.

Maintaining the same order of arrangement, the next variables correspond to the steering angle of the wheel hubs around the king pin, and they measure angular displacements in relation to the suspension arms, that is

- q_{11} = front left wheel hub steering angle.
- q_{12} = front right wheel hub steering angle.
- q_{13} = rear left wheel hub steering angle.
- q_{14} = rear right wheel hub steering angle.

The final 4 variables correspond to the rotational degrees of freedom of the wheels around the spindle axis and they are given by

- q_{15} = front left wheel rotation.
- q_{16} = front right wheel rotation.
- q_{17} = rear left wheel rotation.
- q_{18} = rear right wheel rotation.

The state vector is completed with the generalised speeds, which have an equivalent definition to the above coordinates, the difference being that they represent rates instead of positions.

Therefore, the $\boldsymbol{A}$ matrix can be divided in two submatrices, one containing the coefficients of the equations of the derivatives of the generalised coordinates, $\dot{q}_i$ and another containing the coefficients of the derivatives of the generalised speeds, $\dot{u}_i$. The former is obtained from the kinematical differential equations (eq. 3.5) and the latter from the dynamical differential equations (eq. 3.13) presented in chapter 3.

It is worth noting that the expressions for the derivatives of the generalised coordinates involve complex relationships with trigonometric functions (sin and cos) of the generalised coordinates, but they are linear in relation to the the generalised speeds, according to how they have been defined. See equation 3.4. For example, if the system had linear kinematical relationships, the coefficients C_{rs} of equation 3.4 would be constant.

For the majority of the joints in the present model, which are one degree of freedom only joints, either in translation or in rotation, the following relation holds between the derivatives of the generalised coordinates and the generalised speeds, according to the way they are defined in SD/FAST

$$\dot{q}_i = u_i \tag{6.6}$$

For the 3d rotational joint (gimbal), which is used to describe the orientation of the sprung mass, the following relationships hold between the $\dot{q}_i$ and u_i [116]

$$\begin{aligned}\dot{q}_4 &= u_4 + (u_5 s_6 + u_6 c_6) s_5 / c_5 \\ \dot{q}_5 &= u_5 c_6 - u_6 s_6 \\ \dot{q}_6 &= u_6 c_6 / c_5 + u_5 s_6\end{aligned} \tag{6.7}$$

where s_i and c_i stand for $sin(q_i)$ and $cos(q_i)$, respectively.

For a vehicle in a given trim condition, either driving straight ahead or in steady-state turning, if the roll and pitch angles are small, then the Jacobian of the kinematical differential equations is the identity matrix. Actually this situation in which roll and pitch angles are small quantities, corresponds to the great majority of non-emergency, non-accident operating conditions of vehicles, transient part of the manoeuvres included. Even if the vehicle has simultaneous yaw, pitch and roll motions, pitch and roll angles are always small and therefore the approximations $s_i = q_i$ and $c_i = 1$ can be made. If operating points around the origin are assumed for pitch and roll angles, the identity matrix results for the Jacobian of the kinematical differential equations, if the number of generalised coordinates is equal the number of generalised speeds in the linear model.

The $\boldsymbol{A}$ matrix can be further divided into 4 regions (submatrices) according to figure 6.2.

In some circumstances, the number of generalised coordinates and speeds do not coincide. For instance, if quartenions are used to specify body orientation, then there is a fourth coordinate associated with the Euler parameters, which does not have a corresponding generalised speed. Also the existence of nonholonomic constraints would cause the number of independent generalised coordinates and

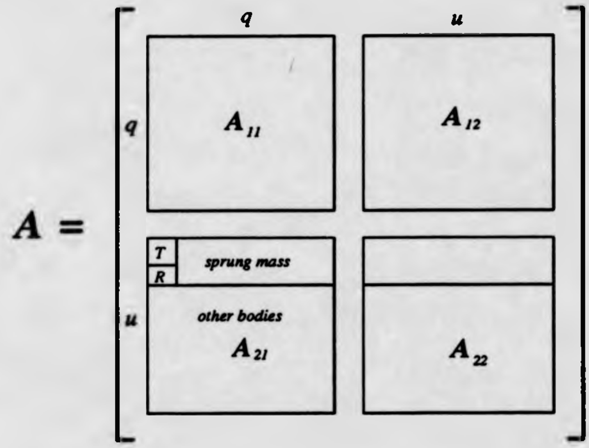

Figure 6.2: Structure of A matrix

speeds to differ. If there are m generalised coordinates and p generalised speeds, the state vector is of dimension $(m+p)$ and submatrices $\boldsymbol{A}_{11}$ and $\boldsymbol{A}_{12}$ correspond to the coefficients of the equations of the derivatives of the generalised coordinates and submatrices $\boldsymbol{A}_{21}$ and $\boldsymbol{A}_{22}$ correspond to the coefficients of the equations of the derivatives of the generalised speeds. According to the previous reasoning about the kinematical equations, submatrix $\boldsymbol{A}_{11}$ is zero and submatrix $\boldsymbol{A}_{12}$ is the identity matrix, if $m = p$. If $m \neq p$ the appropriate rows or columns are filled with zeros.

The elements of the two matrices in the lower region of the $\boldsymbol{A}$ matrix, which correspond to the coefficients of the generalised coordinates and speeds, in the equations of the derivatives of the generalised speeds, results from equations 3.13 and the analysis of their derivation is not so simple. They depend on the evaluation of equation 6.3 for functions obtained from the multiplication of the rows of the inverse of the mass matrix $\boldsymbol{M}^{-1}$, whose elements are given by equation 3.50, and the force array $\boldsymbol{f}$, as described by equation 3.51. It is worth noting that although the derivatives of the generalised speeds do correspond to acceleration terms, they do not describe the absolute acceleration of the bodies they refer to, unless for the translational coordinates of the sprung mass, because they have been defined as relative coordinates in the formalism used by SD/FAST to generate them.

The concept of *stability derivatives*, as has been originated and defined in aeronautics is widely used in vehicle dynamics [255]. They are defined as the partial derivatives of the forcing functions in relation to the states. As far as possible, this discussion will refer to the stability derivative concept in order to help clarify the meaning of the elements of the $\boldsymbol{A}$ matrix being discussed.

Submatrices $\boldsymbol{A}_{21}$ and $\boldsymbol{A}_{22}$ refer to the derivatives of the generalised speeds which can be thought of as acceleration terms. They can also be further subdivided. The first six rows of $\boldsymbol{A}_{21}$ and $\boldsymbol{A}_{22}$ correspond to the six degrees of freedom of the sprung mass. The first three of these correspond to the translational acceleration terms and the last three to the angular acceleration ones. The remaining 12 rows correspond to angular accelerations terms of the swing axles, wheel hubs and wheels.

Submatrix $\boldsymbol{A}_{21}$ contains the coefficients of the generalised coordinates in the equations of the derivatives of the generalised speeds. Because they correspond to acceleration terms which are proportional to the generalised coordinates, which represent either translational or rotational *displacements*, these coefficients can be thought of as *inertia normalised 'spring' constants.*

Submatrix $\boldsymbol{A}_{22}$ describes the coefficients of the generalised speeds in the equations of the derivatives of the generalised speeds. Because they represent proportionality factor in relation to states which correspond to *velocities*, they can be interpreted as *inertia normalised 'damping' constants.*

For the first six rows, which correspond to the six degrees of freedom of rigid body of the sprung mass, these terms possess some analogy to the stability derivative concept previously mentioned. For example, in the equation of $\dot{u}_4$, the proportionality factor of u_4 is related to the damping-in-yaw factor as it is known in vehicle dynamics nomenclature [62, 255]. With this model it is possible to assess the influence on the state variables of many degrees of freedom which are ignored in simple models, either comparatively or by themselves. For instance, in this case one has the front suspension displacement yaw torque coefficient, which has not been previously defined in the literature, and such as this one, there are many others in the present model.

The $\boldsymbol{A}$ matrices are large matrices comprised of many elements. Initially, in order to analyse them qualitatively, the `MESH` command in MATLAB is used to generate 3d plots of them for the various operating conditions. This command plots rows and columns as x and y coordinates and in the z coordinate it plots the values stored at these elements. In this way, it is possible to verify the relative values of the different elements in the matrix, as well as verifying which elements or regions of it varies most as some quantities or parameters are varied in the model. It is worth observing that these matrices have already been reduced to the states which are 'relevant'. The states which have been eliminated are those corresponding to the lateral and forward displacements of the sprung mass, variables q_1 and q_2 because they have zero eigenvalues, which correspond to poles at the origin of the system equation. Also the rotational degrees of freedom of the wheel hubs and wheels, variables q_{11} to q_{18}, with their respective generalised speeds, because they are defined as prescribed motion in SD/FAST. The topic of model order reduction is addressed in more detail in section 6.2.1.4. The yaw angle, q_4 was also expected to have a zero eigenvalue, see equation 6.7. However, it has resulted in a small value, which could not be eliminated from the equations because it is coupled to the yaw rate and lateral velocity and its elimination affected the eigenvalues associated with the handling degrees of freedom. This is a problem which seems inherent to the computer programs being used. It is worth noting that the early version of SD/FAST did not present this behaviour, however sprung mass coordinates then were defined differently from the present version. This fact have been communicated to the software company and apparently, it could be even some problem with the ACSL linear analysis facility. In a private consultation with Dan Rosenthal, one of the developers of SD/FAST, he has stated that the source of this problem might be related to the form ACSL expects the differential equations to be represented, which might not be always be the case with the equations generated by SD/FAST. Although in this particular problem, it should have not occurred according to the opinion of the author. The problem can occur when there is algebraic equations in the model, that represents the constraints equations. However, this situation does not result in invalid values, because they have been checked against the results of the earlier version of the software. One problem is that it worsens the numerical properties of the linearised system.

Although forward velocity is assumed constant, it is left in the equations of motion. Forward dynamics depends on aerodynamic and powertrain characteristics if a nonslipping tyre model is used, and on a combination of the previous together with tyre properties if slipping is considered. In the present simulation model a powertrain with negligible dynamics is considered and with a capacity to accelerate the vehicle with values according to the real vehicle performance specification.

In some of the manoeuvres, however, it is necessary to apply an acceleration force to the wheels, in order to keep the forward velocity constant. In that case it is assumed that the driving force applied is proportional to the error in vehicle speed. Depending on the value of this feedback gain, different values for the eigenvalues related to the forward velocity results. If it is assumed that it can produce a maximum driving force capable of accelerating the vehicle at a rate of 1.0 g, proportional to the error in vehicle speed in relation to the desired value, an eigenvalue of approximately 90 rd/s and damping factor of 1.0 result for the longitudinal dynamics. If lower acceleration levels are adopted, the values of the forward velocity eigenvalue will vary accordingly. For example, if a value of 0.1 g acceleration is desired, an eigenvalue of 4.28 rd/s and also critical damping result.

Therefore the state vector is of dimension 18, of which 8 are generalised co-ordinates and 10 are generalised speeds. The $\boldsymbol{A}$ matrix is of dimension 18×18 and submatrices $\boldsymbol{A}_{11}$, $\boldsymbol{A}_{12}$, $\boldsymbol{A}_{21}$ and $\boldsymbol{A}_{22}$ are of dimensions 8×8, 8×10, 10×8 and 10×10, respectively. When the tyre model includes the time delay, the state vector is increased by the four lateral forces which are included in the first four rows and columns of the $\boldsymbol{A}$ matrix. In such case the $\boldsymbol{A}$ matrix is then of dimension 22×22.

In this case, there are 4 types of $\boldsymbol{A}$ matrix: two for the linear tyre model with and without time delay in the lateral force build-up process and two for the nonlinear tyre model in similar conditions. For these four models, the simulation conditions are similar to some of those described in chapter 5 for the eigenvalue analysis. Therefore, $\boldsymbol{A}$ matrices have been generated for driving straight ahead and in steady-state turning trim conditions. The velocities used were 5, 10, 20 and 40 m/s for the former and 10, 20 and 30 m/s with steering wheel angles of 10, 45, and 90° for the latter. In some situations when model numerical instability occurred or adhesion was lost, intermediary values of steering angles were used. Therefore, angles of 60° for the 20 m/s manoeuvre and 30° for the 30 m/s ones have also been included.

The following nomenclature has been adopted for specifying the $\boldsymbol{A}$ matrices. For the straight ahead trim conditions, the matrix name is comprised of 2 characters and a number, that is

$$\underline{\mathbf{L}}\ \underline{\mathbf{O}}\ \underline{\mathbf{10}}$$

where the first letter can be either **L**, for linear or **N**, for nonlinear tyre model, the second one can be **W**, for with or **O**, for without indicating whether or not the time delay has been included. The number indicates the forward velocity of the vehicle. Therefore, in the above example, the matrix refers to a linear (**L**) tyre model without (**O**) time delay at a speed of 10 m/s.

For the cornering manoeuvres a similar coding system is adopted, but the letter **C** is included in the beginning, in order to differentiate it from the previous one. Also the number has a different meaning, since two pieces of information should be included. The first digit corresponds to the most significant digit of the speed of the vehicle and the second to the most significant digit of the steering angle at which the corner was taken. For instance

$$\underline{\mathbf{C}}\ \underline{\mathbf{N}}\ \underline{\mathbf{W}}\ \underline{\mathbf{24}}$$

indicates a cornering manoeuvre (**C**), of a vehicle incorporating a nonlinear (**N**) tyre with (**W**) time delay, travelling at a speed of 20 m/s and taking a corner with a 45° steering angle.

For the operating conditions previously described, for the models with linear and nonlinear lateral tyre force descriptions and incorporating or not the time delay in the lateral force build-up process, the 3d plots of the $\boldsymbol{A}$ matrices are presented in section D.1 of appendix D and their numerical values together with the list of the state vector is included in section D.2 of the same appendix. Section D.1.1 presents the full matrices, section D.1.2 the coefficients of the derivatives of the generalised speeds of the sprung mass, herein called 'acceleration' coefficients, while sections D.1.3 and D.1.4 depict the same coefficients for the sprung mass but separate in those related to the generalised coordinates and speeds, therefore called 'spring' and 'damping' coefficients, respectively. Within these sections the plots are divided for straight ahead and cornering trim conditions and for the tyre model with and without the time delay. Each related section presents the corresponding heading to which operating condition it refers.

Next, the qualitative analysis about the system matrices is performed. This analysis will be looking at characteristics of the full matrices, before narrowing to the various relevant sections of these matrices.

For the tyre models, linear and nonlinear, without time delay it can be observed that the largest coefficients of the $\boldsymbol{A}$ matrix are those corresponding to the generalised coordinates of the sprung mass in the equations of the derivatives of the generalised speeds of the suspension arms, figures D.1 and D.2, corresponding to the lower part of matrix $\boldsymbol{A}_{21}$ of figure 6.2. From these plots it can be observed that the variation in forward velocity does not result in any visible qualitative change in the matrices, which means that these coefficients are unaffected by variation in vehicle speed. For the tyre models with time delay, the largest coefficients belong to the yaw angle q_4, and to lateral velocity, u_2 and yaw rate, u_4. As vehicle speed is increased, the yaw angle coefficients increase proportionally while the lateral velocity and yaw rate ones remain unchanged. Figures D.3 and D.4 illustrate this situation. When the rows and columns corresponding to the lateral force states are eliminated from the $\boldsymbol{A}$ matrix, the behaviour previously observed is repeated, figures D.5 and D.6.

The behaviour for the vehicle in cornering is similar for the linear tyre model without delay, see figures D.7 and D.8, but it looks quite irregular for the nonlinear tyre with time delay. If one verifies the numerical values for those matrices, it can be observed that the yaw angle, lateral velocity and yaw rate present similar behaviour to the previously described circumstances, however the largest changes are undergone by the vertical displacement and pitch and roll angles, showing the stronger coupling effect due to the tyre model characteristics. Figures D.9 and D.10 depict this situation. Again the elimination of the lateral force states results in very similar matrices to all the previous ones, clearly indicating that the coefficients of the generalised coordinates in the equations of the derivatives of the suspension speeds is also not sensitive to changes in steady-state steering angle. These matrices are illustrated in figures D.11 and D.12.

For the coefficients of the derivatives of the generalised speeds of the sprung mass, that is the 'acceleration' terms, it can be observed that there are differences between matrices of different tyre models, but very little visible change for variations of speed and steering conditions for the same tyre model. Figures D.13 and D.14 depict these sections for linear and nonlinear tyre models without delay and figures D.15 and D.16 for the the same models, with delay. In figures D.17 to D.20 this situation is illustrated for the cornering trim conditions. For the tyre models without delay, either linear or nonlinear and for the vehicle in straight ahead or cornering trim condition, the elements corresponding to the yaw rate and lateral velocity in the equation of the derivative of the yaw rate undergo the most visible changes as vehicle speed is varied. The variation is in an inversely proportional manner for both coefficients, according to the numerical values presented in section D.2 of appendix D. The matrices corresponding to the model with time delay, do not show any significant variation in the 3d plots. Possibly because the magnitude of the largest coefficients are too large compared to the variation of the above mentioned coefficients with vehicle speed. In order to evaluate in more details these effects, the matrices of the derivatives of the generalised speeds of the sprung mass have been further subdivided. One set of matrices refers to the coefficients of the generalised coordinates and the other to the coefficients of the generalised speeds in the equations of the 'accelerations' of the sprung mass.

Concerning the coefficients of the generalised coordinates in the equations of the derivatives of the sprung mass generalised speeds, it can be stated that because the largest values of the 'acceleration' terms are associated with the generalised coordinates, similar conclusions to the previous ones are reached, that is, there are differences between sections corresponding to models with and without delay, but no visible difference for equivalent models as speed and steering angle are varied. This fact indicates that these coefficients are not sensitive to variation in these parameters. Figures D.21 to D.28 depict these facts. The only exception is for the nonlinear tyre model with delay in cornering trim condition, in which they change slightly in a irregular fashion as it can be observed in figures D.27 and D.28.

Finally, the sprung mass 'acceleration' coefficients which multiply the generalised speeds and have been named 'damping' coefficients are discussed. They are illustrated in figures D.29 to D.36. Again it can be observed from these pictures that the largest difference in coefficients are due to the existence of the time delay in the tyre model. Equivalent models, in similar conditions present quite distinct behaviour according to whether or not the time delay is included. Even between linear and nonlinear tyre models there is great similarity in the 'shape' of the matrices as illustrated by figures D.29 and D.30. For the effect of the time delay see for example figures D.29 and D.31 for the linear tyre model and D.30 and D.32 for the nonlinear one. For the models without delay, the behaviour previously mentioned about the variation of the lateral velocity and yaw rate in the equation of the yaw rate derivative is confirmed. Because of that, the relative magnitude between these coefficients and those of the speeds of the suspension arms is smaller and the latter seem to 'increase'. Figures D.29 and D.30 illustrate this situation. In the models which include the time delay, the coefficients for the sprung mass generalised speeds are much smaller or nonexistent. For these matrices, it seems that the effect of the lateral velocity and the yaw rate are confined to the terms in the equations of the derivatives of the lateral tyre forces.

6.2.1.2 Other System Matrices

This section presents and describes the $\boldsymbol{B}$,$\boldsymbol{C}$,$\boldsymbol{D}$ matrices. The $\boldsymbol{B}$ matrix is of dimension equal to the number of states by the number of inputs. The number of states depends whether or not the tyre time delay is included in the model. The number of inputs depends on what studies are intended to be undertaken. If the time delay is included, it has 22 rows, corresponding to the states, otherwise it has 18. If all the inputs to the vehicle are considered, that is, steering angle on all wheels, torques on all wheels, suspension actuator forces on all four suspension arms road vertical disturbances at the wheels and aerodynamic disturbances at the sprung mass, the total number of inputs is 17 and $\boldsymbol{B}$ ia of dimension 18×17 or 22×17. It is presented in the appendix D, section D.2.

The $\boldsymbol{C}$ matrix depends on the observability of the states and which variables are defined as outputs. The concept of observability is discussed in more detail in a future section. In our case, it was assumed full observability and the translational velocities of the sprung mass were adopted to be those expressed in the sprung mass frame, instead of those corresponding to the states which are velocities relative to an inertial frame. In this case, the matrix listed in section D.2 of appendix D results.

The $\boldsymbol{D}$ matrix, because no output was defined to depend on the inputs, is the null matrix. In some situations, depending on the definition of the output variables, it might not be zero, for example if one defines the tyre deflections to be an output variable, the equivalent terms in the $\boldsymbol{D}$ matrix corresponding to the road input would be different from zero.

6.2.1.3 Conditioning

When working with low order SISO models ($n < 5$) computers are quite forgiving and insensitive to numerical problems. For high order models and MIMO systems the finite precision arithmetic of a computer requires caution to be exercised. Since computation with real numbers can be performed only with limited precision, and it is constrained to use a particular machine number system, numerical algorithms almost always give the wrong answer. Numerical analysts have expended much effort in devising algorithms which are *good* in the sense that they don't give unnecessarily large errors. Problems whose solution is so sensitive to specification errors that large errors result simply from expressing the input data with finite precision, are said to be *ill-conditioned*. In order to get an accurate (numerically) answer from a computer, one needs

- a well conditioned problem,
- an algorithm that is numerically stable in finite precision arithmetic,
- a good software implementation of the algorithm.

A problem is said to be *well-conditioned* if small changes in the data cause only small corresponding changes in the solution. An algorithm is said to be numerically stable if it does not introduce any more sensitivity to perturbation than is already inherent to the problem. Thus a stable algorithm cannot be expected to solve an ill-conditioned problem any more accurately than the data warrant, but an unstable algorithm can produce poor solutions even for well-conditioned problems. The use of thoroughly tested and widely used computer packages relieves the research engineer of the task of defining and implementing algorithms, and allows the concentration on the problem at hand itself. For this study, the packages used for the evaluation of the system's properties, the improvement of its numerical characteristics and the simulation and control design stages were ACSL and MATLAB.

One characteristic of large vehicle models when the linearised equations are obtained is their low condition number, indicating sensitivity of the data to numerical disturbances. This is so because the typical range of natural frequencies in such models is approximately one order of magnitude. Since the condition number is associated with the singular values of the $\boldsymbol{A}$ matrix, which is related to the positive square root of the eigenvalues of the matrix squared when the matrix is real, one notices that low conditioning may result. In order to evaluate the conditioning of a matrix, the concept of *singular values* is important.

For every complex matrix $\boldsymbol{A}$ of dimension $m \times n$, the singular values of $\boldsymbol{A}$ are defined as the positive square roots of the eigenvalues of the product $\boldsymbol{A}^H\boldsymbol{A}$, if $m \geq n$ and of $\boldsymbol{A}\boldsymbol{A}^H$, if $m \leq n$, where $\boldsymbol{A}^H$ is the transpose of the complex conjugate of $\boldsymbol{A}$. One can obtain the singular value decomposition (SVD) of $\boldsymbol{A}$ as

$$\boldsymbol{A} = \boldsymbol{U}\Sigma\boldsymbol{V}^H \tag{6.8}$$

where $\boldsymbol{U}$ and $\boldsymbol{V}$ are unitary matrices of dimension $m \times m$ and $n \times n$, respectively, and

$$\Sigma = \begin{bmatrix} \Sigma_r & 0 \\ 0 & 0 \end{bmatrix} \tag{6.9}$$

where $\Sigma_r = \mathrm{diag}\{\sigma_1, \sigma_2, ..., \sigma_r\}$ and $\sigma_1 \geq \sigma_2 \geq ... \geq \sigma_r > 0$. The diagonal elements of Σ_r are the singular values of $\boldsymbol{A}$ and the columns of $\boldsymbol{U}$ and $\boldsymbol{V}$ are called left and right singular vectors, respectively.

The value of σ_r gives a measure of how close the matrix $\boldsymbol{A}$ is to a matrix of rank $(r-1)$. Examination of the singular values, therefore, also gives a reliable way of estimating the rank of a matrix. In practice, the singular values are unlikely to be exactly zero, but one normally considers them to be zero if their magnitudes are comparable to the precision of the machine one is using. If $\boldsymbol{A}$ is full rank and square, then

$$\boldsymbol{A}^{-1} = \boldsymbol{V}\Sigma^{-1}\boldsymbol{U}^H \tag{6.10}$$

The condition number of a matrix is defined as

$$cond(\boldsymbol{A}) = \frac{\overline{\sigma}(\boldsymbol{A})}{\underline{\sigma}(\boldsymbol{A})} \tag{6.11}$$

where $\overline{\sigma}(\boldsymbol{A})$ is the largest and $\underline{\sigma}(\boldsymbol{A})$ is the smallest singular value of $\boldsymbol{A}$. Note that $\mathrm{cond}(\boldsymbol{A}) \geq 1$. A useful rule of thumb in using the condition number for assessing numerical accuracy of calculations performed with the matrix is that the machine may loose the last $log_{10}[cond(\boldsymbol{A})]$ decimal places of a solution because of round-off errors during Gaussian elimination [170, 153].

The SVD of the $\boldsymbol{A}$ matrix of the linearised model in the reference configuration as described in chapter 5, results for the largest and the smallest singular values

$$\begin{aligned} \overline{\sigma}(\boldsymbol{A}) &= 30370 \\ \underline{\sigma}(\boldsymbol{A}) &= 0.00015 \end{aligned} \tag{6.12}$$

giving a condition number of about $2.0 * 10^8$, which according to the previous rule indicates that for numerical accuracy the computer may loose up to 8 decimal places (in our case out of 16) due to round-off errors. The condition numbers for the $\boldsymbol{A}$ matrices of the remaining operating conditions are presented in appendix D, section D.2.

One way of improving this situation is to find another realisation of the system, through a similarity transformation which results in a *balanced realisation.* A balanced realisation of a system is one for which the the Lyapunov equations below result in diagonal and equal controllability and observability gramians. If $\boldsymbol{A}_b$,$\boldsymbol{B}_b$,$\boldsymbol{C}_b$,$\boldsymbol{D}$, is a balanced realisation, the equations

$$\boldsymbol{A}_b\boldsymbol{P} + \boldsymbol{P}\boldsymbol{A}_b^t + \boldsymbol{B}_b\boldsymbol{B}_b^t = 0 \tag{6.13}$$

and

$$\boldsymbol{A}_b^t\boldsymbol{Q} + \boldsymbol{Q}\boldsymbol{A}_b^t + \boldsymbol{C}_b^t\boldsymbol{C}_b = 0 \tag{6.14}$$

have a common solution

$$\boldsymbol{P} = \boldsymbol{Q} = \boldsymbol{\Sigma} \tag{6.15}$$

and $\boldsymbol{\Sigma}$ is diagonal. In this case, each state is, in a certain sense, equally strongly coupled to the input and to the output and this lies in the root of the usefulness of these realisations, particularly for model order reduction. The matrices $\boldsymbol{P}$ and $\boldsymbol{Q}$ are called controllability and observability gramians, respectively, and they are defined in a later section.

One way of determining a balanced realisation is as follow. Let $\boldsymbol{A}$,$\boldsymbol{B}$,$\boldsymbol{C}$,$\boldsymbol{D}$ be a minimal realisation of the system and let $\boldsymbol{P}$ and $\boldsymbol{Q}$ be the solutions of equations

6.13 and 6.14. It can be shown that $\boldsymbol{P}$ and $\boldsymbol{Q}$ are positive definite matrices. Next, obtain the the factorisation $\boldsymbol{Q}=\boldsymbol{R}\boldsymbol{R}^T$ and then find the SVD

$$\boldsymbol{R}\boldsymbol{P}\boldsymbol{R}^T = \boldsymbol{U}\boldsymbol{\Sigma}\boldsymbol{U}^T \tag{6.16}$$

The balanced realisation is then

$$(\boldsymbol{T}\boldsymbol{A}\boldsymbol{T}^{-1}, \boldsymbol{T}\boldsymbol{B}, \boldsymbol{C}\boldsymbol{T}^{-1}, \boldsymbol{D}) \tag{6.17}$$

where

$$\boldsymbol{T} = \boldsymbol{\Sigma}^{-1/2}\boldsymbol{U}^T\boldsymbol{R} \tag{6.18}$$

Other procedures are also mentioned in the literature [153] for obtaining the similarity transformation matrix $\boldsymbol{T}$.

Using the BALREAL command in MATLAB which implements the solution of the Lyapunov equations, results in a representation of the system with condition number equal to 21.1767, for the $\boldsymbol{A}$ matrix of the system in the reference configuration, which shows a much better realisation of the system from a numerical point of view. The output quantities of the BALREAL command are the equivalent realisation of the system, $\boldsymbol{A}_b$,$\boldsymbol{B}_b$,$\boldsymbol{C}_b$,$\boldsymbol{D}$, the diagonal of the resulting controllability and observability gramian, $\boldsymbol{\Sigma}$ and the similarity transformation matrix, $\boldsymbol{T}$. One problem with these realisations, however, is that the system matrices loose their structural and block properties, since through similarity transformation they become full matrices, therefore making the physical interpretation of the coefficients impossible, since they loose their physical meaning. For this reason, it was preferred to work directly with the system matrices themselves. In our case, the loss of 4 decimal places does not represent any major drawback, and in a number of situations calculation using both realisation have been performed and the results have always agreed to an adequate degree, from an engineering point of view. For example, the resulting eigenanalysis furnished the same eigenvalues and eigenvectors for both realisation. The joint gramian $\boldsymbol{\Sigma}$, however, can still be used for model order reduction as it is discussed in a forthcoming section.

6.2.1.4 Controllability/Observability

In order to be able to achieve control systems design which have optimal behaviour in some sense, it is necessary to assess the controllability and observability properties of the open-loop system. Also, these properties establish the conditions for complete equivalence between the state space and the transfer function representations of a system.

A system is said to be completely *state-controllable* if, for any initial time t_o, each initial state $\boldsymbol{x}_o$ can be transferred to any final state $\boldsymbol{x}_f$, in a finite time, $t_f > t_o$, by means of an unconstrained input vector $\boldsymbol{u}(t)$. It means that each mode of the system must be directly affected by the input $\boldsymbol{u}(t)$ and it requires that the controllability matrix, given by equation 6.19 below has full rank.

$$\boldsymbol{M}_c = [\boldsymbol{B}|\boldsymbol{AB}|...|\boldsymbol{A}^{n-1}\boldsymbol{B}] \tag{6.19}$$

A system is said to be completely *state-observable* if every initial state $\boldsymbol{x}_o$ can be exactly determined from the measurements of the output $\boldsymbol{y}(t)$ over the finite time interval $t_o \leq t \leq t_f$. It means that for observability the output must be influenced by each state x_i. It can be shown that the system is completely observable if the observability matrix given by equation 6.20 below has full rank.

$$\boldsymbol{M}_o = [\boldsymbol{C}^T|\boldsymbol{A}^T\boldsymbol{C}^T|...|(\boldsymbol{A}^T)^{n-1}\boldsymbol{C}^T] \tag{6.20}$$

If the system is completely controllable and observable, then a state space and the transfer function matrix representations are equivalent and accurately represent the system. There are various ways of determining the controllability and observability characteristics of a system. Common approaches include rank analysis of the controllability and observability matrices previously defined, the use of similarity transformations if the system has no repeated roots, Popov-Belevitch-Hautus (PBH) rank test, use of gramians, etc [18, 49, 153]. From a numerical point of view, the use of the controllability and observability matrices should be done with considerable caution, since they have a tendency to become highly ill-conditioned [170]. The use of the controllability and observability gramians is more recommended from a numerical point of view. The gramians are symmetric matrices defined as

$$P = \int_0^\infty e^{\tau A} B B^T (e^{\tau A})^T d\tau \tag{6.21}$$

for controllability and

$$Q = \int_0^\infty (e^{\tau A})^T C^T C e^{\tau A} d\tau \tag{6.22}$$

for observability. Because the gramians satisfy the Lyapunov equation, the combined use of them with SVD allows the evaluation of these properties without performing any calculation which could result in ill-conditioned matrices. Also the application of SVD to the gramians allows the degree of controllability and observability to become a choice of threshold, since the number of uncontrollable/unobservable states corresponds the number of 'small' singular values. Just as an example, the controllability and observability matrices as well as their ranks of both the original and balanced realisations for the system in its reference configuration have been calculated in MATLAB and the value for the rank returned for both of them was 8. This meant that in both representations there were only 8 linearly independent (LI) rows/columns, indicating the degree of controllability/observability of the system. On the other hand, the use of the gramians, for both representations, returned full ranks (18), indicating complete observability and controllability, as opposed to the previous result. This has occurred because the `RANK` function in MATLAB is defined as the number of singular values of the x matrix which are larger than

$$maxsize(x) * norm(x) * eps \tag{6.23}$$

where $maxsize(x)$ is the largest dimension of x, $norm(x)$ is its Euclidean norm, and eps is a machine internal variable used in determining singularities and ranks and is given by [169]

$$eps = 2^{-52} = 2.2 * 10^{-16} \tag{6.24}$$

Because the controllability and observability matrices involve powers of the order of $(n-1)$, where n is the order of the system, in this case 18, extremely small and large values for the singular values were obtained. Therefore showing that the use of the controllability and observability matrices is not adequate for this model. In practice, it is unlikely that the singular values will be zero, but one considers them to be zero if their relative magnitude are comparable to the precision of the machine one is using. Another important point to observe is that standards ways of computing singular values are always well-conditioned.

6.2.1.5 Model Order Reduction

One of the main uses of balanced realisations as previously described is in the approximation of state space systems by simpler ones which closely reproduces the input/output behaviour of the original system. The need for such approximations arises constantly when working with multivariable systems. Some formal techniques can be used, such as Hankel approximation [153] which are optimal in the sense that they minimise the Hankel norm of the approximation error, where balanced realisations play a crucial role in obtaining these approximations.

But in practice, very good simplifications can be obtained by using a rather simple procedure in which a balanced realisation of the original system is derived and then simply truncated by discarding those parts relating to the state variables most weakly coupled to the inputs and outputs.

For this purpose the joint gramian can be used to reduce the order of the model, if the system is normalised properly. Since the joint gramian reflects the combined controllability and observability of individual states, it is reasonable to remove those states from the model to which correspond a small singular value in the joint gramian. Elimination of these states therefore retains the most important input/output characteristics of the original system, as initially intended.

For the problem of this thesis, two alternatives to model order reduction exist. The first one is performed in ACSL and the other in MATLAB using the method previously discussed. The approach to be adopted in this work will make use of both ways, at different stages of the analysis.

The elimination of some states from the state variable vector is performed in ACSL through the use of the `ANALYZ` command with the `'freeze'` option. In this case, the rows and columns corresponding to these variables are eliminated from the Jacobian or the system matrices calculations. As has been discussed before, in the representation of constrained mechanical systems, geometric constraints such as loop and prescribed motion constraints originate algebraic equations, giving rise to differential-algebraic equations for the system. A technique used to overcome this situation is the transformation of the algebraic constraint into a differential equation. With this procedure, new states variables are introduced. Because these variables are not states in *stricto sensus* they should be eliminated from the state vector. Such variables are known and quite amenable to elimination procedures in which the variables intended to be eliminated are known beforehand. In this case, therefore, the 8 degrees of freedom associated with the wheel rotation (because it is assumed no rotation) and the steering angle state variables (because they are inputs) have been deleted from the state vector. The eigenvalues corresponding to these variables can be easily perceived through the eigenanalysis in ACSL, because the user chooses the Baumgarte stabilisation constants himself. Because Baumgarte method involves feeding back the errors in the enforcement of the velocity and position constraints, second order dynamics results with natural frequency and damping factor dependent on the constants adopted. Also, as has been mentioned before, states which are obtained by pure integration, that is, they are represented by a pole at the origin have not been retained because it would result in a singular $\boldsymbol{A}$ matrix. However, they could also be included in case it is desirable, for example, in trajectory studies.

Later, after this is completed in ACSL, the joint gramians of the balanced realisation of the system matrices are used to determine further model order reduction possibilities. In the resulting model, for the reference configuration, observation of the joint gramian of the balanced realisation for the $\boldsymbol{A}$ matrix shows that there is only one order of magnitude difference between the largest and the smallest singular value, clearly indicating that all states in the present model, according to its present scaling, are important and should be retained.

6.3 Transfer Function Matrix

Another form of representing a linear multivariable system is using the concept of ***transfer function matrix***. This is simply a matrix $\boldsymbol{G}(s)$ of transfer functions in which the (i,j) element $g_{ij}(s)$ is the transfer function relating the i^{th} output to the j^{th} input. The transfer function matrix of a system with multiple inputs, multiple outputs (MIMO) can be determined from the $\boldsymbol{A}$,$\boldsymbol{B}$,$\boldsymbol{C}$,$\boldsymbol{D}$ matrices of the system as follows. Taking the Laplace transform of equation 6.4 and solving for the output vector $\boldsymbol{y}(s)$ as a function of the input vector $\boldsymbol{u}(s)$, results

$$\boldsymbol{y}(s) = \boldsymbol{G}(s)\boldsymbol{u}(s) \tag{6.25}$$

where $\boldsymbol{G}(s)$ is the transfer function matrix and it is given by

$$\boldsymbol{G}(s) = \boldsymbol{C}(s\boldsymbol{I} - \boldsymbol{A})^{-1}\boldsymbol{B} + \boldsymbol{D} \tag{6.26}$$

It is worth noting that $\boldsymbol{G}(s)$ can also be expressed as

$$\boldsymbol{G}(s) = \frac{\boldsymbol{C}.adj(s\boldsymbol{I} - \boldsymbol{A})\boldsymbol{B}}{det(s\boldsymbol{I} - \boldsymbol{A})} + \boldsymbol{D} \tag{6.27}$$

Since $\boldsymbol{C}.adj(s\boldsymbol{I} - \boldsymbol{A})$ is a polynomial matrix, it is clear that every pole of $\boldsymbol{G}(s)$ must be a zero of $det(s\boldsymbol{I} - \boldsymbol{A})$, that is, an eigenvalue of $\boldsymbol{A}$. The converse is not always true, because zeros of $det(s\boldsymbol{I} - \boldsymbol{A})$ may be cancelled in equation 6.27 and not appear as poles of $\boldsymbol{G}(s)$. Such cancellations occur precisely when the realisation $\boldsymbol{A}$,$\boldsymbol{B}$,$\boldsymbol{C}$,$\boldsymbol{D}$ is uncontrollable or unobservable. When the system is controllable and observable, then the poles of $\boldsymbol{G}(s)$ are the eigenvalues of $\boldsymbol{A}$. Because in this case $\boldsymbol{A}$ is of least dimension, that is, it involves the smallest possible number of state variables, the realisation is called *minimal.* Note that a realisation is not unique, because $\boldsymbol{A}^*, \boldsymbol{B}^*, \boldsymbol{C}^*, \boldsymbol{D}$ is also a realisation if $\boldsymbol{T}$ is any invertible matrix and they are defined as

$$\begin{aligned} \boldsymbol{A}^* &= \boldsymbol{T}^{-1}\boldsymbol{A}\boldsymbol{T} \\ \boldsymbol{B}^* &= \boldsymbol{T}^{-1}\boldsymbol{B} \\ \boldsymbol{C}^* &= \boldsymbol{C}\boldsymbol{T} \end{aligned} \tag{6.28}$$

Transfer function matrices sometimes are a convenient way of describing a system, for instance, when there are time delays through transmission paths in the system. In this case, the elements of $\boldsymbol{G}(s)$ contain transcendental functions and no state space realisation of the system exists. These functions can be easily represented in terms of frequency responses or alternatively, if state space techniques are to be used, then transfer functions of time delays should be approximated by rational functions.

In terms of control system design, the use of the transfer function matrix or the state space representation embodies the old discussion of classical versus modern control approaches. During the 1960's the classical approach to control system design was partially eclipsed by the successful applications of state space methods, particularly in the aerospace industry.

However, the performance in solving linear multivariable problems arising from more traditional process industries was less satisfactory. New interest in classical control appeared and techniques were sought to generalise to the multivariable case, frequency domain concepts such as poles, zeros, Nyquist and Root Locus diagrams, together with related design procedures [18]. Nowadays, advanced MIMO techniques and CAE tools for multivariable control design have been developed based on the transfer function concept originated by Rosenbrock [18, 153, 172, 229].

6.3.1 Poles and Zeros

In order to generalise the ideas and results of SISO systems, to multivariable feedback it is necessary to find an appropriate generalisation of poles and zeros. In SISO systems, poles and zeros of transfer functions are found as the roots of the denominator polynomial and as the roots of the numerator polynomial, respectively. Hence, the transfer function presents a singularity at a pole and it vanishes at a zero. For multivariable systems, however, it is not necessary to look for values of s for which the entire matrix becomes unbounded or zero. It is known that every rational transfer function matrix can be expressed as a polynomial matrix, divided by a common denominator polynomial [153]. This form of expressing a rational matrix is known as Smith-McMillan form. That is

$$\boldsymbol{G}(s) = \frac{1}{d(s)}\boldsymbol{P}(s) \tag{6.29}$$

where $d(s)$ is called the *pole polynomial* of $\boldsymbol{G}(s)$ and it is defined as the least common denominator of all nonzero *minors* (not elements!) of $\boldsymbol{G}(s)$. The poles of $\boldsymbol{G}(s)$ are then defined to be the roots of the pole polynomial. The matrix $\boldsymbol{P}(s)$ is a polynomial matrix of normal rank r, that is, of rank r for almost all s. It can be transformed to its equivalent Smith form such that

$$\boldsymbol{P}(s) \sim \boldsymbol{S}(s) = diag\{\varepsilon_1(s), \varepsilon_2(s), ..., \varepsilon_r(s), 0, ..., 0\} \tag{6.30}$$

in which each $\varepsilon_i(s)$ $(i = 1, ..., r)$ is a monic polynomial (it has leading coefficient 1) satisfying the divisibility property, which states that each $\varepsilon_i(s)$ divides $\varepsilon_{i+1}(s)$ without remainder. From equations 6.29 and 6.30 it follows that

$$\boldsymbol{G}(s) = \frac{1}{d(s)}\boldsymbol{S}(s) = \boldsymbol{M}(s) \tag{6.31}$$

where $\boldsymbol{M}(s)$ is the Smith-McMillan form of $\boldsymbol{G}(s)$. Note that the location of the poles of $\boldsymbol{G}(s)$ can be determined simply by examining the elements of the transfer function matrix - poles occur at those points at which the elements of $\boldsymbol{G}(s)$ have poles, although their multiplicity cannot be determined so easily. However, it is sometimes important to establish their number, for example if there exist right half-plane poles, it is necessary to know how many of them there are before applying Nyquist-like theorems.

The zeros are defined as the the roots of the zero polynomial, which in the case of a nonsingular transfer function is given by

$$z(s) = d(s).det\boldsymbol{G}(s) \tag{6.32}$$

The zero polynomial is therefore, the greatest common divisor of the numerator of all minors of $\boldsymbol{G}(s)$ of order r, where r is the rank of $\boldsymbol{G}(s)$, after these minors have been adjusted to have $d(s)$ as their common denominator. It is important to note that these zeros cannot be located by inspection of the transfer function matrix elements. Zeros defined via the Smith-McMillan form are often called *transmissions zeros.* If $\boldsymbol{G}(s)$ is not square, it is extremely unlikely that it has any transmission zeros, that is, points in the complex plane at which it loses rank, since it is necessary that several of its minors must simultaneously become singular at these points [153]. The linearised model transfer functions (for each operating condition) of the present work, with the list of input and output variables defined is non-square and present no transmission zeros.

However, it is interesting to know the characteristics of some *elements* of the transfer function matrix in terms of its poles and zeros. Because the (i, j) element,

g_{ij} represents the transfer function between the i^{th} output and j^{th} input, its poles and zeros are a form of describing its dynamic characteristics. In this case, there are transfer functions associated mainly with vehicle ride modes, whilst there are others mainly associated with handling modes. The other transfer functions describe the coupling effect between the various modes in vehicle motion.

The poles and zeros of some of the ride and handling modes transfer functions are presented in appendix D, section D.4. They are for the vehicle in the reference configuration and they are presented for the linear tyre model with and without delay, for comparative purposes. It is interesting to notice that most poles are cancelled out by a zero, but those related to the output/input pair are not cancelled. For example, in the pitch angle and front road transfer function, all poles but those related to front-end bounce and in-phase wheel hop are cancelled. For the ride modes the usual pattern is to have remaining two poles and one zero, irrespective of whether the output is vertical displacement, pitch or roll angle, subject to road or suspension force inputs. section D.4.1.

For the handling modes there is not such a regular pattern, however the poles which correspond to the modes associated to the output/input pair are retained. For example in the roll angle/front steer transfer function the rear and front out-of-phase wheel hop, the roll on springs and the lateral velocity and yaw rate poles are present. In the pitch and front steer, the front-end poles are maintained and for the pitch angle and rear steer the rear-end ones are not cancelled.

Another interesting result which confirms the type of time domain behaviour observed in the lateral velocity responses for medium and high speeds in the transient manoeuvres illustrated, in figures C.2 to C.32, is the existence of a right half-plane zero, indicating non-minimum phase behaviour for the lateral velocity/front steer transfer function. These results are presented in section D.4.2.

6.3.2 Frequency Response

The response of dynamical systems to sinusoidal inputs, called *frequency response*, is the basis for many important analysis and design methods for dynamic systems of all kinds, including control systems. By definition, a system's frequency response refers to the sinusoidal steady-state response of the system. Its interest is only the *gain* or *amplitude ratio* and the *phase angle* of the output variable in relation to the corresponding input. Both amplitude ratio and phase angle change as different frequencies of excitation, ω are used. The desired results are graphs of these two quantities plotted against frequency, resulting in the so called frequency response curves.

Although for linear models the frequency response curves may be obtained by getting the particular solution to the differential equation for many different values of ω, the sinusoidal transfer function provides a much better method. It can be shown that if s in any Laplace transfer function is replaced by $i\omega$, ($i = \sqrt{-1}$), a complex number results whose *magnitude* is the gain (or amplitude ratio) and its *phase* is the phase angle between output and input quantities (Negative phase indicating that the output lags the input). What is not obvious, but it is extremely important, is that the frequency response curves are a complete description of the system's dynamic behaviour and allow one to compute the system's response for any input and not only just sine waves.

Evaluation of the frequency response of a system requires the computation of the transfer function matrix $G(s)$ at a number of points on the complex plane. If the model is a continuous-time system these points are the upper part of the imaginary axis and if the system is a discrete-time one, they lie on the upper half of the origin centred unit circle. Practical implementations of frequency response evaluation have to be careful about the algorithm chosen as well as the system representation form, since accurate evaluation of polynomials of high degree can be surprisingly difficult. Maciejwoski [153] discusses different approaches with regard to accuracy and number of operations and suggests that obtaining the frequency response from the state space representation of the system leads to efficient and stable calculations.

For the vehicle model in the reference configuration, with the linear tyre model, with and without the time delay, a number of frequency response curves have been obtained for various pairs of output/input variables. These results are presented in appendix D, section D.5. These results correspond to the frequency response of the elements of the transfer function matrix whose poles and zeros have been presented in the previous section. The frequency response is another alternative way of describing the dynamics of the vehicle system and it works as complementary information to the previous one. For example, the poles and zeros description does not allow for the evaluation of the static (ω =0) gain which is a measure of the strength of the coupling between the various modes. This information is provided by the frequency response plots and it can be stated that variables with low gain are weakly coupled. This implies that the input produces small amounts of the corresponding output. Many other frequency response curves could have been generated, (actually $17 \times 20 = 340$!!) however, it was chosen to illustrate only the most important ones. Other plots, for example for left/right or front/rear input variables have presented similar behaviour as those already presented with the characteristics described in the previous section and were chosen to be omitted. It is interesting to observe that the existence of the time delay in the tyre model causes the transfer function of the lateral velocity/front road to have an extra pole, located at the inverse of the time constant of the lateral force dynamics, which is not cancelled. This effect can be clearly observed in its phase angle plot in which it tends to -180°, instead of -90° as it is the case for the model without time delay.

6.3.3 Principal Gains (Singular Values)

A very brief discussion about multivariable feedback control is included in this section since this is necessary to allow the the discussion of the meaning and utility of principal gains. In a SISO system, the performance of a feedback loop can be determined by the variation of the open-loop gain with frequency. If the open-loop transfer function has no right half-plane zeros (minimum phase) and assuming stability has been achieved then stability margins and closed-loop transient response are determined by the open-loop transfer function characteristics.

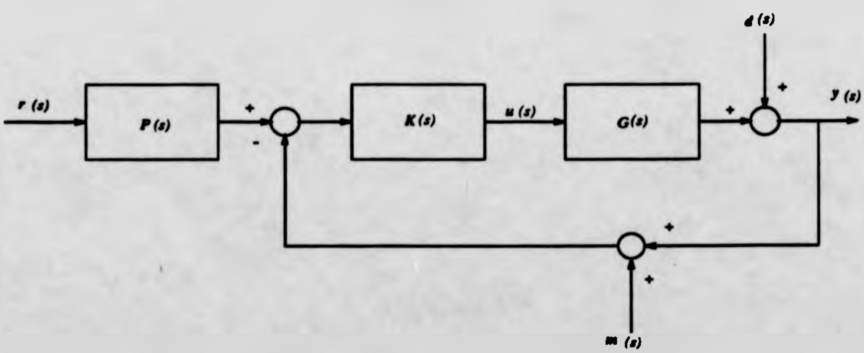

Figure 6.3: Standard feedback configuration

Similarly to SISO systems, for the multivariable feedback control system illustrated in figure 6.3 the following relationships apply

$$\boldsymbol{y}(s) = \boldsymbol{S}(s)\boldsymbol{d}(s) + [\boldsymbol{I} - \boldsymbol{S}(s)]\boldsymbol{P}(s)\boldsymbol{r}(s) - [\boldsymbol{I} - \boldsymbol{S}(s)]\boldsymbol{m}(s) \tag{6.33}$$

where

$$\boldsymbol{S}(s) = [\boldsymbol{I} + \boldsymbol{G}(s)\boldsymbol{K}(s)]^{-1} \tag{6.34}$$

is called the *output sensitivity*, or alternatively

$$\boldsymbol{y}(s) = [\boldsymbol{I} - \boldsymbol{T}(s)]\boldsymbol{d}(s) + \boldsymbol{T}(s)\boldsymbol{P}(s)\boldsymbol{r}(s) - \boldsymbol{T}(s)\boldsymbol{m}(s) \tag{6.35}$$

where

$$\boldsymbol{T}(s) = \boldsymbol{S}(s)\boldsymbol{G}(s)\boldsymbol{K}(s) \tag{6.36}$$

is the *closed loop transfer function.*

From equation 6.33 it can be concluded that if the feedback system is to attenuate the effects of disturbances, $\boldsymbol{S}(s)$ should be small and if it is desired that the output is insensitive to measurement noise it can be observed from equation 6.35 that $\boldsymbol{T}(s)$ should be made small. If tracking of reference signal is desired, $\boldsymbol{T}(s)$ should be made nearly unity in the range of the frequency content of $\boldsymbol{r}(s)$.

For a multivariable feedback system, the norm of $||\boldsymbol{G}(s)\boldsymbol{u}(s)||$ depends on the directions of the vector $\boldsymbol{u}(s)$ and a single gain cannot be defined as it is the case for SISO systems. However, the ratios

$$\frac{||\boldsymbol{G}(s)\boldsymbol{u}(s)||}{||\boldsymbol{u}(s)||} \tag{6.37}$$

and

$$\frac{||\boldsymbol{G}^{-1}(s)\boldsymbol{y}(s)||}{||\boldsymbol{y}(s)||} \tag{6.38}$$

can be bounded using matrix norms. This fact leads to the idea of replacing the concept of a single gain by the notion of a range of gains, this range been bounded above and below. Substituting s for $i\omega$ $(0 \leq \omega \leq \infty)$ in $\boldsymbol{G}(s)$ then the singular values of $\boldsymbol{G}(i\omega)$ are functions of ω and they are called the *principal gains* of $\boldsymbol{G}(s)$. If $\boldsymbol{G}$ is square and non-singular, it can be shown that

$$\underline{\sigma}(\omega) \leq \frac{||G(i\omega)u(i\omega)||}{||u(i\omega)||} \leq \overline{\sigma}(\omega) \tag{6.39}$$

which shows that the gain of a multivariable system is bounded by the smallest and largest principal gains.

With each principal gain it can be associated a pair of principal directions, defined as the columns and rows of the unitary matrices defined in section 6.2.1.2, U and Y^H, respectively. The *input principal directions* are defined as the columns of U and the rows of Y^H are the *output principal directions.* Let the rows of Y^H be y_i^H and the columns of U be u_i, then it can be shown [153] that the system gain is precisely σ_i if the input signal is in the direction of u_i. In particular, the greatest possible gain, $\overline{\sigma} = \sigma_1$, occurs if the input signal is in the direction of u_1 and the smallest possible gain, $\underline{\sigma} = \sigma_l$, occurs if it is in the direction of u_l. It can also be shown that if the input is in the direction of u_i, the output vector is in the direction of y_i.

Plotting the principal gains for the the system with and without delay results in the diagrams of figure 6.4. From this diagram it can be noted that there is a large band between the smallest and largest singular values of about 100 db, if it is considered that the singular values in the region of -250 to -400 db are actually zero. This shows that there is coupling in the system and that compensation techniques are necessary to achieve diagonal dominance, in order to be able to apply single-loop control design techniques.

In the closed loop configuration, what should be done to assess the disturbance rejection properties of the closed loop is to examine the principal gains of $S(s)$. If the region between $\overline{\sigma}(S)$ and $\underline{\sigma}(S)$ is narrow then it is almost a SISO control system design. If the region is not narrow, it is the upper boundary that it is important in order to keep sensitivity small. For the rejection of measurement noise it is the upper boundary of the principal gains of $T(s)$ that is important. For the tracking of reference signal, the lower and upper boundaries of the closed loop transfer function should be kept as close to unit as possible. Summarising, the closed-loop requirements are

- Sensitivity: keep $\overline{\sigma}(S)$ as small as possible.
- Noise propagation: keep $\overline{\sigma}(I - S)$ as small as possible.

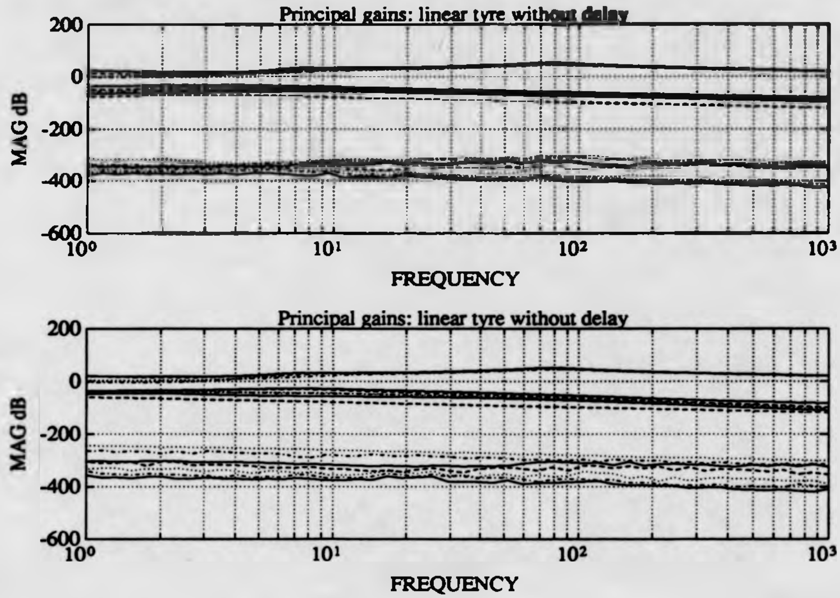

Figure 6.4: Principal gains

- Tracking of reference signal: keep $\underline{\sigma}(\boldsymbol{T}) \simeq 1$ and $\overline{\sigma}(\boldsymbol{T}) \simeq 1$.

- Minimisation of control energy: keep $\overline{\sigma}(\boldsymbol{K})$ as small as possible.

As it can be observed from these requirements, compromises are always involved in the design of control systems.

Plotting the principal gains for the the system with and without delay results in the diagrams of figure 6.4. From this diagram it can be noted that there is a large band between the smallest and largest singular values of about 100 db, if it is considered that the singular values in the region of -250 to -400 db are actually zero. This shows that there is coupling in the system and that compensation techniques are necessary to achieve diagonal dominance, in order to be able to apply single-loop control design techniques.

The present model description as it stands is not in a final form for control design, since the disturbances at the moment are included in the input vector. Only a minor change would be necessary to make it adequate for the application of control concepts. However, to actually perform control studies are beyond the objectives of this thesis. Nevertheless, some preliminary investigations have been carried out in the area of suspension control using controllable dampers and pole-placement techniques and the results achieved seem encouraging. Suspension control design results for a quarter car model using the same set of tools as the ones of this thesis are presented by Haycock [92]. In her work she presents the results comparing a number of control techniques, including optimal control, pole-placement and frequency domain multivariable.

Also, the combination of ACSL and MATLAB for obtaining the linearised model, designing the controller and simulating it controlling the nonlinear model, interactively, provide a very powerful tool for the analysis and design of control systems.

6.4 Conclusions

In this chapter the linearised models for the nonlinear vehicle model have been determined and analysed in operating conditions which have been previously defined as relevant to cover all aspects of vehicle motion in a reasonable range of quantities variation. For the purpose of the analysis to be carried out in this chapter, state space and transfer function representations have been used. State space representation have been obtained in ACSL and transfer function matrix representation were derived using the Multivariable Control Toolbox of MATLAB. It was analysed and discussed numerical and control theoretical properties of the linearised systems.The effect of forward velocity and tyre model on the system characteristics have also been addressed. It could be observed that although these representations are equivalent from a system theory point of view, each presents different aspects of system behaviour which adds together and help clarify the understanding the overall behaviour of the linearised system and the effects of system variables and parameters on its characteristics. Because the representation adopted was minimal, the eigenvalues of the system matrix allowed not only the determination of the system modal behaviour, but also open-loop stability analysis for control design. A brief mention about the possibilities and ways of carrying out control analysis and design have also been made, in which the integration of ACSL and MATLAB for that purpose is highlighted.

Chapter 7

Conclusions

This thesis has proposed the study of integrated vehicle control under the motion management heading. For that purpose, this work has revised the application of automatic control to automotive vehicles. It could be observed that most control applications to date adopted a piecemeal approach, whereby vehicle subsystems were studied in isolation. It has realised that the motor car is a complex system made of interacting subsystems and that these interactions are complex. It has observed that there are appropriate circumstances to initiate investigations into integrated vehicle control. These circumstances include market competition amongst vehicles manufacturers, the availability of present day state-of-the-art computer packages for the simulation and design of vehicle systems, the present development of microelectronics and the widespread use of the concept of active system. A discussion concerning possible approaches for obtaining and solving mathematical models for the study of automotive vehicle control has also been carried out.

Modelling approaches used to date in most situations are of two types, either hand derived or computer generated codes. Hand derived models are usually simple models and they are obtained from the application of first principles. In this thesis, the computer generated codes have been addressed in relation to the mathematical formalism adopted and the nature of the computer implementation used for the generation of the equations of motion. Computer generated models, of which the most common to date are those derived using the code ADAMS, are generally very complex and do not present the flexibility to include in an easy and modular way vehicle subsystems. In this case, all the equations for the vehicle have to be generated and solved together. However, to date this and other similar codes are the most widely used by the automotive research community and automotive manufacturers.

Just as an example, the number of ADAMS installations at Ford, Michigan, USA, outnumbers all the installations of the remaining codes commercially available worldwide!! However, in the research of MBS computer codes, there is a trend towards the use of Kane's Method as the mathematical formalism and symbolic algebra computation for the generation of the equations of motion in the form of computer codes. Also, there is an increasing tendency of adopting an object oriented programming approach to the description of the MBS. The advantage of these approaches is that with Kane's Method the equations of motion are generated as a set of scalar equations, in minimal form and the use of symbolic algebra makes use of the particular characteristics of the system been modelled. The use of object oriented approach is that MBS elements can be divided according to the classification of object-oriented programming. It has been proved in some specific examples, that the combination of Kane's Method and symbolic programming generates the fastest codes.

However, the code generated by these programs, even the symbolic generated ones, when all aspects of vehicle dynamics and suspension details are taken into consideration, it results in computationally expensive computer codes, mainly due to the existence of kinematical closed loops. Experience of the author with the execution of the road vehicle IAVSD benchmarks, has shown that models with up to 78 states, 20000 lines of FORTRAN code an involving approximately 140000 operations may be generated, for the complete description of a Canadian military jeep. Such simulation can take up to 6000 seconds of a SUN 4/330 machine for the simulation of 1 second of real time. For control analysis and design this situation makes the development too slow. For this reason, a technique which accounts for the suspension geometric characteristics in an approximate manner, using differential geometry concepts, but still is adequate to describe a vehicle capable of performing all aspects of vehicle motion have been developed. The resulting code due to the fact that it only involved open loop topology, presented a smaller number of states and therefore, runs much faster. In this way control analysis and design can be carried out more efficiently. Just for comparative purposes, the basic model described in this work, has 20 states, 5000 lines of FORTRAN code but only 4000 operations and executes 1 second of real time simulation in 7.5 seconds, in an equivalent machine.

Therefore, a large number of operating conditions were feasible to be simulated in order to test and explore the model in a wide range of vehicle manoeuvres, embracing all aspects of its motion. These simulations have shown for the present model that the largest influencing factor in vehicle behaviour is the tyre model and the largest influencing variable of motion is forward velocity. The influence of the tyre model is represented by two aspects: load transfer effects and the time delay in the lateral force build-up process. It is also known from the literature that the coupling between longitudinal and lateral slip is important, but at the present model this fact has not been incorporated yet. This remains an area of continuing research with the present model. The influence of forward velocity is a consequence of the nature of the physical laws which govern the motion of rigid bodies, and the way chosen to represent them.

The simulation results have also shown that decoupling of the ride modes for suspension design is possible, under the simplifying assumptions of the present model, but for the handling modes, variables related to the ride and performance modes are important to be retained. For example, in suspension control design, steering wheel displacement and wheel torque are not important but for vehicle handling and performance design, suspension variables and parameters should be considered.

In the analysis of the linear model it was observed that even for 'simple' full vehicle models the system representation is ill-conditioned in some sense, and therefore requires to be carefully monitored throughout the analysis and design stages. It has also been shown that the dependence of the linear model on forward velocity makes it unsuitable for the complete range of manoeuvres, since lateral dynamics are strongly influenced by vehicle speed. However, the linearised model with constant parameters is adequate for simulating manoeuvres in which large variation of vehicle speed does not occur during the length of the simulation. Many real life situations such as vehicle transversing road profile at constant speed or steady-state cornering or even short period emergency manoeuvres fall in such category. Nevertheless, the linear analysis has shown that for vehicle representation including the tyre models without the time delay in the lateral force build-up process, it is possible to trace the influence of forward velocity to a small number of system variables and, most importantly, this influence seems to be 'well' behaved. This means, for instance that it might be possible to include it in an 'adaptive' type of 'linear' model (piecewise linear model). This aspect of modelling possibility has not been investigated in the present thesis and it is also an area which presents possibilities for future research.

While there is full awareness of the limitations of the present model, mainly with regard to the tyre model, as far as combined slip is concerned, and suspension geometry in relation to camber and toe-in effects; the primary purpose of the model developed in this work was not to derive a fully comprehensive vehicle model. This can and has been done somewhere else by the author, whom has shown that the research facilities existing at Warwick are adequate for this purpose.

This thesis proposed to address the motion management issue, in which a model complex enough for describing the complete motion of a full vehicle in the normal range of operation and yet simple enough from a control point of view was sought. The purpose was to verify the possibility of using such model for integrated vehicle control herein named motion management. For that reason it has been presented in a formal way an approach to approximate suspension geometry effects. Although such approach has been mentioned and used before in the literature and by vehicle manufacturer's it has not been so comprehensively presented in any publication in the English language, to the knowledge of this author. It has used state-of-art computer programs for performing the task of obtaining a executable model and has executed extensive simulation for model validation and analysis. Another step taken in this thesis towards the study of the motion mangement topic was the through analysis performed with the resulting linear models. It is felt by this author that the objectives established in this thesis have been achieved and that more knowledge is now available in order to give a contribution to future researches in the area of motion management.

This work also presents a tutorial characteristic in its structure and organisation in a number of aspects related to vehicle modelling. Therefore it gives a contribution to the initiation of newcomers to the area of automotive vehicle motion analysis and control. Finally, one of the main contributions of this work is the extensive simulation analysis and consequent number of *linearised* models describing the full motion of an automotive vehicle. In this sense, these models might be used for control design studies by the control community in a similar way to the well known aircraft linear models (F14(6)) available in the literature, in the form of the $\boldsymbol{A}$,$\boldsymbol{B}$,$\boldsymbol{C}$,$\boldsymbol{D}$ matrices presented in appendix D.

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TITLE ... Application of Multibody System (MBS) Techniques to Automotive Vehicle Chassis Simulation for Motion Control Studies

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Application of Multibody System (MBS) Techniques to Automotive Vehicle Chassis Simulation for Motion Control Studies

by

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A thesis submitted for the degree of Doctor of Philosophy

Engineering Department

University of Warwick

Coventry, UK.

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Volume 2

Appendix A

SD/FAST Input File

```
#Susp4g.sd
#Input file for suspension study of an 8 bodies automotive vehicle -
#Three massless.
#Third  model- Vehicle in the 'normal' position.
#Use of gimbal joints to capture tumble motion and also work
#with 3-2-1 Euler angles.

#preamble

single
gravity  =  0.0    0.0     -9.81

#Bodies description

Body  = Longitudinal        inb = $ground        joint = slider

          mass        =  0.0
          inertia     =  0.0        0.0     0.0
          inbtojoint  =  0.0        0.0     0.0
          bodytojoint =  0.0        0.0     0.0
          pin         =  1.0        0.0     0.0


Body  =  Lateral           inb = Longitudinal      joint = slider

          mass        =  0.0
          inertia     =  0.0        0.0     0.0
          inbtojoint  =  0.0        0.0     0.0
          bodytojoint =  0.0        0.0     0.0
          pin         =  0.0        1.0     0.0


Body  =  Vertical        inb = Lateral       joint = slider

          mass        =  0.0
          inertia     =  0.0        0.0     0.0
          inbtojoint  =  0.0        0.0     0.0
          bodytojoint =  0.0        0.0     0.0
          pin         =  0.0        0.0     1.0


Body  =  Sprung_mass     inb = Vertical  joint - gimbal

           mass       =  1600
          inertia     =  500      3000     3000
          inbtojoint  =  0.0        0.0     0.50
          bodytojoint =  0.0        0.0     0.0
          pin         =  0.0        0.0     1.0
          pin         =  0.0        1.0     0.0
          pin         =  1.0        0.0     0.0


Body  =  Left_front_wheel   inb = Sprung_mass   joint - pin

          mass        =   50
          inertia     =    0         0        0

          inbtojoint  =    1.44090214828721   0.30292586469092
                          -0.24179219374438

          bodytojoint =    0.12090214828721  -0.44707413530908
                           0.05820780625562

          pin         =    0.96586483194595   0.25738705686472
                          -0.02927506393730

```



```

Body  =  Right_front_wheel   inb = Sprung_mass   joint = pin

          mass        =   50
          inertia     =    0         0        0

          inbtojoint  =    1.44090214828721  -0.30292586469092
                          -0.24179219374438

          bodytojoint =    0.12090214828721   0.44707413530908
                           0.05820780625562

          pin         =    0.96586483194595  -0.25738705686472
                          -0.02927506393730


Body  -  Left_rear_wheel   inb = Sprung_mass   joint - pin

          mass        =   40
          inertia     =    0         0        0

          inbtojoint  =   -1.51628978297744   0.10455266370603
                          -0.15664990979858

          bodytojoint =   -0.01628978297744  -0.64544733629397
                           0.14335009020142

          pin         =    0.99360525478972   0.0000000
                           0.11290968804429


Body  =  Right_rear_wheel   inb = Sprung_mass   joint = pin

          mass        =   40
          inertia     =    0         0        0

          inbtojoint  =   -1.51628978297744  -0.10455266370603
                          -0.15664990979858

          bodytojoint =   -0.01628978297744   0.64544733629397
                           0.14335009020142

          pin         =    0.99360525478972   0.0000000
                           0.11290968804429
```

```
#susp6g.sd
#Input file for suspension study of a 16 bodies automotive vehicle
#9 with mass and 7 massless
#Use of gimbal joint to capture tumble motion and also
#to work with 3-2-1 Euler angles.
#model includes massless wheel hubs and rotating wheels.

#preamble

single
grounded
gravity  = 0.0   0.0  -9.81

#Bodies description

Body  =  Horizontal        inb = $ground        joint = slider

        mass          =  0.0
        inertia       =  0.0        0.0      0.0
        bodytojoint   =  0.0        0.0      0.0
        pin           =  1.0        0.0      0.0

Body  =  Lateral          inb = Horizontal      joint = slider

        mass          =  0.0
        inertia       =  0.0        0.0      0.0
        inbtojoint    =  0.0        0.0      0.0
        bodytojoint   =  0.0        0.0      0.0
        pin           =  0.0        1.0      0.0

Body  =  Vertical          inb = 'Lateral       joint = slider

        mass          =  0.0
        inertia       =  0.0        0.0      0.0
        inbtojoint    =  0.0        0.0      0.0
        bodytojoint   =  0.0        0.0     -0.50
        pin           =  0.0        0.0      1.0

Body-Sprung_mass         inb = Vertical         joint = gimbal

        mass          = 1600.0
        inertia       = 500.0     3000.0   3000.0
        inbtojoint    =  0.0         0.0      0.0
        bodytojoint   =  0.0         0.0      0.0
        pin           =  0.0         0.0      1.0
        pin           =  0.0         1.0      0.0
        pin           =  1.0         0.0      0.0

Body = Left_front_susp_arm  inb = Sprung_mass   joint = pin

        mass          =   10.0
        inertia       =    0.0         0.0            0.0
        inbtojoint    =    1.4409022   0.3029258     -0.2417922
        bodytojoint   =    0.0604511  -0.2235371      0.0291039
        pin = 0.96586481805904   0.25738710481244   -0.02927510054736

Body = Right_front_susp_arm  inb = Sprung_mass  joint = pin

        mass          =   10.0
        inertia       =    0.0         0.0            0.0
        inbtojoint    =    1.4409022  -0.3029258     -0.2417922
        bodytojoint   =    0.0604511   0.2235371      0.0291039
        pin = 0.96586481805904  -0.25738710481244   -0.02927510054736

Body = Left_rear_susp_arm  inb = Sprung_mass    joint = pin

```



```
        mass          =    5.0
        inertia       =    0.0          0.0          0.0
        inbtojoint    =   -1.5162898    0.10455266  -0.15664990
        bodytojoint   =   -0.0081449   -0.32272367   0.07167505
        pin = 0.99360525402480                   0   0.11290969477555

Body = Right_rear_susp_arm inb = Sprung_mass    joint = pin

        mass          =    5.0
        inertia       =    0.0          0.0          0.0
        inbtojoint    =   -1.5162898   -0.10455266  -0.15664990
        bodytojoint   =   -0.0081449    0.32272367   0.07167505
        pin = 0.99360525402480                   0   0.11290969477555

Body = Left_front_hub  inb = Left_front_susp_arm    joint = pin
                                                    prescribed =0?

        mass          =    0.0
        inertia       =    0.0          0.0          0.0
        inbtojoint    =   -0.0604511    0.2235371   -0.0291039
        bodytojoint   =    0.0          0.0          0.0
        pin = 0.06899390125287   0.14745720267769   0.98665911791681

Body=Right_front_hub  inb=Right_front_susp_arm  joint=pin
                                                prescribed =0?

        mass          =    0.0
        inertia       =    0.0          0.0          0.0
        inbtojoint    =   -0.0604511   -0.2235371   -0.0291039
        bodytojoint   =    0.0          0.0          0.0
        pin = 0.06899390125287  -0.14745720267769   0.98665911791681

Body=Left_rear_hub  inb=Left_rear_susp_arm  joint=pin
                                            prescribed =0?

        mass          =    0.0
        inertia       =    0.0          0.0          0.0
        inbtojoint    =    0.0081449    0.32272367  -0.07167505
        bodytojoint   =    0.0          0.0          0.0
        pin           =    0.0          0.0          1.0


Body=Right_rear_hub  inb=Right_rear_susp_arm  joint=pin
                                              prescribed =0?

        mass          =    0.0
        inertia       =    0.0          0.0          0.0
        inbtojoint    =    0.0081449   -0.32272367  -0.07167505
        bodytojoint   =    0.0          0.0          0.0
        pin           =    0.0          0.0          1.0


Body=Left_front_wheel  inb=Left_front_hub  joint=pin
                                           prescribed =0?

        mass          =   40.0
        inertia       =    0.75        1.50        0.75
        inbtojoint    =    0.0         0.0         0.0
        bodytojoint   =    0.0         0.0         0.0
        pin           =    0.0         1.0         0.0

Body=Right_front_wheel  inb=Right_front_hub  joint=pin
                                             prescribed =0?

        mass          =   40.0
        inertia       =    0.75        1.50        0.75
```

```
        inbtojoint  =    0.0          0.0          0.0
        bodytojoint =    0.0          0.0          0.0
        pin         =    0.0          1.0          0.0


Body=Left_rear_wheel  inb=Left_rear_hub  joint=pin
                                         prescribed =0?

        mass        =   35.0
        inertia     =    0.75         1.50         0.75
        inbtojoint  =    0.0          0.0          0.0
        bodytojoint =    0.0          0.0          0.0
        pin = 0   0.99991432751132   0.01308960036014


Body=Right_rear_wheel  inb=Right_rear_hub  joint=pin
                                           prescribed =0?

        mass        =   35.0
        inertia     =   0.75          1.50         0.75
        inbtojoint  =    0.0          0.0          0.0
        bodytojoint =    0.0          0.0          0.0
        pin = 0   0.99991432751132   0.01308960036014
```

Appendix B

ACSL Language Listings

B.1 ACSL Program

Jan 16 1992 10:21:36 **susp4g.acsl** Page 1

```
'===================================================================='

'.........1.........2.........3.........4.........5.........6.......'
'ACSL program for simulating an automotive vehicle consisting...
 of a 5(8) body, 10 DOF model, incorporating suspension geometry ...
 effects, for handling and ride study purposes.'

'Program includes variables initialization for acsl new version.'

'Simple linear  vertical, longitundinal and lateral tire dynamics.'

'Aerodynamics effects included are drag, lateral and lift forces...
 and  roll, pitch and yaw torques.'

'This program is the implementation of the new version of ...
 sdfast. (sdfast.beta). '

'It considers the connection between the ground and the sprung ...
 mass to be made by 3 massless bodies which have the translational...
 degrees of freedom and a 3-2-1 gimbal joint.'

'================================================================'


PROGRAM SUSP4G    $ ' SD/FAST.BETA  '


'================= DATA SECTION ================================'

  INTEGER ROW, GROUND, SPMASS, ARM(4)
  INTEGER LONG, LAT, VERT, YAW, PITCH, ROLL, PIN
  ARRAY CB(4)
  ARRAY KLAF(4)
  ARRAY KLA21(4), KLA22(4), KLA23(4)
  ARRAY KLAS(4)
  ARRAY RAMPZ(4)
  ARRAY RAMPX(4)
  ARRAY UZ(4)
  ARRAY US(4)

'Names of sdfast joints in susp4g_info.'

  CONSTANT GROUND  = 0
  CONSTANT LONG    = 1
  CONSTANT LAT     = 2
  CONSTANT VERT    = 3
  CONSTANT SPMASS  = 4
  CONSTANT ARM   = 5, 6, 7, 8

'Names of sdfast axis for joint 4 (sprung mass orientation.)'

  CONSTANT YAW     = 1
  CONSTANT PITCH   = 2
  CONSTANT ROLL    = 3

  CONSTANT PIN     = 1

'Other program constants.'

  CONSTANT KLAF    = 4*0.0     $ 'Constant for linear anal. sforce.'
  CONSTANT KLA21   = 4*0.0     $ 'Constant for linear anal. zroad.'
  CONSTANT KLA22   = 4*0.0     $ 'Constant for linear anal. zroad.'
  CONSTANT KLA23   = 4*0.0     $ 'Constant for linear anal. zroad.'
  CONSTANT KLAS    = 4*1.0     $ 'Constant for linear anal. steer.'
```

Jan 16 1992 10:21:36 **susp4g.acsl** Page 2

```
  CONSTANT SR      = 20        $ 'Steering/road wheel ratio.'
  CONSTANT RAMPZ   = 4*0.0     $ 'Ramp height [m].'
  CONSTANT RAMPX   = 4*2.0     $ 'Ramp length [m].'
  CONSTANT UFA     =12*0.0     $ 'Pert. term for lin. anal. sforce.'
  CONSTANT UZ      = 4*0.0     $ 'Pert. term for lin. anal. zroad.'
  CONSTANT US      = 4*0.0     $ 'Pert. term for lin. anal. steer.'
  CONSTANT SIMLEN  = 5.0       $ 'Simulation length [s].'
  CONSTANT CB      = 4*0.0     $ 'Braking coefficient [% of G].'
  CONSTANT TS      = 0.5       $ 'Inital time of steering input'
  CONSTANT TB      = 0.5       $ 'Intial time of brake input.'
  CONSTANT UMAX    = 25.0      $ 'Maximum forward velocity [m/s].'
  CONSTANT UMIN    = 1.0       $ 'Minimum forward velocity [m/s].'
  CONSTANT TDF     = 0.0       $ 'Initial time for driving force.'
  CONSTANT KA      = 0.0       $ 'Actuator proportional gain.'
  CONSTANT BA      = 0.0       $ 'Actuator derivative gain.'
  CONSTANT ROW     = 1         $ 'Index for matrices.'
  CONSTANT G       = 9.81      $ 'Gravity acceleration [N/m^2]'
  CONSTANT MASS    = 1600      $ 'Vehicle mass [Kg].'
  CONSTANT PI=3.14159265358979 $ 'pi.'
  CONSTANT SF      = 10       $ ' Force scaling factor.'



'Vehicle geometric data'

'Wheelbase, track and tire characteristics [m].'

  ARRAY WB(4),TR(4),TH(4)
  CONSTANT WB = 1.32,  1.32, 1.50,  1.50
  CONSTANT TR = 0.75,  0.75,  0.75,  0.75
  CONSTANT TH = 4*0.20

'Position of the center of gravity for each wheel in their ...
 local frame.'

  ARRAY CG(3)
  CONSTANT CG  =3*0.0

'Suspension connecting points in the sprung mass.'

  ARRAY SSP(3,4)
  CONSTANT SSP = 1.32,   0.75,  -0.30, ...
                 1.32,  -0.75,  -0.30, ...
                -1.50,   0.75,  -0.30, ...
                -1.50,  -0.75,  -0.30

'Spring and damping constants of vehicle suspension.'

'K(I) [N/m].'
'B(I) [N.s/m].'

'K(1), B(1) - Front left'
'K(2), B(2) - Front right'
'K(3), B(3) - Rear left'
'K(4), B(4) - Rear right'

  ARRAY K(4), B(4), KAR(4)
  CONSTANT K =   20000,  20000,  27000,  27000
  CONSTANT B =    1400,   1400,   2000,   2000
  CONSTANT KAR = 20000,  20000,    0.0,    0.0

'Tire characteristics'

  ARRAY KT(3,4)
  CONSTANT KT = 0.0,           66000.0,       250000.0,    ...
                0.0,           66000.0,       250000.0,    ...
```

```
                  0.0,          70000.0,       250000.0,    ...
                  0.0,          70000.0,       250000.0

  'Steering system compliance.'

     ARRAY CS(4)
     CONSTANT  CS = 5.1E-6, 5.1E-6, 0.0, 0.0

  'Feedback constants'

    CONSTANT KP = 160000  $ 'Feedback gain [kg/s].'
    CONSTANT SPEED = 20.0 $ ' Desired speed [m/s].'

  'Pole placement gain for ride control.'

    ARRAY KPP(4,20)
    CONSTANT KPP = 80*0.0

  'Aerodynamic data'

    ARRAY PAR(3)
    CONSTANT PAR = -0.09,0.0,-0.50 $ 'Origin of aerodynamic frame.'
    CONSTANT CDX = 0.39              $ 'Drag coefficient'
    CONSTANT CDY = 1.63              $ 'Lateral force coefficient'
    CONSTANT CDZF = 0.33             $ 'Front lift coeffficient.'
    CONSTANT CDZR = 0.72             $ 'Rear lift coeffficient.'
    CONSTANT CMX = 0.43              $ 'Roll torque coefficient [N.m].'
    CONSTANT CMY = -0.01             $ 'Pitch torque coefficient [N.m].'
    CONSTANT CMZ = 0.38              $ 'Yaw torque coefficient [N.m].'
    CONSTANT RO = 1.202              $ 'Air density [kg/m^3]'
    CONSTANT AF = 2.060              $ 'Frontal area [m^2]'
    CONSTANT VEL= 0.0                $ 'Wind gust velocity [m/s].'
    CONSTANT WID = 2.0               $ 'Wind gust duration.'
    CONSTANT PER = 20.0              $ 'Pulse train period.'
    CONSTANT TP = 0.5                $ 'Pulse train initial time.'

  'Initial conditions'

    ARRAY QO(10), UO(10), ZO(3,4), ZTO(4), ZRO(4), SACCO(4), SVELO(4)

  'Pre-deformation of the suspension.'

    CONSTANT ZO =  0.0,           0.0,       0.208723768592, ...
                   0.0,           0.0,       0.208723768592, ...
                   0.0,           0.0,       0.136056706309, ...
                   0.0,           0.0,       0.136056706309

  'Pre-deformation of the tyre.'

    CONSTANT ZTO= ...
    0.0184980184, 0.0184980184, 0.0160862356, 0.0160862356

  'Initial conditions for the states.'

    CONSTANT QO = 10*0.0

    CONSTANT UO = 20.0, 9*0.0

  'Initial condition for the integrators.'

    CONSTANT ZRO = 4*0.0

    CONSTANT SACCO = 4*0.0

    CONSTANT SVELO = 4*0.0

```



```
 'Integration information'
   ALGORITHM IALG = 5       $ 'Integration algorithm'
   CINTERVAL CINT = 0.02    $ '250 points'
   NSTEPS    NSTP = 2       $ '2 calculations/cinterval'


 '================= INITIAL SECTION ====================='

 INITIAL

   INTEGER I, J

 'Call SDFAST (sdfast.beta) subroutine SDINIT...
  to initialize the system states and variables.'

   CALL SDINIT

 'Initialize all the other system variables'

  UERR = 0.0
  UW = 0.0
  ABETA = 0.0
  VWIND = 0.0
  VRES = 0.0
  QW = 0.0

  DO 88 I=1,3
    VBB(I)=0.0
    ABB(I)=0.0
    FA(I)=0.0
    MA(I)=0.0
    VGB(I)=0.0
    AGB(I)=0.0
    FD(I)=0.0
  88..CONTINUE

  DO 81 I=1,4
    KCE(I)=0.0
    RAMPIN(I)=0.0
    TRAMP(I)=0.0
    ZROAD1(I)=0.0
    ZROAD2(I)=0.0
    ZROAD3(I)=0.0
    ZT(I) = 0.0
    SACC(I)=0.0
    SVEL(I)=0.0
    ALFA(I)=0.0
    SDELR(I)=0.0
    SPOS(I)=0.0
    SVEL1(I)=0.0
    DELTA(I)=0.0
  81..CONTINUE

 DO 82 I=1,4
   DO 83 J=1,3
     FW(J,I)=0.0
     FSW(J,I)=0.0
     CP(J,I)=0.0
     FAT(J,I)=0.0
     FT(J,I)=0.0
     PGW(J,I)=0.0
     PGS(J,I)=0.0
     AGW(J,I)=0.0
     VGW(J,I)=0.0
     VGS(J,I)=0.0
     Z(J,I)=0.0
```

```
      ZS(J,I)=0.0
      ZDOT(J,I)=0.0
      FS(J,I)=0.0
      FB(J,I)=0.0
      FSB(J,I)=0.0
      UFL(J,I)=0.0
      UF1(J,I)=0.0
      UF2(J,I)=0.0
      UF2W(J,I)=0.0
      UFW(J,I)=0.0
    83..CONTINUE
  82..CONTINUE
  DO 85 I=1,10
    QDOT(I)=0.0
    UDOT(I)=0.0
  85..CONTINUE
  END  $ 'OF INITIAL'
  '=====================DYNAMIC SECTION ========================'
  DYNAMIC
    ARRAY Q(10), U(10), QDOT(10), UDOT(10)
  DERIVATIVE
    INTEGER I, J, SDINDX
    ARRAY FA(3), MA(3), FD(3), VBB(3), ABB(3)
    ARRAY DELTA(4), RAMPIN(4), RAMPST(4),SPOS(4)
    ARRAY ALFA(4), KCE(4), ZT(4),SACC(4),SVEL(4), SVEL1(4)
    ARRAY Z(3,4), ZS(3,4), ZDOT(3,4), CP(3,4), TRAMP(4)
    ARRAY FS(3,4), FB(3,4), FAT(3,4), FSB(3,4), FT(3,4), FW(3,4)
    ARRAY USDEL(4), SDELR(4), UF1(3,4), UFA(3,4), UFL(3,4), AGB(3)
    ARRAY FSW(3,4), PGW(3,4), PGS(3,4), VGW(3,4), VGS(3,4)
    ARRAY VGB(3),AGW(3,4), UF2(3,4),UF2W(3,4), UFW(3,4)
    ARRAY ZROAD1(4), ZROAD2(4), ZROAD3(4)
  PROCEDURAL
    CALL SDSTATE(T,Q,U)
  '============================================================'
  ' Call all sdfast subroutines which calculates all kinematical ...
    quantities necessary to perform all the other calucations in ...
    sdfast.'
  '============================================================'
    DO 10 I=1,4
  'Calculate the position of each wheel cg in relation to the ground.'
      CALL SDPOS(ARM(I), CG, PGW(ROW,I))
  'Calculate the position of each suspension attaching point on ...
   the sprung mass in relation to the ground.'
     CALL SDPOS(SPMASS, SSP(ROW,I), PGS(ROW,I))
  'Calculate the velocity of each wheel cg in relation to the ground.'
     CALL SDVEL(ARM(I), CG, VGW(ROW,I))
  'Calculate the velocity of each suspension attaching point on  ...
   the sprung mass in relation to the ground.'
```



```
     CALL SDVEL(SPMASS, SSP(ROW,I),VGS(ROW,I))
   10..CONTINUE
  'Calculate the velocity of the sprung mass cg in relation to the ...
   ground.'
    CALL SDVEL(SPMASS, CG, VGB)
  'Transform the velocity and acceleration of the sprung mass ...
   from the ground frame to the sprung mass frame.'
    CALL SDTRANS(GROUND, VGB, SPMASS, VBB)
    CALL SDTRANS(GROUND, AGB, SPMASS, ABB)
  '============================================================'
  'Calculate all the inputs to the system.'
  '============================================================'
  'Road vertical input.'
  'Calculate dead time for road vertical input'
    DEADT = (WB(2)+WB(4))/VBB(LONG)
  'Calculate equivalent obstacle (ramp) in time domain.'
   DO 70 I=1,4
     RAMPIN(I) = RAMPZ(I)*VBB(LONG)/RAMPX(I)     $ 'Ramp inclination'
     TRAMP(I) = RAMPX(I)/VBB(LONG)               $ 'Ramp duration'
   70..CONTINUE
  'Calculate initial time of obstacle input'
    RAMPST(1)=.50
    RAMPST(2)=.60
    RAMPST(3)=0.0
    RAMPST(4)=0.0
    DO 80 I=3,4
      RAMPST(I)=RAMPST(I-2)+DEADT
    80..CONTINUE
  'Calculate the ramp input in the time domain for...
   the vertical displacement of the road.'
    DO 90 I=1,4
  'Out of phase sleeping policeman. (Triangular bump.)'
      ZROAD1(I) = KLAZ1(I)*RAMPIN(I)*(RAMP(RAMPST(I)) ...
                            - 2*RAMP(RAMPST(I) + TRAMP(I)) ...
                            + RAMP(RAMPST(I) +2*TRAMP(I)))...
                  + UZ(I)
  'Curb input. (Terminated ramp.) '
      ZROAD2(I) = KLAZ2(I)*RAMPIN(I)*(RAMP(RAMPST(I)) - ...
                              RAMP(RAMPST(I) + TRAMP(I)))  ...
                 + UZ(I)
      ZROAD3(I) = KLAZ3(I)*RAMPZ(I)*STEP(RAMPST(I)) + UZ(I)
```

```
   90..CONTINUE

'=================================================================='

'Steering input.'

'Steering wheel angle input function.'

'Step input.'

'Step size.'

     CONSTANT USDEL = 4*0.0

'Steering wheel angle in degrees.'

  DO 98 I=1,4
     SACC(I) = USDEL(I)*(16.0*STEP(TS) - 32.0*STEP(TS+0.25)   ...
             + 16.0*STEP(TS+0.5))
  98..CONTINUE

     SVEL = INTVC(SACC,SACCO)

  DO 99 I=1,4
    SVEL1(I)=SVEL(I)
  99..CONTINUE

     SPOS = INTVC(SVEL1,SVELO)

'Steering wheel angle in radians.'

  DO 92 I=1,4

     SDELR(I) =  (SPOS(I) + US(I))*PI/180.0

'Equivalent road wheel angle in radians.'

     DELTA(I) = SDELR(I)/SR

  92..CONTINUE

'=================================================================='

'Longitudinal forces input: braking and acceleration on the wheels.'

' (Braking: Negative CB.)'
' (Acceleration: Positive CB.)'

'Terminated ramp input with 0.5s ramp time.'

   DO 94 I=1,4
    UFL(1,I) =    RSW((VBB(LONG).LT.UMIN.OR.VBB(LONG).GT.UMAX), ...
                  0.0, 2.0*CB(I)*MASS*G * (RAMP(TB) - RAMP(TB+0.5)))
   94..CONTINUE

'=================================================================='

'Force input: actuator force on the suspension.'

'Expression to generate B matrix.'

   DO 96 I=1,4
     DO 97 J=3,3
        UF1(J,I) = - KLAF(I)*(KA*Z(J,I) + BA*ZDOT(J,I)) - UFA(J,I)
     97..CONTINUE
```



```
   96..CONTINUE

'Pole placement gain implementation for ride studies.'

   DO 106 I=1,4
     DO 107 J=1,10
       UF2(3,I) = - KPP(I,J)*Q(J) - KPP(I,J+10)*U(J)
     107..CONTINUE
   106..CONTINUE

'=================================================================='
'Calculate suspension displacements and forces'
'=================================================================='

   DO 100 I=1,4

'Calculate the relative displacement of each suspension.'

    DO 110 J=1,3
      Z(J,I) =  PGS(J,I) - PGW(J,I)
    110..CONTINUE

'Calculate the relative velocity of each suspension.'

    DO 120 J=1,3
      ZDOT(J,I) =  VGS(J,I) - VGW(J,I)
    120..CONTINUE

'=================================================================='

'Suspension forces'

    DO 130 J=3,3
      ZS(J,I) =  Z(J,I) - ZO(J,I)
      FS(J,I) = - K(I)*ZS(J,I)
      FB(J,I) = - B(I)*ZDOT(J,I)
    130..CONTINUE

   100..CONTINUE

'Calculate antiroll torsion bar force. Front suspension only.'

   DO 101 I=1,2
     DO 140 J=3,3
       FAT(J,I) = (-1)**(I-1)*KAR(I)*(Z(J,2) - Z(J,1))/2.0
     140..CONTINUE
   101..CONTINUE

'Total suspension force.'

  DO 102 I=1,4
    DO 150 J=1,3
      FSB(J,I) = FS(J,I) + FB(J,I) + FAT(J,I) + UF1(J,I)
    150..CONTINUE
  102..CONTINUE

'=================================================================='
' Calculate tyre displacements and forces.'
'=================================================================='

'Tyre deflection.'

   DO 103 I=1,4

     ZT(I) =  PGW(3,I) - (ZROAD1(I) + ZROAD2(I) + ZROAD3(I)) ...
            - ZTO(I) - TH(I)
```

```
'Determination of the contact point between the wheel and the ...
 ground, expressed in the wheel frame.'

   CP(3,I) = -TH(I) - ZT(I)
   CP(2,I) = 0.0
   CP(1,I) = 0.0

'Vertical tyre force.'

    FT(3,I) = - KT(3,I)*ZT(I)

'============================================================'

'Longitudinal tire force.'

    FT(1,I) = UFL(1,I)

'============================================================'

'Lateral tire force.'

'Sideslip angle of the wheels [rd].'

    ALFA(I) =  ATAN2((VBB(LAT) + SSP(1,I)*U(SDINDX(SPMASS,YAW)))...
               ,(VBB(LONG) - SSP(2,I)*U(SDINDX(SPMASS,YAW)))) - ...
DELTA(I)

'Effective cornering coefficient.'

     KCE(I) = KT(2,I)/(1+KT(2,I)*CS(I))

'Cornering forces. Includes steering system compliance.'

    FT(2,I) = - KCE(I)*ALFA(I)

  103..CONTINUE

'Sideslip  angle of the car [rd].'

    BETA = ATAN2(VBB(LAT),VBB(LONG))

'=============================================================='
'Determination of aerodynamics effects'
'=============================================================='

'Side wind velocity. Considered  in the lateral direction  of ...
the base body frame.'

    VWIND =  VEL*PULSE(TP,PER,WID) + UW

'Module of the resultant air velocity.'

    VRES = SQRT(VWIND**2 + VBB(LONG)**2)

'Aerodynamic sideslip angle .'

    ABETA =  ATAN2(VWIND,VBB(LONG))

'Aerodynamic pressure.'

    QW = RO*VRES**2/2.0

'Aerodynamic drag.'

```



```
    FA(1) = - CDX*AF*QW*SIGN(1.0,VBB(LONG))

'Aerodynamic side force.'

    FA(2) =  CDY*ABETA*AF*QW

'Aerodynamic lift.'

    FA(3) =  (CDZF + CDZR*ABETA)*AF*QW

'Aerodynamic roll torque.'

    MA(1) = - CMX*ABETA*AF*QW

'Aerodynamic pitch torque.'

    MA(2) = CMY*AF*QW

'Aerodynamic yaw torque.'

    MA(3) =  CMZ*ABETA*AF*QW

'=============================================================='

'Determination of driving force to maintain constant speed ...
 during cornering.'

'Proportional controller of speed with high gain.'

'Error in the forward velocity.'

    UERR= SPEED - VBB(LONG)

'Driving force.'

    FD(3) = 0.0
    FD(2) = 0.0
    FD(1) = KP*UERR*(STEP(TDF)-STEP(TB))

'============================================================'
'Apply forces and torques to the system.'
'============================================================'

'External forces on the sprung mass.'

'Suspension forces.'

    DO 160 I=1,4
      CALL SDPOINTF(SPMASS,SSP(ROW,I),FSB(ROW,I))
      CALL SDPOINTF(SPMASS,SSP(ROW,I),UF2(ROW,I))
    160..CONTINUE

'Aerodynamic forces.'

    CALL SDPOINTF(SPMASS,PAR,FA)

'Aerodynamic torques.'

    CALL SDBODYT(SPMASS,MA)

'Driving force.'

    CALL SDPOINTF(SPMASS, CG, FD)

'============================================================'

```

```
'Transform suspension forces from sprung mass frame to the ...
 suspension frame.'

    DO 170 I=1,4
      CALL SDTRANS(SPMASS,FSB(ROW,I),ARM(I),FSW(ROW,I))
      CALL SDTRANS(SPMASS,UF2(ROW,I),ARM(I),UF2W(ROW,I))
    170..CONTINUE

    DO 172 I=1,4
      DO 174 J=1,3
        FW(J,I) = - FSW(J,I)
        UFW(J,I)= - UF2W(J,I)
      174..CONTINUE
    172..CONTINUE

'Apply the suspension forces.'

   DO 180 I=1,4

      CALL SDPOINTF(ARM(I), CG, FW(ROW,I))
      CALL SDPOINTF(ARM(I), CG, UFW(ROW,I))

'Apply the tyre forces.'

      CALL SDPOINTF(ARM(I), CP(ROW,I), FT(ROW,I))

   180..CONTINUE

'================================================================'
'Call subroutine SDDERIV which contains the equations of...
 of motion generated by SDFAST.'
'================================================================'


  CALL SDDERIV(QDOT,UDOT=)

  CALL SDACC(SPMASS, CG, AGB)

END $ 'OF PROCEDURAL'


'Integrate system states'

  Q = INTVC(QDOT,QO)
  U = INTVC(UDOT,UO)

END  $ 'OF DERIVATIVE'

  TERMT(T.GE.SIMLEN)

END  $ 'OF DYNAMIC'

  CALL SDPRINTERR(6)

END  $ 'OF PROGRAM'
```

```


'================================================================'

'.........1.........2.........3.........4.........5.........6.......'
'ACSL program for simulating an automotive vehicle consisting...
 of a 16 bodies, 18 DOF model, incorporating suspension geometry ...
 effects, for handling, ride and performance study purposes.'

'Program includes variables initialization for acsl new version.'

'Linear vertical, linear and nonlinear lateral  tyre dynamics,   '

'Aerodynamics effects included are drag, lateral and lift forces...
 and  roll, pitch and yaw torques.'

'This program is the implementation of the new version of ...
 sdfast. (sdfast.beta). '

'It considers the connection between the ground and the sprung ...
 mass to be made by 3 massless bodies which have the translational...
 degrees of freedom and a 3-2-1 gimbal joint.'

'This model has massless wheel hubs for steering and rotating ...
 wheels for traction and braking control.'

'=============================================================='

PROGRAM SUSP6G    $ ' SD/FAST.BETA  '

'=============================================================='



'=====================PROGRAM MACROS=========================='


'=============================================================='
'=             MACRO DEFINITION: DOT PRODUCT                  ='
'=============================================================='

'=============== MACRO INVOCATION: DOT(A,B) ==================='

MACRO MACRO DOT(P,Q)
MACRO RELABEL L2 $ MACRO REDEFINE I
MACRO IF (Q(2)=Q(3)) ML1
MACRO PRINT CONFLICTING DIMENSIONS IN DOT PRODUCT
MACRO EXIT
MACRO ML1..CONTINUE
        PROCEDURAL(P(1)=P(2),P(3))
        P(1)=0.0
        DO L1 I=1,Q(2)
          P(1) = P(1) + P(2)(I)*P(3)(I)
        L1..CONTINUE
        END $ 'OF PROCEDURAL'
MACRO END

'=============================================================='
'=               MACRO DEFINITION: LINEAR TYRE                ='
'=============================================================='

'=  MACRO INVOCATION:LTYRE(FT=KT,UFL,ALFA,ZT,J)                ='

MACRO MACRO LTYRE(P,Q,R)

        PROCEDURAL(P(1)=P(2),P(3),P(4),P(5),P(6))

```



```
        P(1)(1,P(6)) =   P(3)(1,P(6))

        P(1)(3,P(6)) =   P(2)(3,P(6))*P(5)(P(6))

        P(1)(2,P(6)) = - P(2)(2,P(6))*P(4)(P(6))

        END $ 'OF PROCEDURAL'
MACRO END

'=============================================================='
'=              MACRO DEFINITION:NONLINEAR TYRE               ='
'=============================================================='

'=    MACRO INVOCATION:NTYRE(FT,TP=KT,CFL,CTP,UFL,ALFA,ZT,J)    ='

MACRO MACRO NTYRE(P,Q,R)

PROCEDURAL(P(1),P(2)=P(3),P(4),P(5),P(6),P(7),P(8),P(9))

P(1)(1,P(9))=P(6)(1,P(9))

P(1)(3,P(9)) = P(3)(3,P(9))*P(8)(P(9))

P(1)(2,P(9))= - ((P(4)(1)*P(1)(3,P(9))...
              +P(4)(2)*P(1)(3,P(9))**2...
              + P(4)(3)*P(1)(3,P(9))**3 + ...
                P(4)(4)*P(1)(3,P(9))**4)*P(7)(P(9))  +...
              (P(4)(5)*P(1)(3,P(9))+P(4)(6)*P(1)(3,P(9))**2...
              + P(4)(7)*P(1)(3,P(9))**3 + ...
                P(4)(8)*P(1)(3,P(9))**4)*P(7)(P(9))**2 +...
               (P(4)(9)*P(1)(3,P(9))+P(4)(10)*P(1)(3,P(9))**2...
              + P(4)(11)*P(1)(3,P(9))**3 + ...
                P(4)(12)*P(1)(3,P(9))**4)*P(7)(P(9))**3 +...
              (P(4)(13)*P(1)(3,P(9))+P(4)(14)*P(1)(3,P(9))**2...
              + P(4)(15)*P(1)(3,P(9))**3 + ...
                P(4)(16)*P(1)(3,P(9))**4)*P(7)(P(9))**4 + ...
              (P(4)(17)*P(1)(3,P(9))+P(4)(18)*P(1)(3,P(9))**2...
              + P(4)(19)*P(1)(3,P(9))**3 + ...
                P(4)(20)*P(1)(3,P(9))**4)*P(7)(P(9))**5)


P(2)(P(9))=P(5)(1)+P(5)(2)*(-P(7)(P(9)))+P(5)(3)*P(1)(3,P(9))...
         + P(5)(4)*P(1)(3,P(9))**2 ...
         + P(5)(5)*(-P(7)(P(9)))*P(1)(3,P(9))...
         + P(5)(6)*(-P(7)(P(9)))**2*P(1)(3,P(9))

END $ 'OF PROCEDURAL'

MACRO END

'================= DATA SECTION ==============================='

  INTEGER ROW, GROUND, SPMASS, LONG, LAT, VERT, YAW, PITCH, ROLL
  INTEGER ARM(4), HUB(4), WHEEL(4), PIN
  ARRAY CB(4), CFL(20), CTP(6), XO(2)
  ARRAY KLAF(4), KP(4)
  ARRAY KLAZ1(4), KLAZ2(4), KLAZ3(4), KLAZ4(4)
  ARRAY KLAS(4)
  ARRAY RAMPZ(4)
  ARRAY RAMPX(4)
  ARRAY UZ(4)
  ARRAY US(4)
  ARRAY UT(4)
  ARRAY UFD(4)

'Names of sdfast joints in susp6_info.'
```

```
    CONSTANT GROUND  = 0
    CONSTANT LONG    = 1
    CONSTANT LAT     = 2
    CONSTANT VERT    = 3
    CONSTANT SPMASS  = 4
    CONSTANT ARM     = 5, 6, 7, 8
    CONSTANT HUB     = 9, 10, 11, 12
    CONSTANT WHEEL   = 13, 14, 15, 16

  'Names of sdfast axes for joint 4 (sprung mass orientation.)'

    CONSTANT YAW     = 1
    CONSTANT PITCH   = 2
    CONSTANT ROLL    = 3

  'Name of the other joints.'

    CONSTANT PIN     = 1

  'Other program constants.'

    CONSTANT KLAF    = 4*0.0      $ 'Constant for linear anal. sforce.'
    CONSTANT KLAZ1   = 4*0.0      $ 'Constant for linear anal. zroad.'
    CONSTANT KLAZ2   = 4*0.0      $ 'Constant for linear anal. zroad.'
    CONSTANT KLAZ3   = 4*0.0      $ 'Constant for linear anal. zroad.'
    CONSTANT KLAZ4   = 4*0.0      $ 'Constant for linear anal. zroad.'
    CONSTANT KLAS    = 4*1.0      $ 'Constant for linear anal. steer.'
    CONSTANT SR      = 20         $ 'Steering/road wheel ratio.'
    CONSTANT RAMPZ   = 4*0.0      $ 'Ramp height [m].'
    CONSTANT RAMPX   = 4*2.0      $ 'Ramp length [m].'
    CONSTANT UFA     =12*0.0      $ 'Pert. term for lin. anal. sforce.'
    CONSTANT UZ      = 4*0.0      $ 'Pert. term for lin. anal. zroad.'
    CONSTANT UW      = 0.0        $ 'Pert. term for lin. anal. wind.'
    CONSTANT US      = 4*0.0      $ 'Pert. term for lin. anal. steer.'
    CONSTANT UT      = 4*0.0      $ 'Pert. term for lin. anal. torque.'
    CONSTANT UFD     = 4*0.0      $ 'Pert. term for lin. anal. driving.'
    CONSTANT SIMLEN  = 5.0        $ 'Simulation length [s].'
    CONSTANT CB      = 4*0.0      $ 'Braking coefficient [% of G].'
    CONSTANT TS      = 0.5        $ 'Inital time of steering input'
    CONSTANT TBO     = 50.0       $ 'Intial time of brake input.'
    CONSTANT TB      = 0.5        $ 'Total time of brake input.'
    CONSTANT UMAX    = 25.0       $ 'Maximum forward velocity [m/s].'
    CONSTANT UMIN    = 1.0        $ 'Minimum forward velocity [m/s].'
    CONSTANT TDF     = 0.0        $ 'Initial time for driving force.'
    CONSTANT KA      = 0.0        $ 'Actuator proportional gain.'
    CONSTANT KTAU    = 0.0        $ 'Wheel torque stiffness.'
    CONSTANT BA      = 0.0        $ 'Actuator derivative gain.'
    CONSTANT BTAU    = 0.0        $ 'Wheel torque damper.'
    CONSTANT ROW     = 1          $ 'Index for matrices.'
    CONSTANT G       = 9.81       $ 'Gravity acceleration [N/m^2].'
    CONSTANT MASS    = 1780       $ 'Vehicle mass [Kg].'
    CONSTANT ASTAB   = 1.0        $ 'Baumgarten position constant.'
    CONSTANT PI=3.14159265358979 $ 'pi.'
    CONSTANT SF      = 10         $ 'Force scaling factor.'
    CONSTANT MU      = 0.98       $ 'Road adhesion coefficient.'
    CONSTANT MUDYN   = 0.80       $ 'Road dynamic adhesion coefficient.'
    CONSTANT XO      = 2.5,2.5    $ 'Initial distance of road input [m].'
    CONSTANT FL      = 0.0        $ 'Lateral force to check  over/under.'

  'Nonlinear tyre model sideslip angle coefficients.'

    CONSTANT CFL     = 15.014522, 0.00417075, -0.00000132838, ...
                       9.14832e-11, 503.258991, -0.339215, ...
                       0.0000672330999999999, -4.21518E-9, ...
                      -19291.713, 9.901508, -0.00171675, ...
```



```
                       9.94122E-8, 219991.02, -113.39167, ...
                       0.0193546, -0.00000109341, -835912.2, ...
                       443.517648, -0.0765168000000001, ...
                       0.000004312270000000001

  'Nonlinear tyre model pneumatic trail coefficients.'

    CONSTANT CTP     = 0.000194631, -0.105903, 0.0000080097, ...
                       1.69201E-11, -0.000043221, 0.000121753

  'Orthonormal base.'

     ARRAY E(3,3)
     CONSTANT E = 1, 0, 0, ...
                  0, 1, 0, ...
                  0, 0, 1

  'Vehicle geometric data'

  'Wheelbase, track and tire characteristics [m].'

    ARRAY WB(4),TR(4),TH(4),CLEN(4)
    CONSTANT WB = 1.32,  1.32,  1.50,  1.50
    CONSTANT TR = 0.75,  0.75,  0.75,  0.75
    CONSTANT TH = 4*0.20
    CONSTANT CLEN = 4*0.30

  'Position of the center of gravity for each suspension part ...
   in their local frame.'

    ARRAY CG(3)
    CONSTANT CG  = 3*0.0

  'Suspension connecting points in the sprung mass.'

    ARRAY SSP(3,4)
    CONSTANT SSP = 1.3200,   0.75,  -0.3000, ...
                   1.3200,  -0.75,  -0.3000, ...
                  -1.5000,   0.75,  -0.3000, ...
                  -1.5000,  -0.75,  -0.3000

  'Length of the suspension arms.'

    ARRAY ARML(4)
    CONSTANT ARML = 0.466777, 0.466777,  0.66137500,  0.66137500



  'Spring and damping constants of vehicle suspension.'

  'K(I) [N/m].'
  'B(I) [N.s/m].'

  'K(2), B(2) - Front left'
  'K(3), B(3) - Front right'
  'K(4), B(4) - Rear left'
  'K(5), B(5) - Rear right'

    ARRAY K(4), B(4), KAR(4)
    CONSTANT K = 20000.0,  20000.0,  27000.0,  27000.0
    CONSTANT B =  1400.0,   1400.0,   2000.0,   2000.0
    CONSTANT KAR = 20000.0, 20000.0,  0.0,      0.0

  'Tyre characteristics [N/m].'

    ARRAY KT(3,4)
```

```
    CONSTANT KT = 0.0,      66000.0,      250000.0,     ...
                  0.0,      66000.0,      250000.0,     ...
                  0.0,      70000.0,      250000.0,     ...
                  0.0,      70000.0,      250000.0

 'Steering system compliance.'

    ARRAY CS(4)
    CONSTANT  CS = 5.1E-6, 5.1E-6, 0.0, 0.0

 'Feedback constants for driving force.'

    CONSTANT KP =     4*0.0 $ 'Feedback gain [kg/s].'
    CONSTANT SPEED =  20.0  $ ' Desired speed [m/s].'

 'Pole placement gain for ride control.'

    ARRAY KPP(4,36)
    CONSTANT KPP = 144*0.0

 'Aerodynamic data'

    ARRAY PAR(3)
    CONSTANT PAR = -0.09,0.0,-0.50  $ 'Origin of aerodynamic frame.'
    CONSTANT CDX = 0.39             $ 'Drag coefficient'
    CONSTANT CDY = 1.63             $ 'Lateral force coefficient'
    CONSTANT CDZF = 0.33            $ 'Front lift coeffficient.'
    CONSTANT CDZR = 0.72            $ 'Rear lift coeffficient.'
    CONSTANT CMX = 0.43             $ 'Roll torque coefficient [N.m].'
    CONSTANT CMY = -0.01            $ 'Pitch torque coefficient [N.m].'
    CONSTANT CMZ = 0.38             $ 'Yaw torque coefficient [N.m].'
    CONSTANT RO = 1.202             $ 'Air density [kg/m^3]'
    CONSTANT AF = 2.060             $ 'Frontal area [m^2]'
    CONSTANT WINVEL= 0.0            $ 'Wind gust velocity [m/s].'
    CONSTANT WID = 2.0              $ 'Wind gust duration.'
    CONSTANT PER = 20.0             $ 'Pulse train period.'
    CONSTANT TPUL = 0.5             $ 'Pulse train initial time.'

 'Initial conditions.'

    ARRAY QO(18), UO(18), ZO(3,4), ZTO(4), ZRO(4), SACCO(4), SVELO(4)
    ARRAY FTLO(4)

    CONSTANT ZO =  0.0, 0.0, 0.209311048592, ...
                   0.0, 0.0, 0.209311048592, ...
                   0.0, 0.0, 0.134969066309, ...
                   0.0, 0.0, 0.134969066309

    CONSTANT ZTO= ...
    0.0184980184, 0.0184980184, 0.0160862356, 0.0160862356

    CONSTANT QO = 18*0.0

    CONSTANT UO = 20.0,17*0.0

    CONSTANT ZRO = 4*0.0

    CONSTANT SACCO = 4*0.0

    CONSTANT SVELO = 4*0.0

    CONSTANT FTLO = 4*0.0

 'Integration information'
    ALGORITHM IALG = 5        $ 'Integration algorithm'
    CINTERVAL CINT = 0.02     $ '250 points'
```



```
    NSTEPS    NSTP = 2        $ '2 calculations/cinterval'


 '================= INITIAL SECTION ===================='

 INITIAL

   INTEGER I, J

 'Call SDFAST (sdfast.beta) subroutine SDINIT...
  to initialize the system states and variables.'

   CALL SDINIT

 'Call sdfast stabilisation subroutine for the prescribed motion.'

   CALL SDSTAB(2*ASTAB,ASTAB*ASTAB)

 'Initialize all the other system variables'

   VERR = 0.0
   ABETA = 0.0
   BETA = 0.0
   VWIND = 0.0
   VRES = 0.0
   QW = 0.0

   DO 88 I=1,3
     VBB(I)=0.0
     ABB(I)=0.0
     AGB(I)=0.0
     VGB(I)=0.0
     PGB(I)=0.0
     FA(I)=0.0
     FD1(I)=0.0
     MA(I)=0.0
   88..CONTINUE

   DO 81 I=1,4
     RAMPIN(I)=0.0
     TRAMP(I)=0.0
     ZROAD1(I)=0.0
     ZROAD2(I)=0.0
     ZROAD3(I)=0.0
     ZROAD4(I)=0.0
     ZT(I) = 0.0
     ALFA1(I)=0.0
     ALFA2(I)=0.0
     SACC(I)=0.0
     SACC1(I) =0.0
     SVEL(I)=0.0
     SVEL1(I)=0.0
     HPOS(I)=0.0
     SPOS(I)=0.0
     SDELR(I)=0.0
     DELTA(I)=0.0
     TAU(I)=0.0
     TP(I)=0.0
     DEADT(I)=0.0
     FTL(I)=0.0
     FTLD(I)=0.0
     TAW(I)=0.0
     FTOT(I)=0.0
     FMAX(I)=0.0
     MAGS(I)=0.0
     MAGH(I)=0.0
```

```
      WN(I)=0.0
      MAGV(I)=0.0
      TFIN(I)=0.0
      ALFA3(I)=0.0
    81..CONTINUE


    DO 86 I=1,3
      DO 87 J=1,3
        DCOS(J,I)=0.0
      87..CONTINUE
    86..CONTINUE

    DO 82 I=1,4
      DO 83 J=1,3
        FW(J,I)=0.0
        FSW(J,I)=0.0
        CP(J,I)=0.0
        FAT(J,I)=0.0
        FT(J,I)=0.0
        FTS(J,I)=0.0
        PGW(J,I)=0.0
        PGS(J,I)=0.0
        VGW(J,I)=0.0
        VGP(J,I)=0.0
        VWP(J,I)=0.0
        VGS(J,I)=0.0
        AGW(J,I)=0.0
        Z(J,I)=0.0
        ZS(J,I)=0.0
        ZDOT(J,I)=0.0
        FS(J,I)=0.0
        VWW(J,I)=0.0
        FB(J,I)=0.0
        FD(J,I)=0.0
        FSB(J,I)=0.0
        UFL(J,I)=0.0
        UF(J,I)=0.0
        FSDIR(J,I)=0.0
        FT1(J,I)=0.0
        FSLIDE(J,I)=0.0
        EG(J,I)=0.0
        HEAD(J,I)=0.0
        UVEL(J,I)=0.0
      83..CONTINUE
    82..CONTINUE

    DO 85 I=1,18
      QDOT(I)=0.0
      UDOT(I)=0.0
    85..CONTINUE

  END  $ 'OF INITIAL'

  '======================DYNAMIC SECTION ========================'

  DYNAMIC

    ARRAY Q(18), U(18), QDOT(18), UDOT(18)

  DERIVATIVE

    INTEGER I, J, SDINDX
    ARRAY FA(3), MA(3), FD(3,4), VBB(3), HPOS(4), TP(4), ABB(3)
    ARRAY DELTA(4), RAMPIN(4), RAMPST(4),SVEL(4), SVEL1(4)
    ARRAY ALFA1(4), ZT(4), TAU(4), USDEL(4), SACC(4), SACC1(4)
```



```
    ARRAY Z(3,4), ZS(3,4), ZDOT(3,4), CP(3,4), TRAMP(4), SPOS(4)
    ARRAY FS(3,4), FB(3,4), FAT(3,4), FSB(3,4), FT(3,4), FW(3,4)
    ARRAY  SDELR(4), UF(3,4), UFA(3,4), UFL(3,4), DCOS(3,3), VWP(3,4)
    ARRAY FSW(3,4),PGW(3,4),PGS(3,4),VGW(3,4),VGS(3,4),AGW(3,4)
    ARRAY ALFA2(4), VWW(3,4), FTS(3,4), PGB(3), VGB(3), AGB(3)
    ARRAY DEADT(4), ZROAD1(4), ZROAD2(4), ZROAD3(4), ZROAD4(4), VGP(3,4)
    ARRAY FTL(4), FTLD(4), TAW(4), FTOT(4), FMAX(4), MAGS(4), FSDIR(3,4)
    ARRAY FT1(3,4), FSLIDE(3,4), EG(3,4), UVEL(3,4), HEAD(3,4)
    ARRAY MAGH(4), MAGV(4), ALFA3(4), TFIN(4), WN(4), FD1(3)

  PROCEDURAL

    CALL SDSTATE(T,Q,U)

  '==============================================================='
  ' Call all sdfast subroutines which calculates all kinematical ...
    quantities necessary to perform all the other calculations in ...
    ACSL to be passed on to sdfast.'
  '==============================================================='

    DO 10 I=1,4

  'Calculate the position of each wheel cg in relation to the ground.'

      CALL SDPOS(WHEEL(I), CG, PGW(ROW,I))

  'Calculate the position of each suspension attaching point on ...
   the sprung mass in relation to the ground.'

      CALL SDPOS(SPMASS, SSP(ROW,I), PGS(ROW,I))

  'Calculate the velocity of each wheel cg in relation to the ground.'

      CALL SDVEL(WHEEL(I), CG, VGW(ROW,I))

  'Transform it to its own frame.'

      CALL SDTRANS(GROUND, VGW(ROW,I), WHEEL(I), VWW(ROW,I))

  'Calculate the velocity of the contact point in relation to ground.'

      CALL SDVEL(WHEEL(I), CP(ROW,I), VGP(ROW,I))

  'Transform it to the wheel frame.'

       CALL SDTRANS(GROUND, VGP(ROW,I), WHEEL(I), VWP(ROW,I))

  'Calculate the velocity of each suspension attaching point on  ...
   the sprung mass in relation to the ground.'

      CALL SDVEL(SPMASS, SSP(ROW,I), VGS(ROW,I))

   'Find the wheel heading axis in the ground frame.'

        CALL SDTRANS(WHEEL(I), E(ROW,2), GROUND, EG(ROW,I))

     10..CONTINUE

  'Calculate the velocity and position of the sprung mass cg ...
   in relation to the ground.'

    CALL SDPOS(SPMASS, CG, PGB)
    CALL SDVEL(SPMASS, CG, VGB)

  'Transform the velocity and acceleration of the sprung ...
   mass from the ground frame to the sprung mass frame.'
```

```
   CALL SDTRANS(GROUND, VGB, SPMASS, VBB)
   CALL SDTRANS(GROUND, AGB, SPMASS, ABB)

 '================================================================'
 'Calculate all the inputs to the system.'
 '================================================================'

 'Road vertical input.'

 'Calculate dead time for road vertical input'

   DEADT(1) = 0.0
   DEADT(2) = 0.0
   DEADT(3) = (WB(2)+WB(4))/VBB(LONG)
   DEADT(4) = (WB(2)+WB(4))/VBB(LONG)

 'Calculate equivalent obstacle (ramp) in time domain.'

   DO 70 I=1,4
     RAMPIN(I) = RAMPZ(I)*VBB(LONG)/RAMPX(I)     $ 'Ramp inclination'
     TRAMP(I) = RAMPX(I)/VBB(LONG)               $ 'Ramp duration'
   70..CONTINUE

 'Calculate initial time of obstacle input'

   RAMPST(1)=XO(1)/VBB(LONG)
   RAMPST(2)=XO(2)/VBB(LONG)          $ 'Out of phase obstacle crossing.'
   RAMPST(3)=0.0
   RAMPST(4)=0.0

   DO 80 I=3,4
     RAMPST(I)=RAMPST(I-2)+DEADT(I)
   80..CONTINUE

 'Calculate the road input in the time domain for...
  the vertical displacement of the road.'

   DO 90 I=1,4

 'Out of phase sleeping policeman. (Triangular bump.)'

     ZROAD1(I) = KLAZ1(I)*RAMPIN(I)*(RAMP(RAMPST(I)) ...
                             - 2*RAMP(RAMPST(I) + TRAMP(I)) ...
                             + RAMP(RAMPST(I) +2*TRAMP(I)))...
                 + UZ(I)

 'Curb input. (Terminated ramp.) '


     ZROAD2(I) = KLAZ2(I)*RAMPIN(I)*(RAMP(RAMPST(I)) - ...
                               RAMP(RAMPST(I) + TRAMP(I)))

 ' Sinusoidal waveform.'

     WN(I)= 2*PI/TRAMP(I)

     ZROAD3(I) = KLAZ3(I)*STEP(RAMPST(I))*RAMPZ(I)*(1-COS(WN(I)* ...
                 (T-RAMPST(I))))

 '   ZROAD3(I) = KLAZ3(I)*RAMPZ(I)*HARM(RAMPST(I),WN(I)) '

 'Curb (Step function).'

     ZROAD4(I) = KLAZ4(I)*RAMPZ(I)*STEP(RAMPST(I))

```



```
   90..CONTINUE

 '==============================================================='

 'Steering input.'

 'Steering wheel angle input function.'

 'Calculate the prescribed motion (acceleration, velocity and  ...
  position) for the prescribed joints of the steering system.     '

 ' Steering system prescribed motion in radians at the wheel.'

 'Acceleration. Square double impulse approximation.'

   CONSTANT USDEL=4*0.0

   DO 98 I=1,4

 'Hand wheel acceleration, in degrees.'

     SACC1(I) = USDEL(I)*(400.0*STEP(TS) - 800.0*STEP(TS+0.05)  ...
                + 400.0*STEP(TS+0.1))

 'Road wheel aceleration, in radians.'

     SACC(I) = (PI/(180.0*SR))*SACC1(I)

   98..CONTINUE

 'Velocity. Triangular impulse approximation.'

   SVEL = INTVC(SACC,SACCO)

   DO 99 I=1,4
     SVEL1(I)=SVEL(I)
   99..CONTINUE

 'Position. Parabolic step approximation.'

   SPOS = INTVC(SVEL1,SVELO)

 'Steering wheel angle in degrees.'

   DO 105 I=1,4
     HPOS(I) =  (180.0*SR/PI)*SPOS(I)
   105..CONTINUE

 'Turn prescribed motion in the hubs and wheels on (1) or off (0).'

   DO 30 I = 1,4
     CALL SDPRES(HUB(I), PIN, 1)
     CALL SDPRES(WHEEL(I), PIN, 1)
   30..CONTINUE

 'Especify the prescribed motion at each wheel to be ...
  equal to zero .'

   DO 32 I=1,4
     CALL SDPRESACC(WHEEL(I), PIN, 0.0)
     CALL SDPRESVEL(WHEEL(I), PIN, 0.0)
     CALL SDPRESPOS(WHEEL(I), PIN, 0.0)
   32..CONTINUE

 'Especify the prescribed motion of the front hubs. (steering)'

```

```
    DO 34 I = 1,4
      CALL SDPRESACC(HUB(I), PIN, SACC(I))
      CALL SDPRESVEL(HUB(I), PIN, SVEL(I))
      CALL SDPRESPOS(HUB(I), PIN, SPOS(I))
    34..CONTINUE

'Equivalent road wheel angle in radians.'

    DO 92 I=1,4
      DELTA(I) = Q(SDINDX(HUB(I),PIN)) + PI*US(I)/(180*20)
    92..CONTINUE

'==============================================================='

'Longitudinal forces input: braking and acceleration on the wheels.'

' (Braking: Negative CB.)'
' (Acceleration: Positive CB.)'

    DO 94 I=1,4
      UFL(1,I) =    RSW((VBB(LONG).LT.UMIN.OR.VBB(LONG).GT.UMAX), ...
            0.0, (CB(I)*MASS*G)/(TB-TBO)*(RAMP(TBO) - RAMP(TB))) + UFD(I)
    94..CONTINUE

'==============================================================='

'Force input: actuator force on the suspension.'

'Not implemented at the moment - only to generate B matrix.'

    DO 96 I=1,4
      DO 97 J=1,3
         UF(J,I) = - KLAF(I)*(KA*Z(J,I) + BA*ZDOT(J,I)) - UFA(J,I)
      97..CONTINUE
    96..CONTINUE

'Pole placement gain implementation for ride studies.'

'   DO 116 I=1,4 '
'     DO 118 J=1,18 '
'       UF2(3,I) = - KPP(I,J)*Q(J) - KPP(I,J+18)*U(J) '
'     116..CONTINUE '
'   118..CONTINUE '

'==============================================================='
'Calculate suspension displacements and forces'
'==============================================================='

    DO 100 I=1,4

'Calculate the relative displacement of each suspension.'

      DO 110 J=3,3
        Z(J,I) = ARML(I)*(-1)**(I+1)*Q(SDINDX(ARM(I),PIN))
'       Z(J,I) = PGS(J,I) - PGW(J,I) '
      110..CONTINUE

'Calculate the relative velocity of each suspension.'

      DO 120 J=3,3
        ZDOT(J,I) = ARML(I)*(-1)**(I+1)*U(SDINDX(ARM(I),PIN))
'       ZDOT(J,I) =  VGS(J,I) - VGW(J,I) '
      120..CONTINUE

'==============================================================='
```



```
'Suspension forces. Only in the vertical direction.'

      DO 130 J=3,3
        ZS(J,I) =    Z(J,I) + ZO(J,I)
        FS(J,I) =  K(I)*ZS(J,I)
        FB(J,I) =  B(I)*ZDOT(J,I)
      130..CONTINUE

    100..CONTINUE

'Calculate antiroll torsion bar force. Front suspension only.'

    DO 101 I=1,2
      DO 140 J=3,3
        FAT(J,I) = (-1)**(I)*KAR(I)*(Z(J,2) - Z(J,1))/2.0
      140..CONTINUE
    101..CONTINUE

'Total suspension force.'

    DO 102 I=1,4
      DO 150 J=3,3
        FSB(J,I) = FS(J,I) + FB(J,I) + FAT(J,I) + UF(J,I)
      150..CONTINUE
    102..CONTINUE

'==============================================================='
' Calculate tyre displacements and forces.'
'==============================================================='

'Tyre deflection in the vertical direction.'

    DO 103 I=1,4

      ZT(I) = - PGW(3,I) + (ZROAD1(I) + ZROAD2(I) + ZROAD3(I) + ...
                           ZROAD4(I)) + ZTO(I) + TH(I)

'Sideslip angle of the wheels [rd].'

'My own, old way because of the variables in sdfast-a.'

      ALFA1(I) =  ATAN2((VBB(LAT) + SSP(1,I)*U(SDINDX(SPMASS,YAW))) ...
                    ,(VBB(LONG)   ...
                    - SSP(2,I)*U(SDINDX(SPMASS,YAW)))) - DELTA(I)

'Another way of calculating the sideslip angle, using the velocity ...
 of the contact point expressed in the wheel frame.'

      ALFA2(I) =  ATAN(VWP(2,I)/VWP(1,I))

'Dans way of calculating sideslip angle.'

      MAGH(I) = SQRT(EG(1,I)*EG(1,I) + EG(2,I)*EG(2,I))

'Calculate the heading vector as the cross product between the ...
spin axis and the inertial vertical axis, assuming the ground  ...
to be horizontal.'

      HEAD(1,I) =  EG(2,I)/MAGH(I)
      HEAD(2,I) = -EG(1,I)/MAGH(I)

      MAGV(I) = SQRT(VGW(1,I)*VGW(1,I) + VGW(2,I)*VGW(2,I))

      UVEL(1,I) = VGW(1,I)/MAGV(I)
      UVEL(2,I) = VGW(2,I)/MAGV(I)
```

```
'Sideslip angle calculated using the definition of cross product ...
 between the projection of the velocity and heading angle.'

   ALFA3(I) =  ASIN((UVEL(2,I)*HEAD(1,I)) - UVEL(1,I)*HEAD(2,I))

'Calculate the tyre forces.'


'     NTYRE(FT,TP=KT,CFL,CTP,UFL,ALFA1, ZT,I) '

      LTYRE(FT=KT,UFL,ALFA1,ZT,I)

 'Determination of the contact point between the wheel and the ...
  ground, expressed in the wheel frame.'

   CP(3,I) = - TH(I) + ZT(I)
   CP(2,I) = 0.0
   CP(1,I) = - TP(I)

  103..CONTINUE

'Calculate the delayed lateral tyre force.'

  DO 104 I=1,4
'    TAN(I) = CLEN(I)/MAGV(I) '
'    FTLD(I) = (FT(2,I) - FTL(I))/TAN(I) '
  104..CONTINUE

'  FTL = INTVC(FTLD,FTL0) '

'=================================================================='

'Determination of driving force to maintain constant speed ...
 during cornering.'

'Proportional controller of speed with high gain.'

'Error in the forward velocity.'

  VERR= SPEED - VBB(LONG)

'Driving force.'

  DO 190 I=1,4
    FD(3,I) = 0.0
    FD(2,I) = 0.0
    FD(1,I) = KP(I)*VERR*(STEP(TDF) - STEP(TBO))/2.0
  190..CONTINUE

  DO 106 I=1,4

'Calculate  the lateral tyre force due to steering system ...
 compliance.'

    FT1(1,I) = RSW(ZT(I).LT.0.0, 0.0, FT(1,I) + FD(1,I))
    FT1(2,I) = RSW(ZT(I).LT.0.0, 0.0, FT(2,I)/(1+KT(2,I)*CS(I)))
    FT1(3,I) = RSW(ZT(I).LT.0.0, 0.0, FT(3,I))

  106..CONTINUE

 'Calculate the total force required for moving the vehicle, ...
  for each wheel.'

  DO 112 I=1,4

    FTOT(I) = SQRT(FT1(1,I)**2 +FT1(2,I)**2)
```



```

'Maximum adhesion force available for each wheel.'

    FMAX(I) = MU*FT1(3,I)

'Tyre force if vehicle is sliding.'

'Direction of the sliding force.'

    MAGS(I) = SQRT(VWP(1,I)**2 + VWP(2,I)**2)

    DO 108 J=1,2

      FSDIR(J,I) = VWP(J,I)/MAGS(I)

'Sliding force.'

      FSLIDE(J,I) = - MUDYN*FT1(3,I)*FSDIR(J,I)

    108..CONTINUE

'For the vertical direction, the force must remain the same.'

    FSLIDE(3,I)= FT1(3,I)

'Actual tyre force.'

    DO 107 J=1,3
      FTS(J,I) = FT1(J,I)
'     FTS(J,I) = RSW(FTOT(I) .GT. FMAX(I), FSLIDE(J,I), FT1(J,I)) '
    107..CONTINUE

  112..CONTINUE

'Sideslip angle of the car.'

  BETA = ATAN(VBB(2)/VBB(1))

'=================================================================='
'Determination of aerodynamics effects'
'=================================================================='

'Side wind velocity. Considered  in the lateral direction  of ...
the base body frame.'

  VWIND =  WINVEL*PULSE(TPUL,PER,WID) + UW

'Module of the resultant air velocity.'

  VRES = SQRT((-VWIND+VBB(LAT))**2 + VBB(LONG)**2)

'Aerodynamic sideslip angle .'

  ABETA =  ATAN2((-VWIND+VBB(LAT)),VBB(LONG))

'Aerodynamic pressure.'

  QW = RO*VRES**2/2.0

'Aerodynamic drag.'

  FA(1) = - CDX*AF*QW*SIGN(1.0,VBB(LONG))

'Aerodynamic side force.'

  FA(2) = - CDY*ABETA*AF*QW
```

```
'Aerodynamic lift.'

  FA(3) =   (CDZF - CDZR*ABETA)*AF*QM

'Aerodynamic roll torque.'

  MA(1) = -  CMX*ABETA*AF*QM

'Aerodynamic pitch torque.'

  MA(2) =  - CMY*AF*QM

'Aerodynamic yaw torque.'

  MA(3) =  CMZ*ABETA*AF*QM

'Lateral force to check over/understeer behaviour.'

  FD1(1) = 0.0
  FD1(2) = FL
  FD1(3) = 0.0


'=================================================================='
'Apply forces and torques to the system.'
'=================================================================='

'External forces on the sprung mass.'

'Suspension forces.'

  DO 160 I=1,4
    CALL SDPOINTF(SPMASS,SSP(ROW,I),FSB(ROW,I))
  160..CONTINUE

'Aerodynamic forces.'

  CALL SDPOINTF(SPMASS,PAR,FA)

'Aerodynamic torques.'

  CALL SDBODYT(SPMASS,MA)

'Driving force.'

  CALL SDPOINTF(SPMASS, CG, FD1)

'=================================================================='

'Transform suspension forces from sprung mass frame to the ...
 suspension frame.'

  DO 170 I=1,4
    CALL SDTRANS(SPMASS, FSB(ROW,I), WHEEL(I), FSW(ROW,I))
  170..CONTINUE

  DO 172 I=1,4
    DO 174 J=1,3
        FW(J,I) = - FSW(J,I)
    174..CONTINUE
  172..CONTINUE

'Apply the suspension forces to the suspension arm ends.'

  DO 180 I=1,4
```



```

    CALL SDPOINTF(HUB(I), CG, FW(ROW,I))

'Apply the tyre forces to the wheels.'

    CALL SDPOINTF(WHEEL(I), CP(ROW,I), FTS(ROW,I))

  180..CONTINUE

'=================================================================='
'Call subroutine SDDERIV which contains the equations of...
 of motion generated by SDFAST.'
'=================================================================='

  CALL SDDERIV(QDOT,UDOT=)

'Calculate the sprung mass acceleration in the ground frame.'

  CALL SDACC(SPMASS, CG, AGB)

END $ 'OF PROCEDURAL'

'Integrate system states'

  Q = INTVC(QDOT,QO)
  U = INTVC(UDOT,UO)

  DO 114 I=1,4
    TERMT(T.GE.SIMLEN  .OR. VBB(1) .LT. 0.8 )
  114..CONTINUE

END  $ 'OF DERIVATIVE'

END  $ 'OF DYNAMIC'

  CALL SDPRINTERR(6)

END  $ 'OF PROGRAM'
```

B.2 ACSL Setup file

```
'================================================================'
' This file contains the following procedure blocks corresponding'
' to the cmd files in the directory in order to exercise         '
' the model nsusp6.acsl:                                         '

' acceleration4                                                  '
' deceleration4                                                  '
' corner4                                                        '
' corbra4                                                        '
' obstacle4                                                      '
' sidewind4                                                      '
' abcd4                                                          '
' eigen4                                                         '
' ss24                                                           '
' ss54                                                           '
' ss204                                                          '
' ss404                                                          '

'=============================================================== '
'This part contains the commands for an ACSL proced  block for   '
'runtime in order to simulate an acceleration maneuvre from      '
'an initial velocity of 2 m/s (-4 mph) to 25 m/s (-50 mph).      '

'It is assumed a terminated ramp input rear wheel drive          '
'force with a maximum acceleration of 0.5g.                      '

' Coefficient cb is given as a percentage of g.                  '

'=============================================================== '

proced acceleration4
s title = "DRIVING FORCE INPUT"
output 'clear'
prepar 'clear'
s rampz=4*0.0
s simlen=9.98
s kp=0
s uo(1)=2.0
s umax=25.0
s tb=.5
s usdel=4*0.0
s cb=0,0,.25,.25
prepar t,ft(7),u(1),q(5),q(7),q(9),q(1),q(3)
output t,u(1),q(5)
s cint=.02
s natp=2
s nciout=10
spare$start$spare
end

'================================================================'
'This part contains the commands for an ACSL proced  block for   '
'runtime in order to simulate a deceleration maneuvre from       '
'an initial velocity of 20.0 m/s to 1.0 m/s.                     '

'It is assumed a terminated ramp input rear wheel drive          '
'with a maximum deceleration of 1g, with 40/60 front/rear force  '
'distribuition.                                                  '

'Coefficient cb is given as a percentage of g.                   '

'=============================================================== '

proced deceleration4
s title = "BRAKING FORCE INPUT"
```



```
output 'clear'
prepar 'clear'
s rampz=4*0.0
s simlen=5.98
s kp=0
s uo(1)=20.0
s umax=40.0
s umin=1.0
s tb=.5
s usdel=4*0.0
s cb=-.3,-.3,-.2,-.2
prepar t,ft(1),ft(7),u(1),q(5),q(7),q(9),q(1),q(3)
output t,u(1),q(5)
s cint=.01
s natp=1
s nciout=10
spare$start$spare
end

'================================================================'
' This procedure contains the runtime commands for a combined    '
' cornering and braking maneuvres. The initial speed is 20 m/s   '
' when the steering wheel is moved of an angle of 20d. After 4s  '
' the brakes are applied at the front and rear, with a 60/40     '
' distribuition and a total braking force of 0.5 g.              '
'================================================================'

proced corbra4
s title = "STEERING ANGLE AND BRAKING FORCE INPUTS"
prepar 'clear'
output 'clear'
s simlen=11.98
s kp=160000
s ts-.5
s tb=4.5
s usdel=20,20,0,0
s umin=5
s uo(1)=20.0
s cb=-.15,-.15,-.10,-.10
s rampz=4*0
prepar t,spos(1),u(4),u(2),u(1),q(5),q(6),udot(2),q(2),q(1)
prepar vbb, ft(6), ft(12), beta, abb
s cint=.02
s natp=2
s nciout=5
output t,u(4),u(1)
spare$start$spare
end

'================================================================'
' This procedure describes a cornering maneuvre at constant speed '
' There is a driving force to keep the forward speed constant and '
' the steering input is an approximate step function of 10 d.    '
'================================================================'

proced corner4
s title = "STEERING ANGLE INPUT"
output 'clear'
prepar 'clear'
s cb=4*0.0
s uo(1)=20
s speed=20
s kp=16000
s rampz=4*0
s usdel= 45,45,0,0
s simlen=2.98
```

```
s tb=10
s tdf=0
s ts=.5
s cint=.02
s nciout=5
s nstp=2
prepar t,u(4),q(5),q(6),alfa(1),q(2),q(1),u(2),ft(2)
prepar ft(5),ft(8),ft(11),u(1),udot(2),vbb,spos,abb,beta
output t,u(4),vbb(2)
spare$start$spare
end

'================================================================'
' This part performs a linear analysis and calculate the abcd ma-'
' trices. The states are the generalised coordinates and speeds  '
' and the inputs are the steering angle, road disturbance,       '
' suspension force, wheel torque and wind disturbance.           '
'================================================================'

proced abcd4
s simlen=0
output 'clear'
prepar 'clear'
start
analyz 'clear'
analyz 'freeze'=svel,spos,q(1),q(2)
analyz 'xinc'= 1.0e-3,'contrl'=us,uz,ufa(3),ufa(6),ufa(9),ufa(12),uw
analyz 'observ'=q(1),q(2),q(3),q(4),q(5),q(6),q(7),q(8),q(9),q(10)
analyz 'observ'=u(1),u(2),u(3),u(4),u(5),u(6),u(7),u(8),u(9),u(10)
analyz 'observ'=z(3),z(6),z(9),z(12)
analyz 'observ'= zdot(3),zdot(6),zdot(9),zdot(12),zt
analyz 'jacob'= susp4g
end

'=================================================================='
' This procedure block describes an obstacle crossing maneuvre of  '
' a triangular bump of length 2*rampx and height rampz with a time '
' difference of 0.1s between the left and right hand sides of the  '
' car, corresponding to a distance of 0.5m traveling at 5 m/s.     '
'=================================================================='

proced obstacle4
s title = "ROAD INPUT"
output 'clear'
prepar 'clear'
s simlen=3.48
s kp=0
s cb=4*0
s rampz=4*0.1
s rampx=4*0.5
s usdel-4*0.0
s klazl=4*1.0
s uo(1)=5
prepar t,zroadl,q(3),q(5),q(7),q(8),q(9),q(10),q(6),pgw
prepar uf2(3),uf2(6),uf2(9),uf2(12),zt
output t,zroadl(1),q(3)
s cint=.01
s nstp=1
s nciout-10
spare$start$spare
end

'=================================================================='
'This part contains the commands for the                          '
'sidewind procedure to be implemented  in the program             '
'susp6.acsl in order to simulate a sidewind                       '
```



```
'disturbance.                                                     '
'=================================================================='

proced sidewind4
s title = "SIDEWIND INPUT"
output 'clear'
prepar 'clear'
s rampz=4*0.0
s cb=4*0.0
s usdel=4*0.0
s kp=160000
s simlen=4.98
s tb=25
s tp=0.5
s wid=2.0
s per=25
s speed=40
s uo(1)=40
s umax =45
s umin=1
s vel=7.08
output t,q(2),vwind
prepar t,q(5),q(6),q(1),q(2),q(3),u(4),vbb(2),u(1),fa,vwind
prepar ma,alfa,udot(2),u(2),abb, beta , abeta, q(4)
s cint=.02
s nstp=2
s nciout=5
spare$start$spare
end

'=================================================================='
' This procedure block performs an eigenvalue analysis in order to '
' check whether the model has been assembled correctly or not.     '
'=================================================================='

proced eigen4
s simlen=0
output 'clear'
output 'clear'
analyz 'clear'
start
analyz 'freeze'=spos,svel,'eigen'
analyz 'clear'
end

'================================================================='
' These procedures calculate the steady state for various initial '
' velocities, in order to eliminate transient effects in the      '
' pitch and vertical motions, as well as in the suspension.       '
'================================================================='

'Initial velocity =  2.0 m/s.                                     '

proced ss24
output 'clear'
prepar 'clear'
s simlen=9.98
s rampz=4*0
s usdel=4*0
s tb=25
s cb=4*0
s kp=160000
s speed=2
s uo(1)=2
s cint=.02
s nstp=2
```

```
s nclout=10
output t,u(1),q(3)
spare$start$spare
s q(1)=0
s t=0
save 'ss2'
end

'=================================================================='

'Initial velocity = 5.0 m/s                                        '

proced ss54
output 'clear'
prepar 'clear'
s simlen=9.98
s rampz=4*0
s usdel=4*0
s cb=4*0
s tb=25
s kp=160000
s speed=5
s uo(1)=5
s cint=.02
s nstp=2
s nclout=10
output t,u(1),q(3)
spare$start$spare
s q(1)=0
s t=0
save 'ss5'
end

'=================================================================='

'Initial velocity =  20.0 m/s                                      '

proced ss20
output 'clear'
prepar 'clear'
s simlen=9.98
s rampz=4*0
s usdel=4*0
s cb=4*0
s tb=25
s kp=160000
s speed=20
s uo(1)=20
s cint=.02
s nstp=2
s nclout=10
output t,u(1),q(3)
spare$start$spare
s q(1)=0
s t=0
save 'ss20'
end

'=================================================================='

'Initial velocity = 40.0 m/s                                       '

proced ss40
output 'clear'
prepar 'clear'
s simlen=9.98
```



```
s rampz=4*0
s usdel=4*0
s cb=4*0
s tb=25
s kp=160000
s speed=40
s uo(1)=40
s cint=.02
s nstp=2
s nclout=10
output t,u(1),q(3)
spare$start$spare
s q(1)=0
s t=0
save 'ss40'
end

'=================================================================='
```

```

'==================================================================='
' This file contains the following procedure blocks corresponding'
' to the cmd files in the directory in order to exercise          '
' the model susp6g.acsl:                                          '

' acceleration6                                                   '
' deceleration6                                                   '
' corner6                                                         '
' corbra6                                                         '
' obstacle6                                                       '
' sidewind6                                                       '
' abcd6                                                           '
' eigen6                                                          '
' ss26                                                            '
' ss56                                                            '
' ss206                                                           '
' ss406                                                           '

'================================================================ '
'This part contains the commands for an ACSL proced  block for    '
'runtime in order to simulate an acceleration maneuvre from       '
'an initial velocity of 2 m/s (-4 mph) to 25 m/s (-50 mph).       '

'It is assumed a terminated ramp input rear wheel drive           '
'force with a maximum acceleration of 0.5g.                       '

' Coefficient cb is given as a percentage of g.                   '

'================================================================ '

proced acceleration6
s title = "DRIVING FORCE INPUT"
output 'clear'
prepar 'clear'
s rampz=4*0.0
s simlen=9.98
s kp=4*0
s uo(1)=2.0
s umax=25.0
s tb=1.0
s tbo=0.5
s usdel=4*0.0
s cb=0,0,.25,.25
prepar t,fts,zt,u(1),q(5),q(7),q(9),q(1),q(3)
output t,u(1),q(5)
s cint=.02
s nstp=2
s nciout=10
spare$start$spare
end

'================================================================='
'This part contains the commands for an ACSL proced  block for    '
'runtime in order to simulate a deceleration maneuvre from        '
'an initial velocity of 20.0 m/s to 1.0 m/s.                      '

'It is assumed a terminated ramp input rear wheel drive           '
'with a maximum deceleration of 1g, with 40/60 front/rear force   '
'distribuition.                                                   '

'Coefficient cb is given as a percentage of g.                    '

'---------------------------------------------------------------- '

proced deceleration6
```



```
s title = "BRAKING FORCE INPUT"
output 'clear'
prepar 'clear'
s rampz=4*0.0
s simlen=5.98
s kp=0,0,160000,160000
s uo(1)=20.0
s speed=20.0
s umax=40.0
s umin=1.0
s tbo=.5
s tb=1.0
s usdel=4*0.0
s cb=-0.28,-0.28,-0.12,-0.12
prepar t,fts,zt,u(1),q(5),q(7),q(9),q(1),q(3)
output t,u(1),q(5)
s cint=.01
s nstp=1
s nciout=10
spare$start$spare
end

'================================================================='
' This procedure contains the runtime commands for a combined     '
' cornering and braking maneuvres. The initial speed is 20 m/s    '
' when the steering wheel is moved of an angle of 20d. After 4s   '
' the brakes are applied at the front and rear, with a 60/40      '
' distribuition and a total braking force of 0.5 g.               '
'================================================================='

proced corbra6
s title = "STEERING ANGLE AND BRAKING FORCE INPUTS"
prepar 'clear'
output 'clear'
s simlen=11.98
s kp=0,0,160000,160000
s ts=.5
s tb=4.5
s usdel=20,20,0,0
s umin=5
s uo(1)=20.0
s cb=-.15,-.15,-.10,-.10
s rampz=4*0
prepar t,hpos,u(4),u(2),u(1),q(5),q(6),udot(2),q(2),q(1)
prepar vbb, ft(6), ft(12), beta, abb,alfa1
s cint=.02
s nstp=2
s nciout=5
output t,u(4),u(1)
spare$start$spare
end

'================================================================='
' This procedure describes a cornering maneuvre at constant speed '
' There is a driving force to keep the forward speed constant and '
' the steering input is an approximate step function of 10 d.    '
'================================================================='

proced corner6
s title = "STEERING ANGLE INPUT"
output 'clear'
prepar 'clear'
s cb=4*0.0
s uo(1)=20
s speed=20
s kp=0.0,0.0,8000,8000
```

```
s rampz=4*0
s usdel= 45,45,0,0
s simlen=2.98
s tbo=50
s tdf=0
s ts=.5
s cint=.02
s nciout=5
s nstp=8
prepar t,u(4),q(5),q(6),alfa1,q(2),q(1),u(2),fts,ftot,fmax
prepar w(6),u(1),udot(2),vbb,hpos(1),abb(2),beta ,falide
prepar alfa2,alfa3,q(4), spos ,tp
output t,u(4),vbb(2)
spare$start$spare
end

'=================================================================='
' This procedure block describes an obstacle crossing maneuvre of  '
' a triangular bump of length 2*rampx and height rampz with a time '
' difference of 0.1s between the left and right hand sides of the  '
' car, corresponding to a distance of 0.5m traveling at 5 m/s.     '
'=================================================================='

proced obst16
s title = "ROAD INPUT"
output 'clear'
prepar 'clear'
s simlen=2.48
s kp=4*0
s cb=4*0
s xo=2.5,3.0
s rampz=4*0.1
s rampx=4*0.5
s usdel=4*0.0
s klaz1=4*1.0
s klaz2=4*0.0
s klaz3=4*0.0
s klaz4=4*0.0
s uo(1)=5.0
s speed=5.0
prepar t,zroad1,q(3),q(5),q(7),q(8),q(9),q(10),q(11),q(6),pgw
prepar zt,fts(3),fts(6),fts(9),fts(12),abb(3)
prepar fsb(3), fsb(6),fsb(9),fsb(12)
output t,zroad1(1),zroad1(4)
s cint=.01
s nstp=1
s nciout=10
spare$start$spare
end

'=================================================================='
' This procedure block describes an obstacle crossing maneuvre of  '
' a terminated ramp input with length rampx and height  rampz. The '
' program calculates the equivalent time domain input for          '
' travelling with a forward velocity vbb(long)                     '
'=================================================================='

proced obst26
s title = "ROAD INPUT"
output 'clear'
prepar 'clear'
s simlen=3.48
s kp=4*0
s cb=4*0
s xo=2.5,2.5
s rampz=4*0.05
```



```
s rampx=4*1.0
s usdel=4*0.0
s klaz1=4*0.0
s klaz2=4*1.0
s klaz3=4*0.0
s klaz4=4*0.0
s uo(1)= 5.0
s speed=5.0
prepar t,zroad2,q(3),q(5),q(7),q(8),q(9),q(10),q(11),q(6),pgw
prepar zt, ft1(3), ft1(6), ft1(9), ft1(12), abb(3)
prepar fsb(3),fsb(6),fsb(9),fsb(12)
output t,zroad2(1),zroad2(3)
s cint=.01
s nstp=1
s nciout=10
spare$start$spare
end

'=================================================================='
' This procedure block describes an obstacle crossing maneuvre of  '
' a terminated ramp input with length rampx and height  rampz. The '
' program calculates the equivalent time domain input for          '
' travelling with a forward velocity vbb(long)                     '
'=================================================================='

proced obst46
s title = "ROAD INPUT"
output 'clear'
prepar 'clear'
s simlen=3.48
s kp=4*0
s cb=4*0
s xo=2.5,2.5
s rampz=4*0.05
s rampx=4*1.0
s usdel=4*0.0
s klaz1=4*0.0
s klaz2=4*0.0
s klaz3=4*0.0
s klaz4=4*1.0
s uo(1)= 5.0
s speed=5.0
prepar t,zroad4,q(3),q(5),q(7),q(8),q(9),q(10),q(11),q(6),pgw
prepar zt, ft1(3), ft1(6), ft1(9), ft1(12), abb(3)
prepar fsb(3),fsb(6),fsb(9),fsb(12)
output t,zroad4(1),zroad4(3)
s cint=.01
s nstp=1
s nciout=10
spare$start$spare
end

'=================================================================='
' This procedure block describes an obstacle crossing maneuvre of  '
' a triangular bump of length 2*rampx and height rampz with a time '
' difference of 0.1s between the left and right hand sides of the  '
' car, corresponding to a distance of 0.5m traveling at 5 m/s.     '
'=================================================================='

proced obst36
s title = "ROAD INPUT"
output 'clear'
prepar 'clear'
s simlen=4.98
s kp=0,0,160000,160000
s cb=4*0
```

```
s xo=2.5,2.5
s rampz=4*.005
s rampx=4*5
s usdel=4*0.0
s klaz1=4*0.0
s klaz2=4*0.0
s klaz3=4*1.0
s klaz4=4*0.0
s uo(1)=5.0
s speed=5.0
prepar t,zroad3,q(3),q(5),q(7),q(8),q(9),q(10),q(6),pgw
prepar fts(3), fts(6), fts(9), fts(12), abb(3)
prepar zt, fsb(3),fsb(6),fsb(9),fsb(12)
output t,rampz(1),wn(1)
s cint=.04
s nstp=1
s nciout=25
spare$start$spare
end

'=================================================================='
'This part contains the commands for the                           '
'sidewind procedure to be implemented  in the program              '
'susp6.acsl in order to simulate a sidewind                        '
'disturbance.                                                      '
'=================================================================='

proced sidewind6
s title = "SIDEWIND INPUT"
output 'clear'
prepar 'clear'
s rampz=4*0.0
s cb=4*0.0
s usdel=4*0.0
s kp=0,0,160000,160000
s simlen=4.98
s tb=25
s tp=0.5
s wid=2.0
s per=25
s speed=40
s uo(1)=40
s umax =40
s umin=1
s winvel=7.08
output t,q(2),vwind
prepar t,q(5),q(6),q(1),q(2),q(3),u(4),vbb(2),u(1),fa,vwind
prepar ma,alfa1,udot(2),u(2),abb, beta ,abeta
s cint=.02
s nstp=2
s nciout=5
spare$start$spare
end

'=================================================================='
' This procedure block performs an eigenvalue analysis in order to '
' check whether the model has been assembled correctly or not.     '
'=================================================================='

proced eigen6
s simlen=0
output 'clear'
prepar 'clear'
analyz 'clear'
start
analyz 'freeze'=q(11),q(12),q(13),q(14),q(15),q(16),q(17),q(18)
```



```
analyz 'freeze'=u(11),u(12),u(13),u(14),u(15),u(16),u(17),u(18)
analyz 'freeze'=spos,svel,q(1),q(2),'eigen'
end


'=================================================================='
' This part performs a linear analysis and calculate the abcd ma-'
' trices. The states are the generalised coordinates and speeds  '
' and the inputs are the steering angle, road disturbance,       '
' suspension force, wheel torque and wind disturbance.           '
'=================================================================='

proced abcd6
s simlen=0
output 'clear'
prepar 'clear'
start
analyz 'clear'
analyz 'freeze'=svel,spos
analyz 'freeze'=q(11),q(12),q(13),q(14),q(15),q(16),q(17),q(18)
analyz 'freeze'=u(11),u(12),u(13),u(14),u(15),u(16),u(17),u(18)
analyz 'xinc'= 1.0e-3,'contrl'=us,uz,ufd,ufa(3),ufa(6),ufa(9),ufa(12),uw
analyz 'observ'=q(1),q(2),q(3),q(4),q(5),q(6),q(7),q(8),q(9),q(10)
analyz 'observ'=vbb,u(4),u(5),u(6),u(7),u(8),u(9),u(10)
'analyz 'observ'=udot(2),udot(3),vbb(2) '
'analyz 'observ'= zdot(3),zdot(6),zdot(9),zdot(12),zt '
analyz 'jacob'=susp6g
end

'=================================================================='
' These procedures calculate the steady state for various initial '
' velocities, in order to eliminate transient effects in the      '
' pitch and vertical motions, as well as in the suspension.       '
' ACSL trim facility can also be used.                            '
'=================================================================='

'Initial velocity =  2.0 m/s.                                     '

proced ss26
output 'clear'
prepar 'clear'
s simlen=9.98
s rampz=4*0
s usdel=4*0
s tb=25
s cb=4*0
s kp=0,0,160000,160000
s speed=2
s uo(1)=2
s cint=.02
s nstp=2
s nciout=10
output t,u(1),q(3)
spare$start$spare
s q(1)=0
s t=0
save 'ss2'
end

'=================================================================='

'Initial velocity = 5.0 m/s                                       '

proced ss56
output 'clear'
prepar 'clear'
```

```
s simlen=9.98
s rampz=4*0
s usdel=4*0
s cb=4*0
s tb=25
s kp=0,0,160000,160000
s speed=5
s uo(1)=5
s cint=.02
s nstp=2
s nciout=10
output t,u(1),q(3)
spare$start$spare
s q(1)=0
s t=0
save 'ss5'
end

'================================================================='

'Initial velocity =  20.0 m/s                                      '

proced ss206
output 'clear'
prepar 'clear'
s simlen=9.98
s rampz=4*0
s usdel=4*0
s cb=4*0
s tb=25
s kp=0,0,160000,160000
s speed=20
s uo(1)=20
s cint=.02
s nstp=2
s nciout=10
output t,u(1),q(3)
spare$start$spare
s q(1)=0
s t=0
save 'ss20'
end

'================================================================='

'Initial velocity = 40.0 m/s                                       '

proced ss406
output 'clear'
prepar 'clear'
s simlen=9.98
s rampz=4*0
s usdel=4*0
s cb=4*0
s tb=25
s kp=0,0,160000,160000
s speed=40
s uo(1)=40
s cint=.02
s nstp=2
s nciout=10
output t,u(1),q(3)
spare$start$spare
s q(1)=0
s t=0
save 'ss40'
```



```
end

'================================================================='
' This proced block is used to generate the sinusoidal steady-state'
' response - Generate the data for the simulation frequency       '
' response                                                        '
proced obloo36
s title = "ROAD INPUT"
output 'clear'
prepar 'clear'
's simlen=12.0                                                    '
s kp=0,0,160000,160000
s cb=4*0
's rampz=4*.005                                                   '
's rampx=4*10                                                     '
s usdel=4*0.0
s klaz1=4*0.0
s klaz2=4*0.0
s klaz3=4*1.0
's uo(1)=5.0                                                      '
's speed=5.0                                                      '
's xo=2.5,3                                                       '
prepar t,zroad3,q(3),q(5),q(7),q(8),q(9),q(10),q(6),pqw
prepar fts(3), fts(6), fts(9), fts(12), abb(3)
prepar zt, fsb(3),fsb(6),fsb(9),fsb(12)
output t,rampz(1), wn(1)
's nciout=25                                                      '
's cint=.04                                                       '
's nstp=8                                                         '
'spare'$'start'$'spare                                                '
'matlab                                                           '
's rampz=4*.01                                                    '
'spare'$'start'$'spare                                                '
'matlab                                                           '
's rampz=4*.025                                                   '
'spare'$'start'$'spare                                                '
'matlab                                                           '
's rampz=4*.005                                                   '
's uo(1)=10                                                       '
's speed=10                                                       '
's xo=5,6                                                         '
's simlen=8                                                       '
's cint=0.02                                                      '
's nstp=4                                                         '
's nciout=50                                                      '
'spare'$'start'$'spare                                                '
'matlab                                                           '
's rampz=4*.01                                                    '
'spare'$'start'$'spare                                                '
'matlab                                                           '
's rampz=4*.025                                                   '
'spare'$'start'$'spare                                                '
'matlab                                                           '
's rampz=4*.005                                                   '
's uo(1)=12.5                                                     '
's speed=12.5                                                     '
's xo=6.125,7.375                                                 '
's simlen=7.4                                                     '
's cint=.016                                                      '
's nstp=4                                                         '
's nciout=60                                                      '
'spare'$'start'$'spare                                                '
'matlab                                                           '
's rampz=4*.01                                                    '
'spare'$'start'$'spare                                                '
'matlab                                                           '
```

```
's rampz=4*.025                                   '
'spare'$'start'$'spare                               '
'matlab                                           '
's rampz=4*.005                                   '
's uo(1)=15                                       '
's speed=15                                       '
's xo=7.5,9                                       '
's simlen=7.0                                     '
's cint=.0133                                     '
's nstp=2                                         '
's nclout=75                                      '
'spare'$'start'$'spare                               '
'matlab                                           '
's rampz=4*.01                                    '
'spare'$'start'$'spare                               '
'matlab                                           '
's rampz=4*.025                                   '
'spare'$'start'$'spare                               '
'matlab                                           '
's rampz=4*.005                                   '
's rampx=4*1                                      '
's uo(1)=5                                        '
's speed=5                                        '
's xo=2.5,3                                       '
's simlen=5                                       '
's cint=.004                                      '
's nstp=1                                         '
's nclout=250                                     '
'spare'$'start'$'spare                               '
'matlab                                           '
's rampz=4*.01                                    '
'spare'$'start'$'spare                               '
'matlab                                           '
's rampz=4*.025                                   '
'spare'$'start'$'spare                               '
'matlab                                           '
s rampz=4*.005
s uo(1)=10
s speed=10
s rampx=4*1
s xo=5,6
s simlen=4.5
s cint=.002
s nstp=1
s nclout=500
spare$start$spare
matlab
s rampz=4*.01
spare$start$spare
matlab
s rampz=4*.025
spare$start$spare
matlab
s rampz=4*.005
s uo(1)=12.5
s speed=12.5
s xo=6.125,7.375
s simlen=4.4
s cint=.0016
s nstp=1
s nclout=625
spare$start$spare
matlab
s uo(1)=15
s speed=15
s xo=7.5,9
```



```
s simlen=4.3
s cint=.00133
s nstp=1
s nclout=750
spare$start$spare
matlab
s uo(1)=30
s speed=30
s xo=15,18
s simlen=4.15
s cint=.000667
s nstp=1
s nclout=1500
spare$start$spare
matlab
end
```

Appendix C

Simulation Results

C.1 Transient Manoeuvres

C.1.1 Steering Input

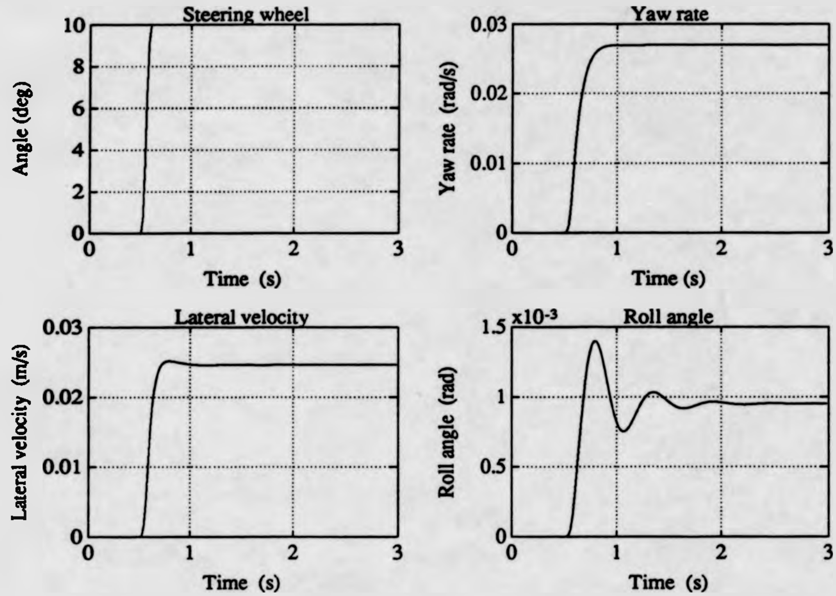


Figure C.1: Linear, without, steering angle: 10°, speed: $10 m/s$

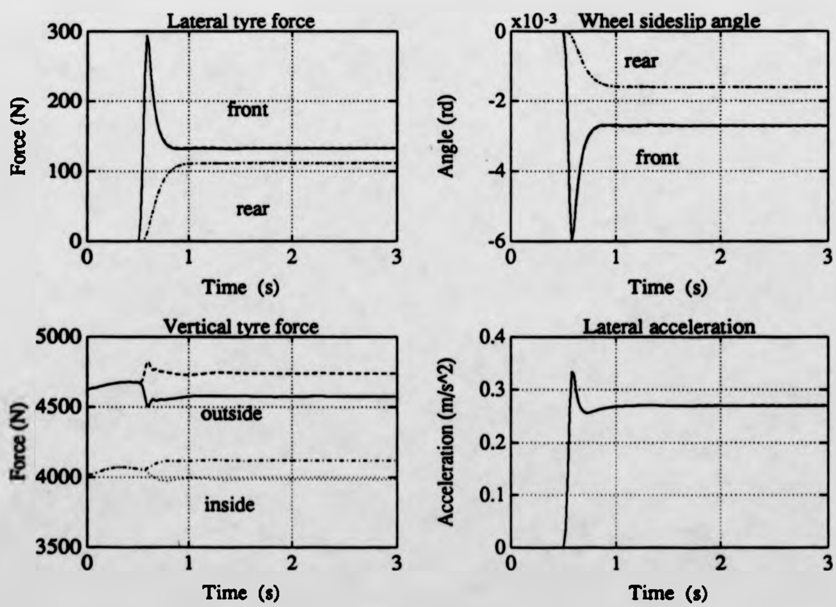


Figure C.2: Linear, without, steering angle: 10°, speed: $10 m/s$

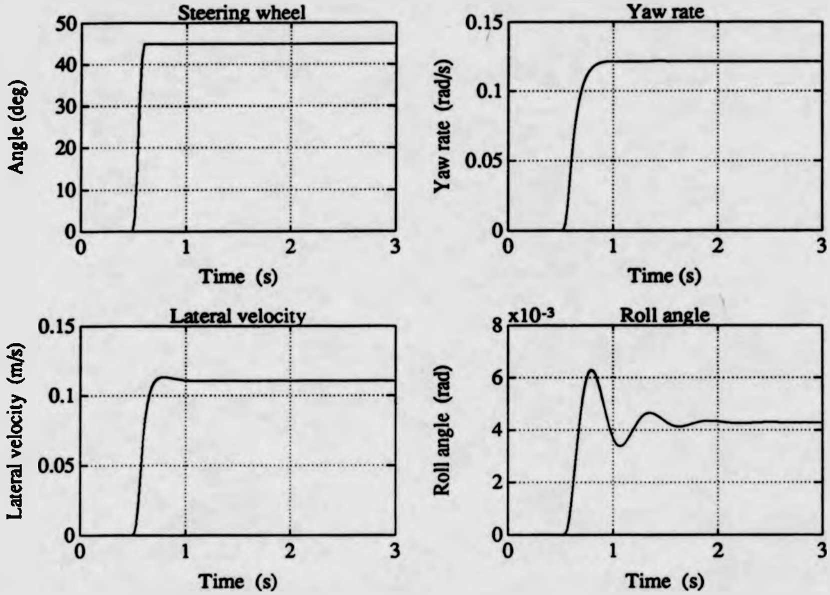


Figure C.3: Linear, without, steering angle: 45°, speed: $10m/s$

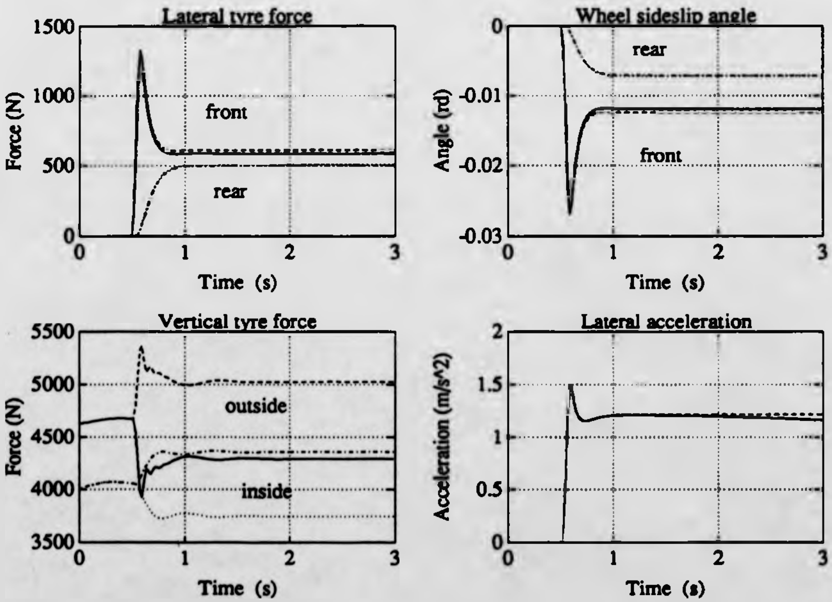


Figure C.4: Linear, without, steering angle: 45°, speed: $10m/s$

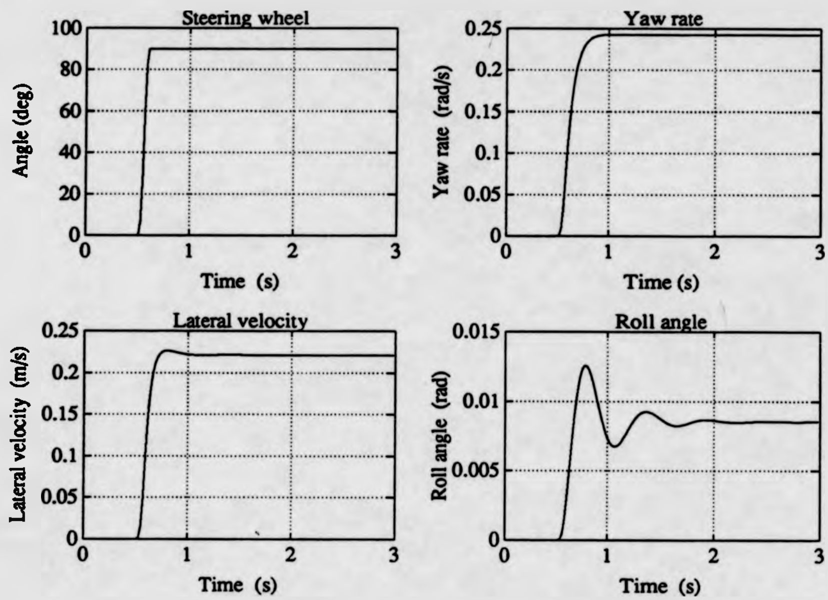


Figure C.5: Linear, without, steering angle: 90°, speed: $10m/s$

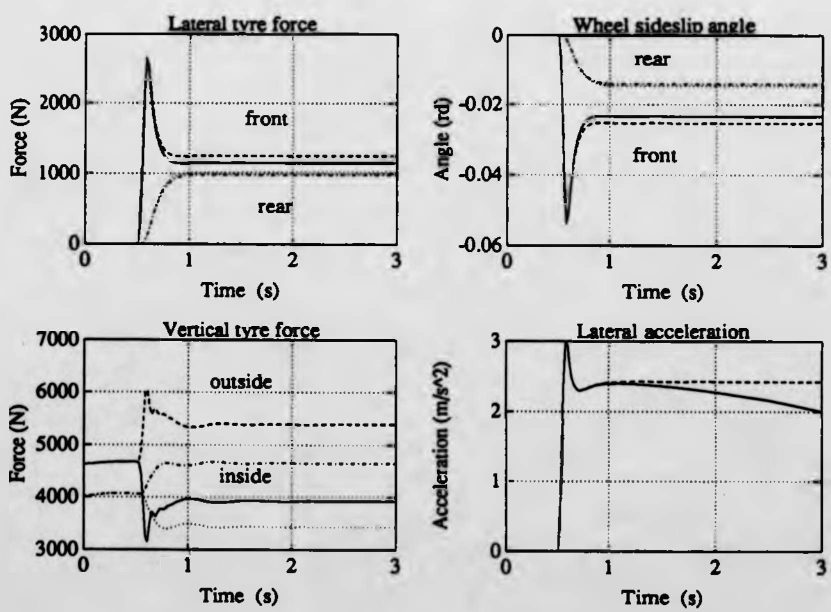


Figure C.6: Linear, without, steering angle: 90°, speed: $10m/s$

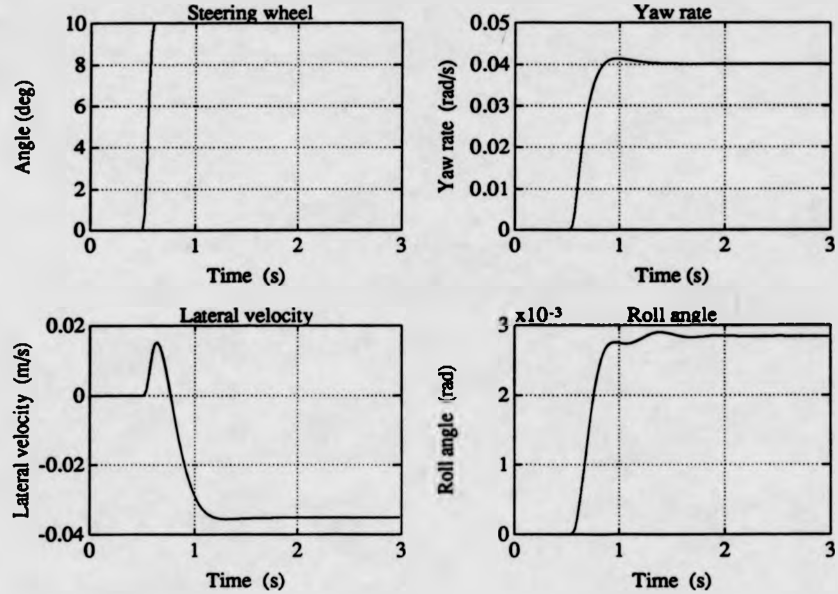


Figure C.7: Linear, without, steering angle: 10°, speed: $20 m/s$

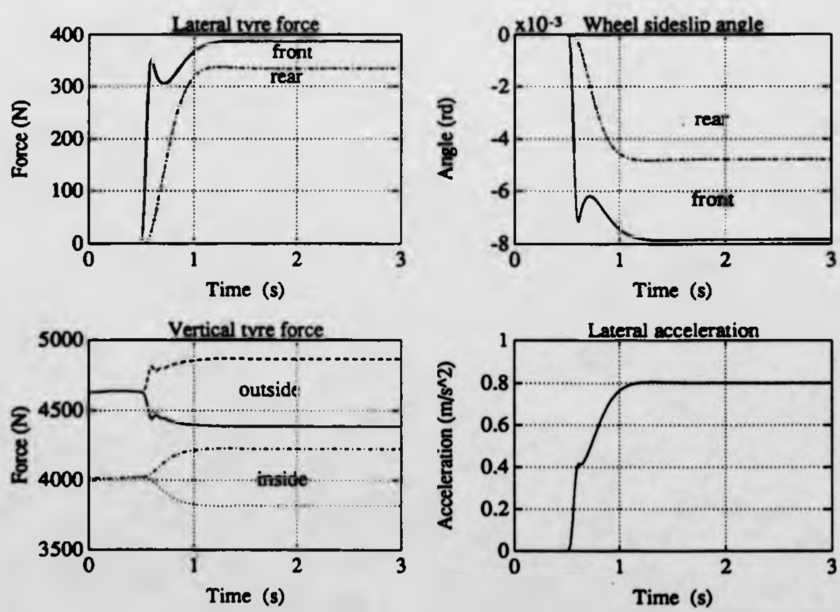


Figure C.8: Linear, without, steering angle: 10°, speed: $20 m/s$

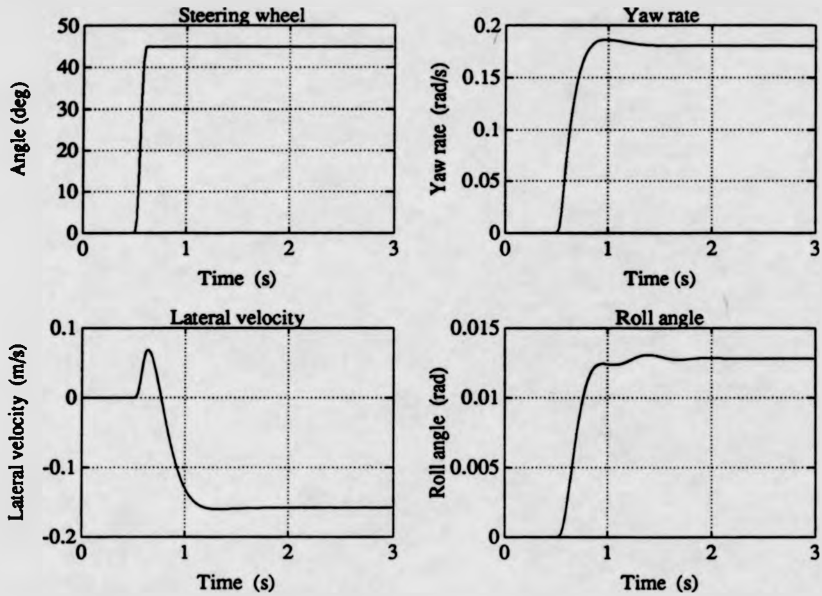


Figure C.9: Linear, without, steering angle: 45°, speed: $20m/s$

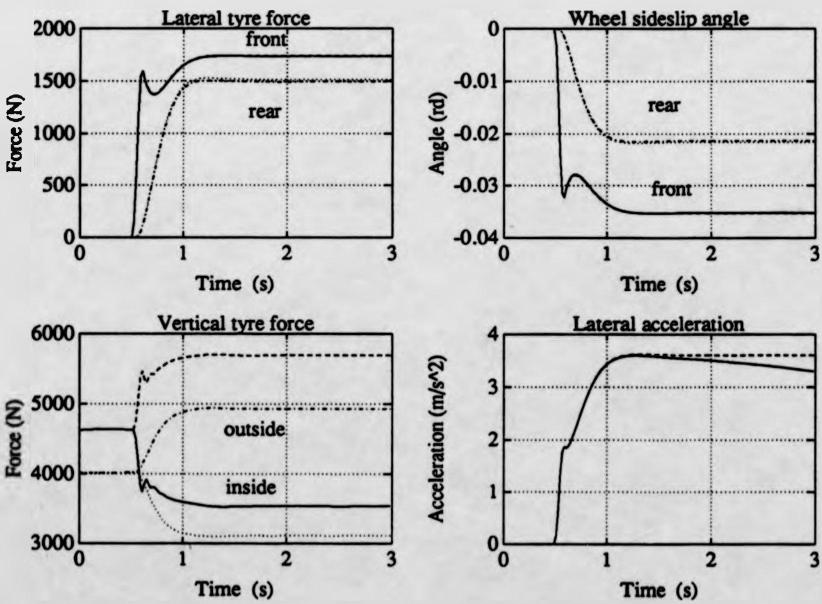


Figure C.10: Linear, without, steering angle: 45°, speed: $20m/s$

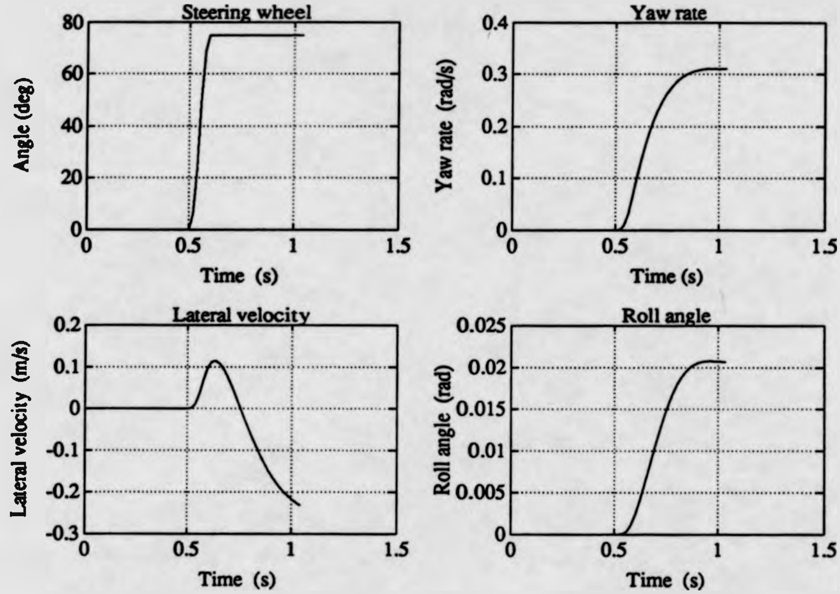


Figure C.11: Linear, without, steering angle: 75°, speed: $20m/s$

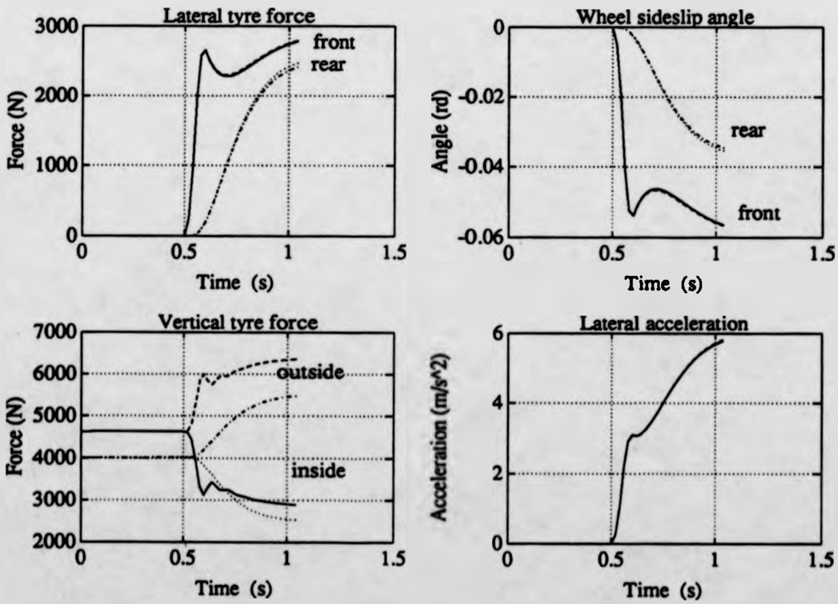


Figure C.12: Linear, without, steering angle: 75°, speed: $20m/s$

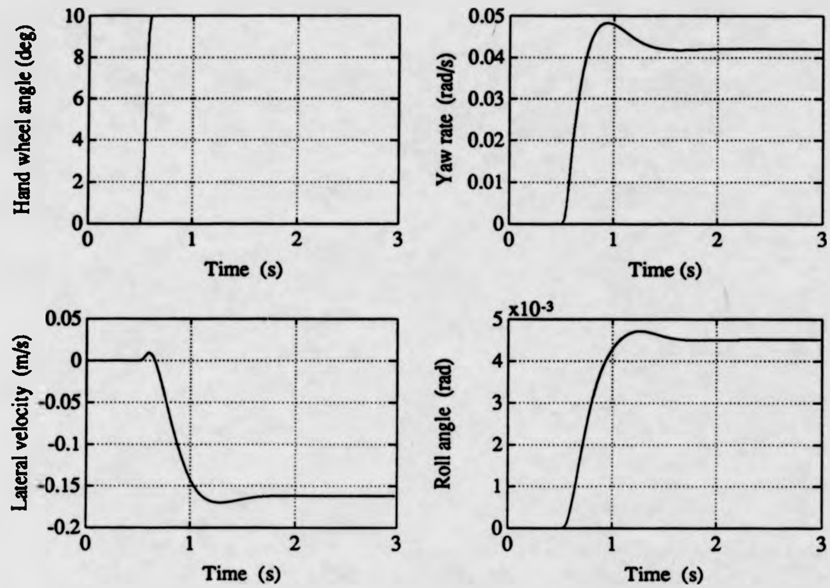


Figure C.13: Linear, without, steering angle: 10^o, speed: $30m/s$

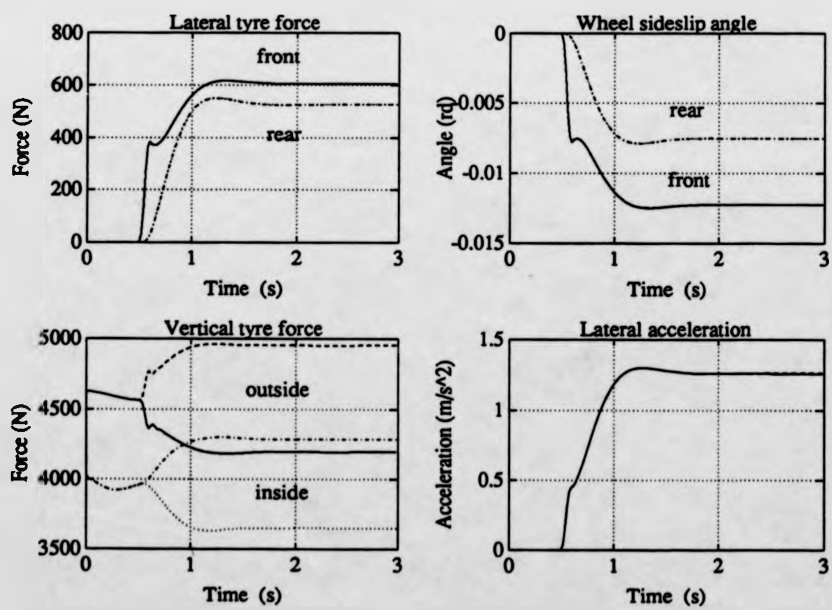


Figure C.14: Linear, without, steering angle: 10^o, speed: $30m/s$

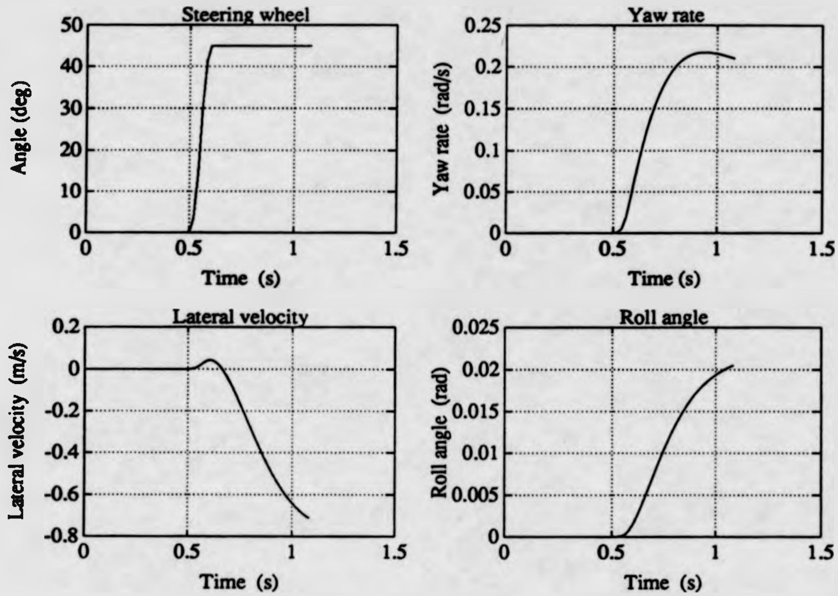


Figure C.15: Linear, without, steering angle: 45°, speed: $30m/s$

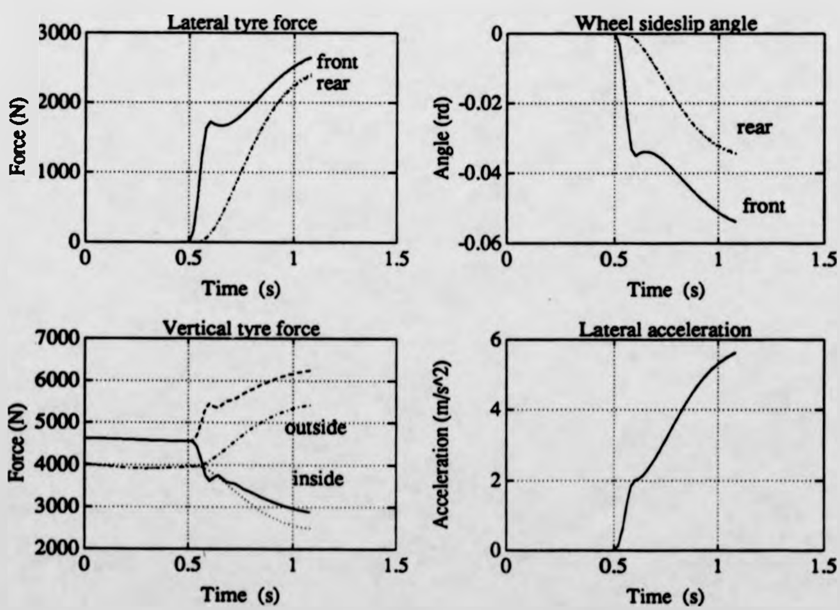


Figure C.16: Linear, without, steering angle: 45°, speed: $30m/s$

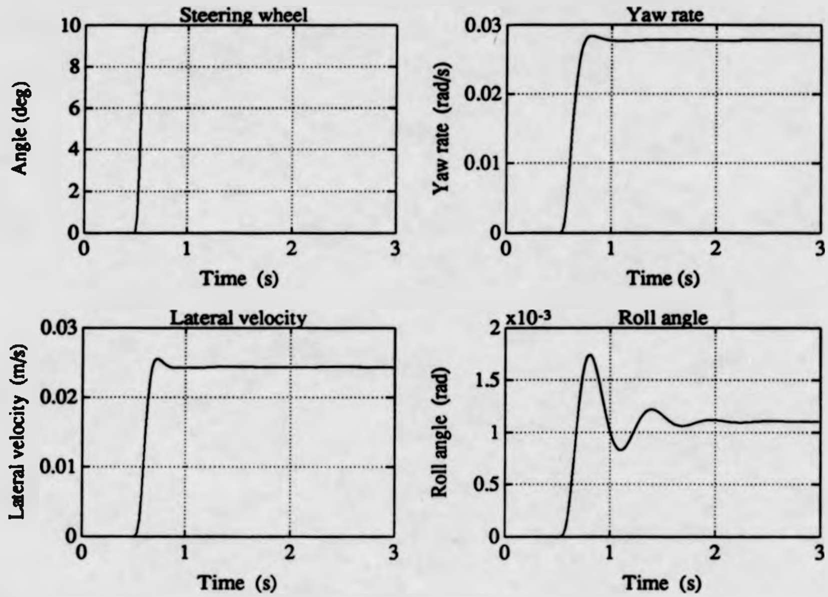


Figure C.17: Nonlinear, with, steering angle: 10^o, speed: $10m/s$

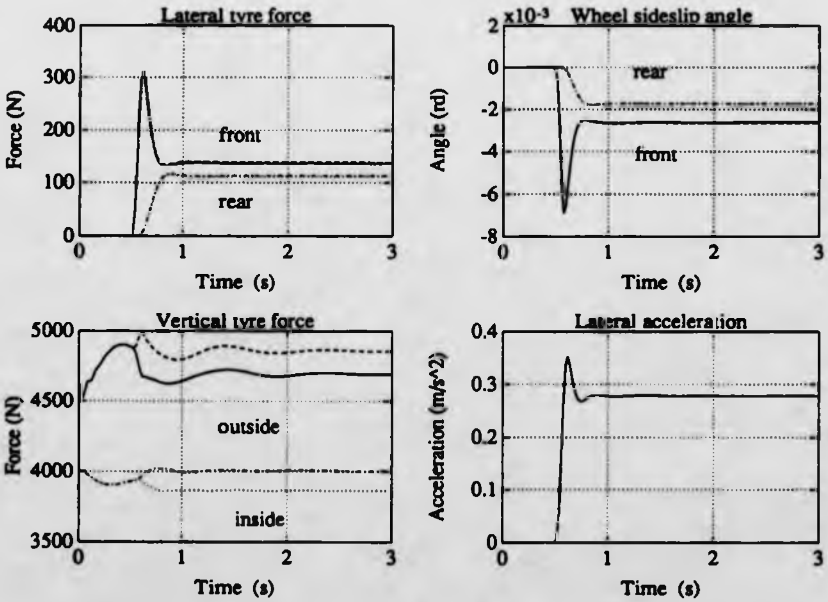


Figure C.18: Nonlinear, with, steering angle: 10^o, speed: $10m/s$

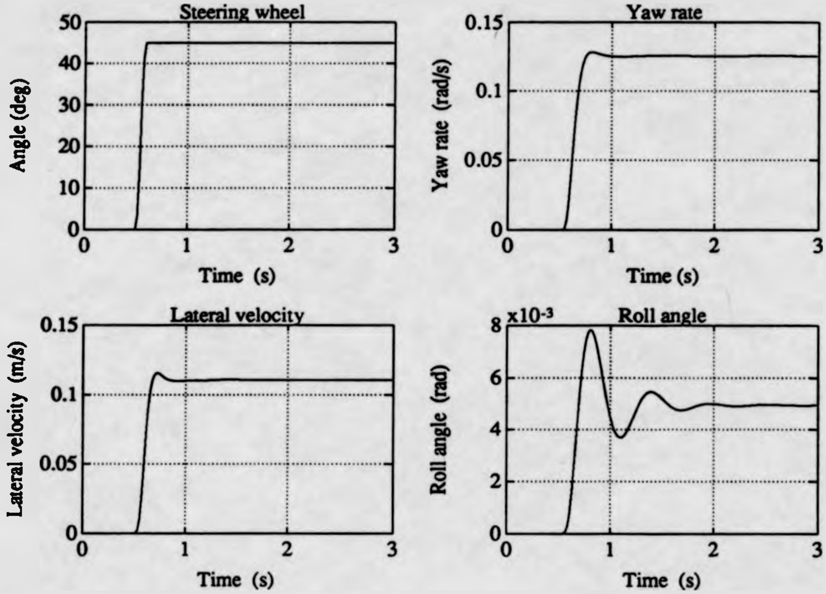


Figure C.19: Nonlinear, with, steering angle: 45°, speed: $10m/s$

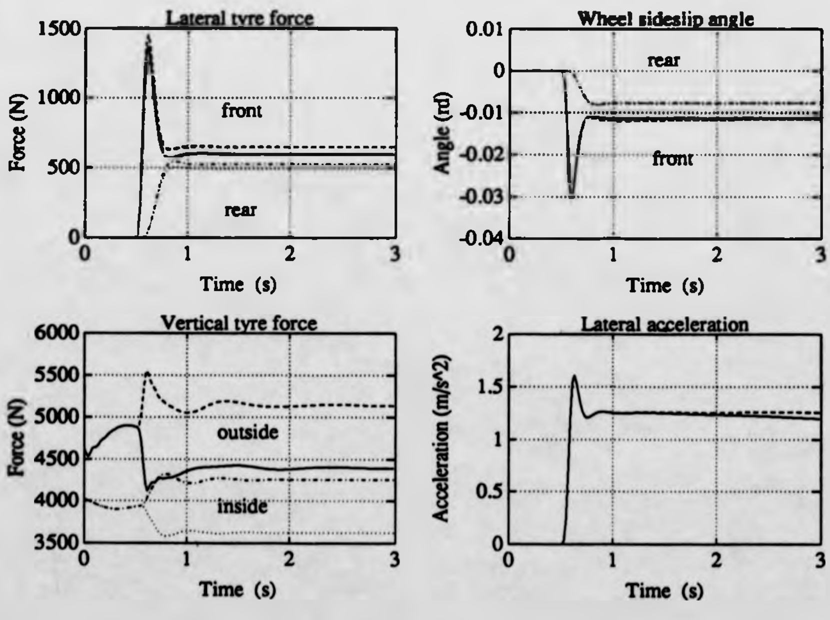


Figure C.20: Nonlinear, with, steering angle: 45°, speed: $10m/s$

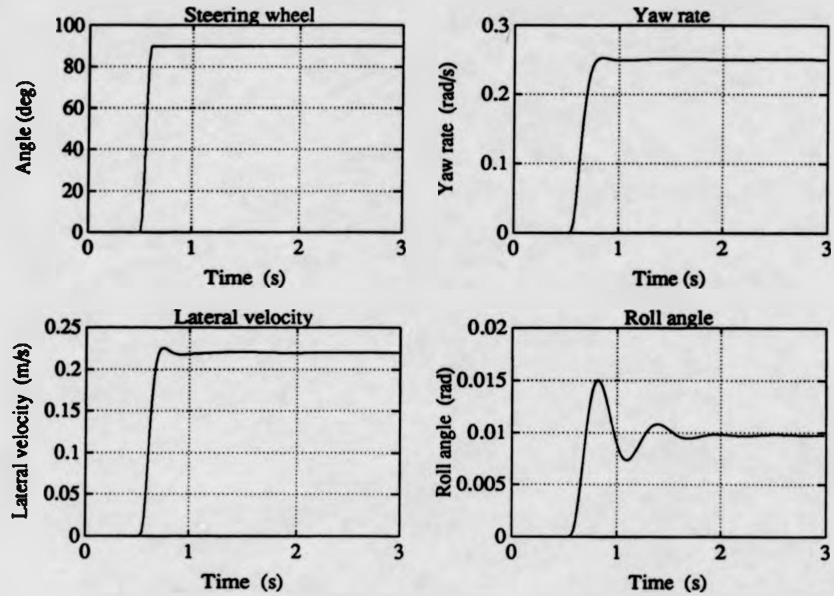


Figure C.21: Nonlinear, with, steering angle: 90°, speed: $10m/s$

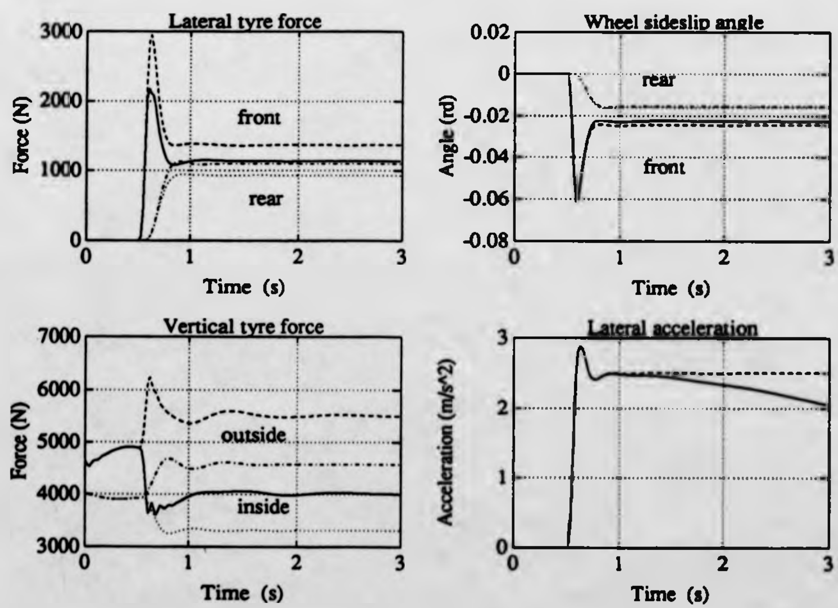


Figure C.22: Nonlinear, with, steering angle: 90°, speed: $10m/s$

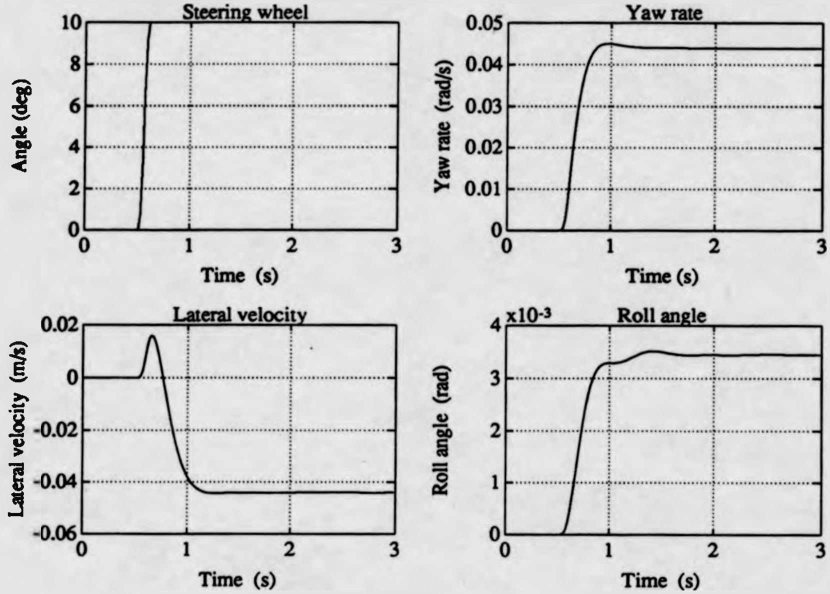


Figure C.23: Nonlinear, with, steering angle: 10°, speed: $20m/s$

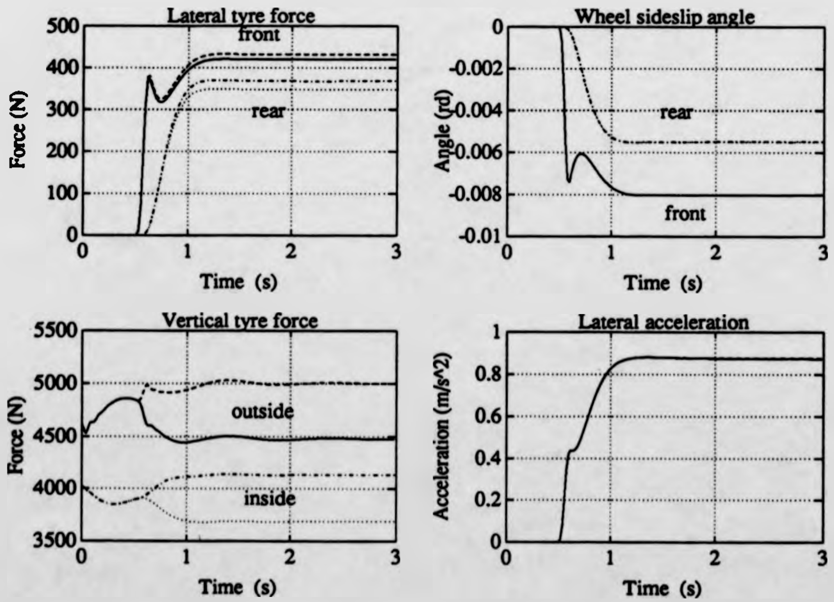


Figure C.24: Nonlinear, with, steering angle: 10°, speed: $20m/s$

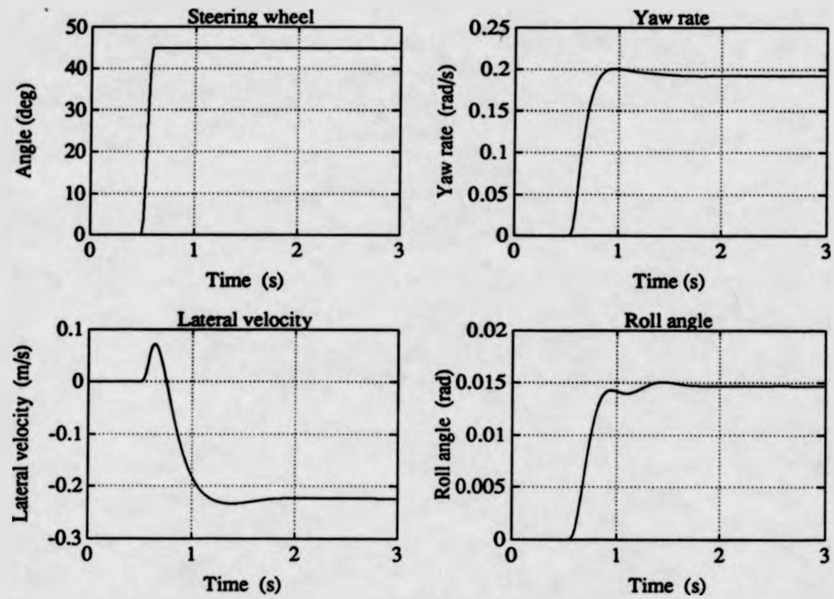


Figure C.25: Nonlinear, with, steering angle: 45°, speed: $20m/s$

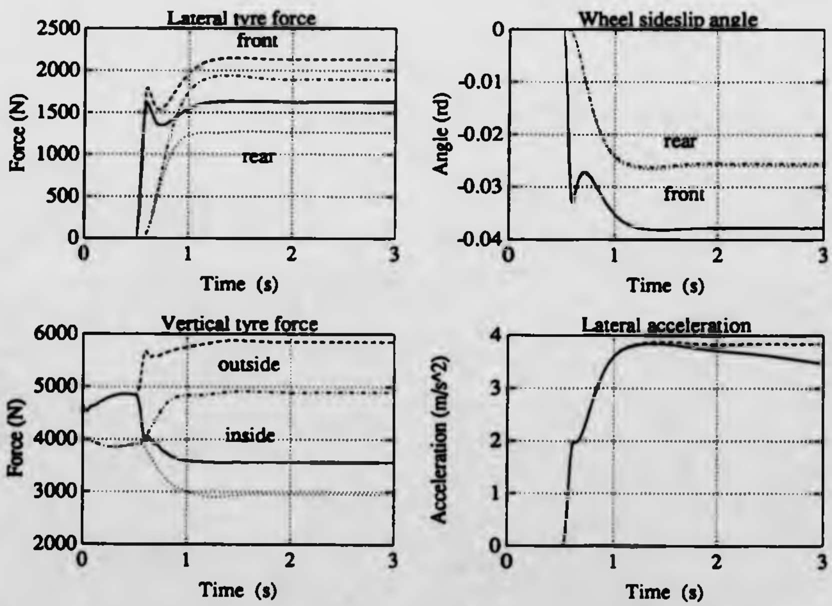


Figure C.26: Nonlinear, with, steering angle: 45°, speed: $20m/s$

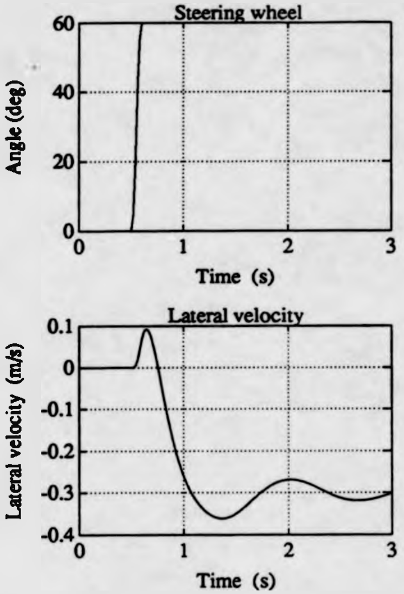


Figure C.27: Nonlinear, with, steering angle: 60°, speed: $20m/s$

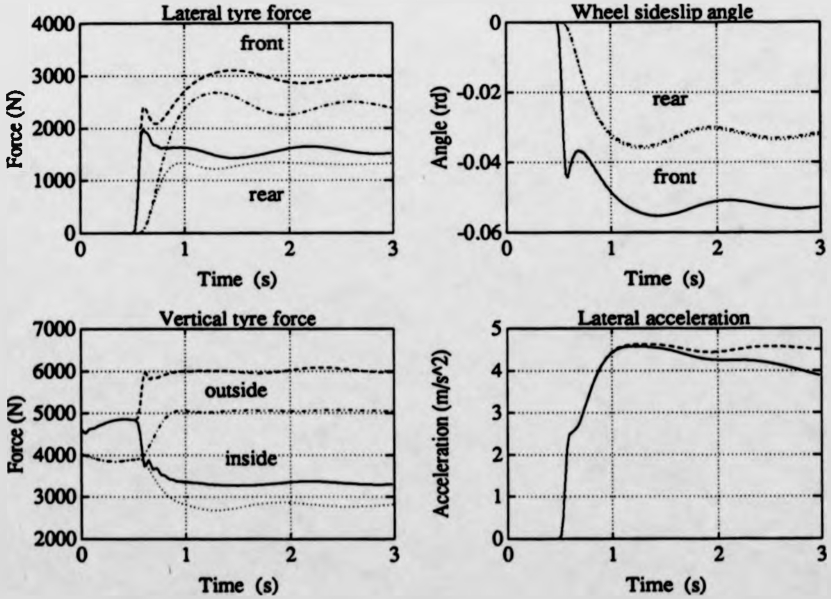


Figure C.28: Nonlinear, with, steering angle: 60°, speed: $20m/s$

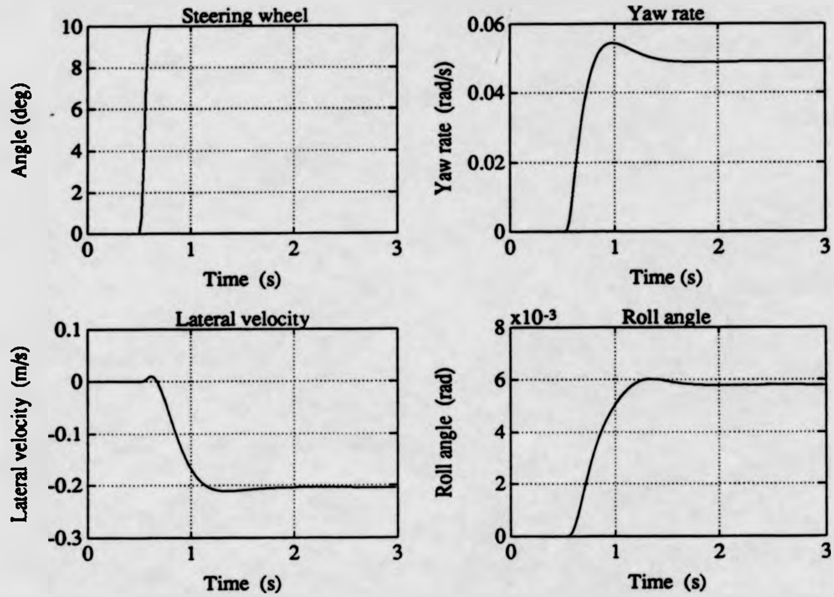


Figure C.29: Nonlinear, with, steering angle: 10°, speed: $30m/s$

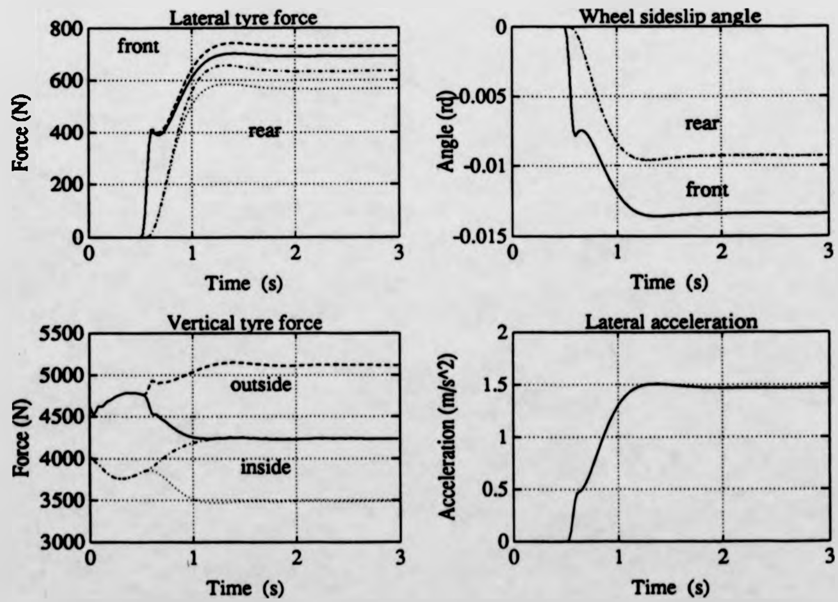


Figure C.30: Nonlinear, with, steering angle: 10°, speed: $30m/s$

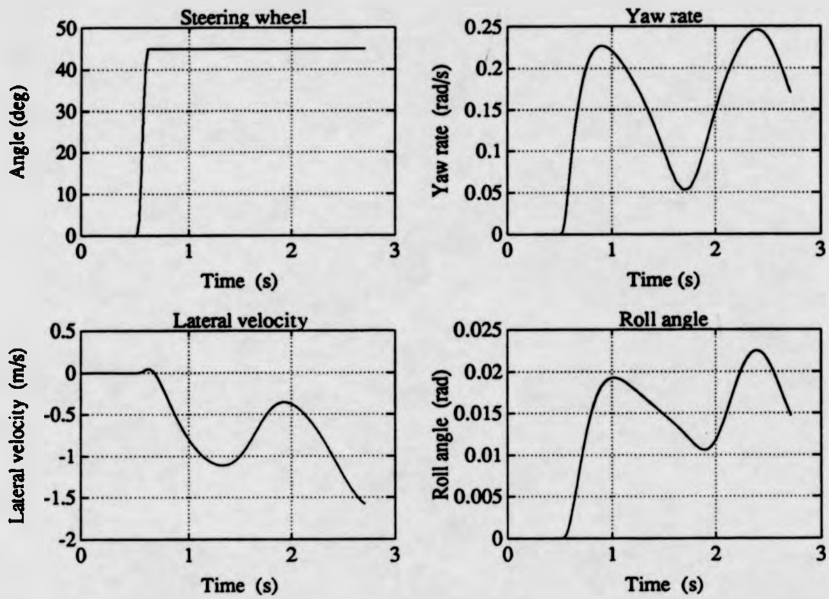


Figure C.31: Nonlinear, with, steering angle: 45°, speed: $30m/s$

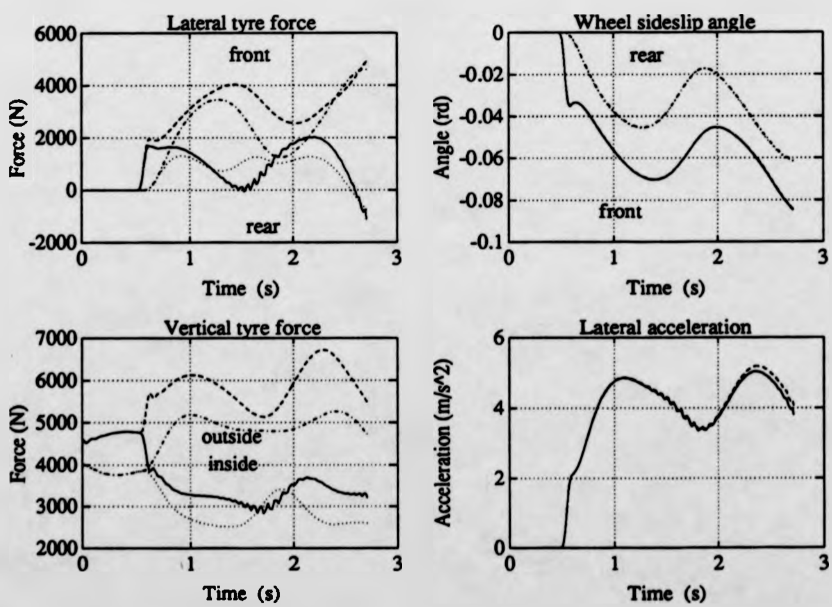


Figure C.32: Nonlinear, with, steering angle: 45°, speed: $30m/s$

C.1.2 Traction/Braking Input

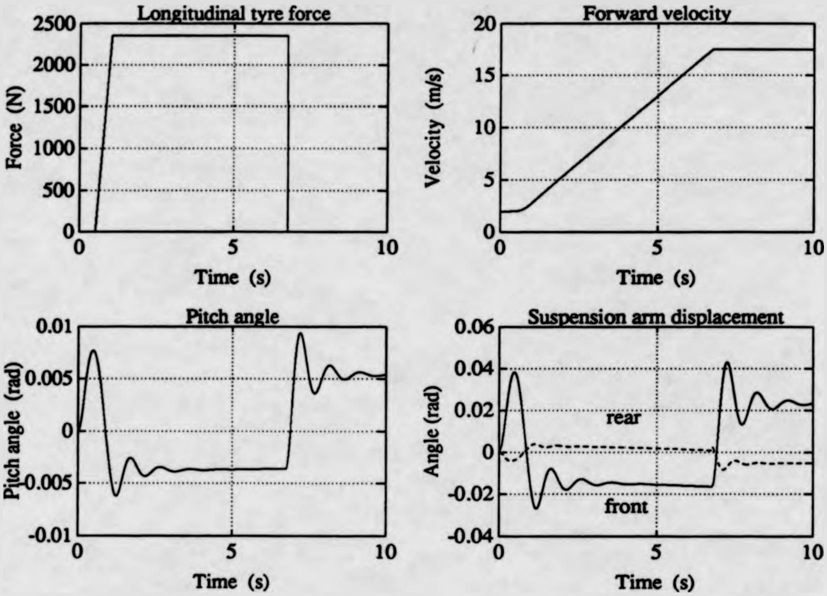


Figure C.33: Acceleration, 0.3g

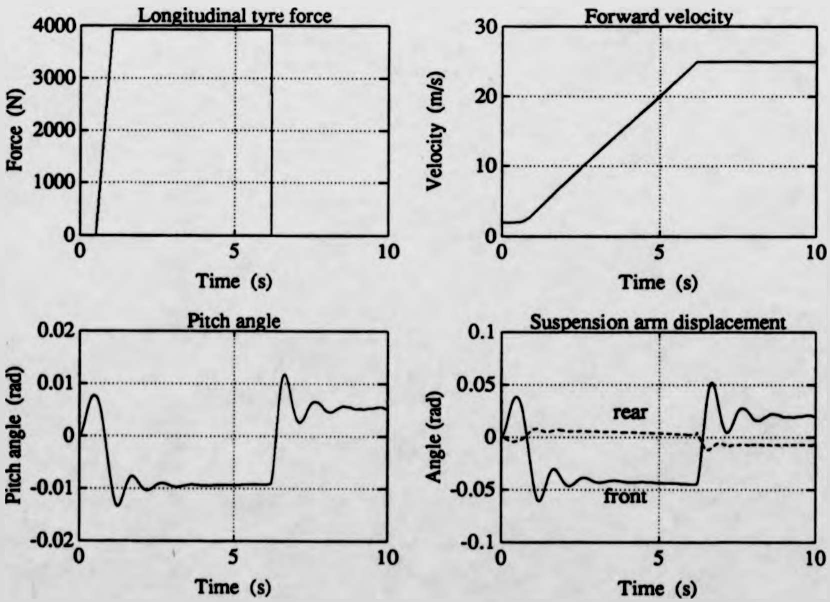


Figure C.34: Acceleration, 0.5g

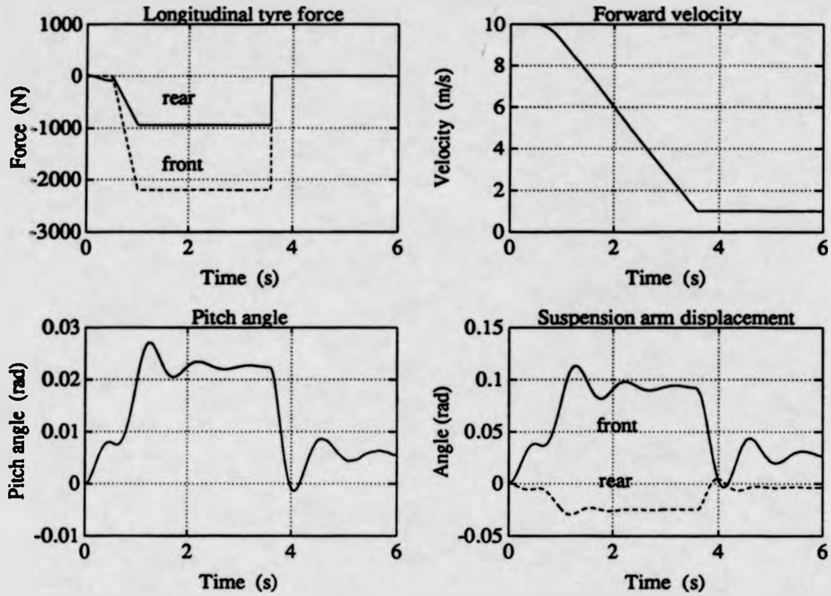


Figure C.35: Deceleration, 0.4g, 70/30 distribuition

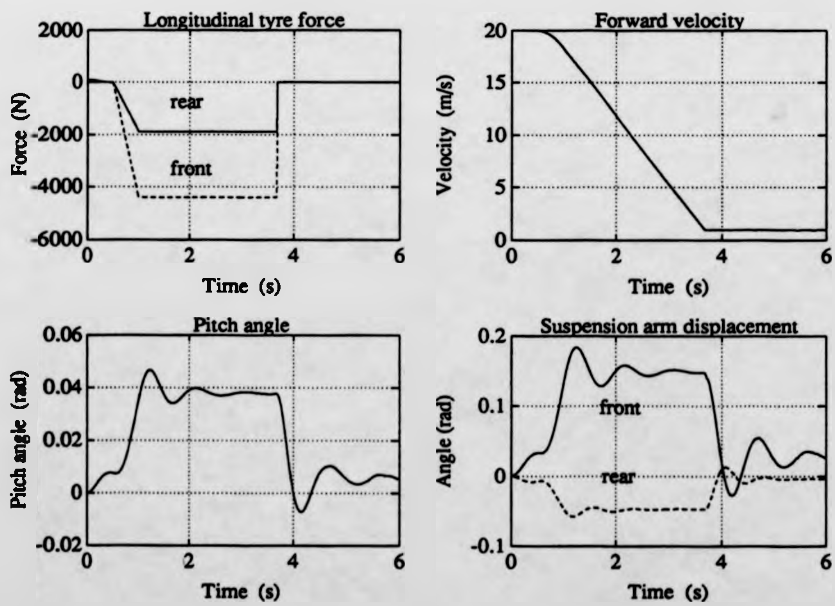


Figure C.36: Deceleration, 0.8g, 70/30 distribuition

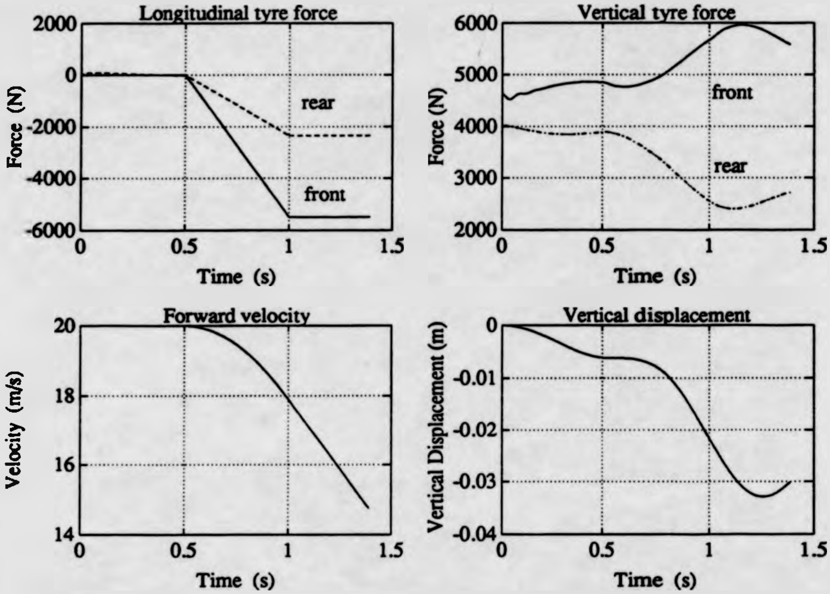


Figure C.37: Deceleration, $1.0g$, 70/30 distribuition, front lock-up

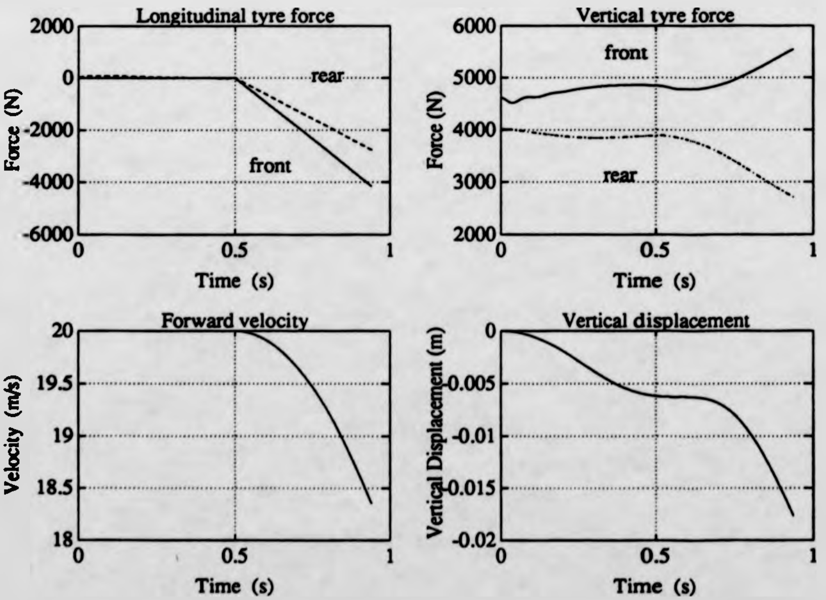


Figure C.38: Deceleration, $1.0g$, 60/40 distribuition, rear lock-up

C.1.3 Road Vertical Disturbances

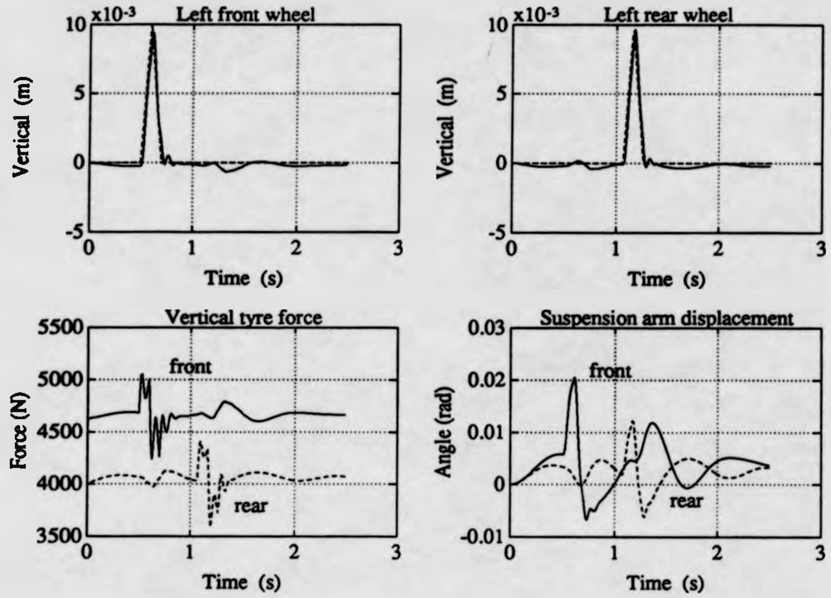


Figure C.39: Triangular bump, height: $0.01m$, speed: $5m/s$

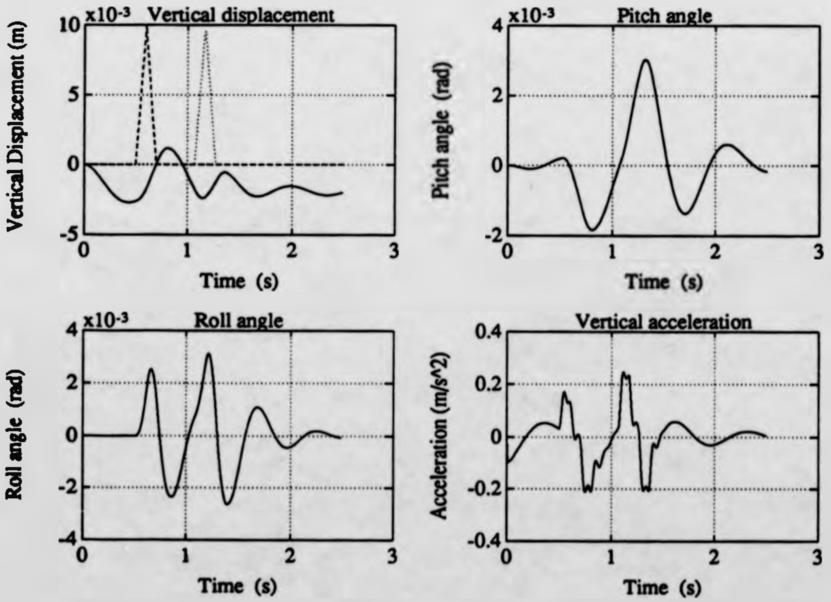


Figure C.40: Triangular bump, height: $0.01m$, speed: $5m/s$

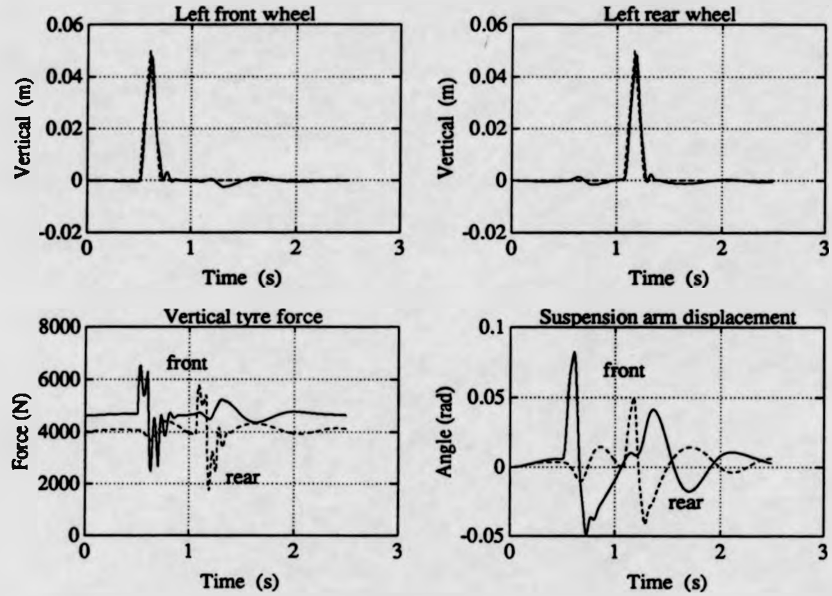


Figure C.41: Triangular bump, height: $0.05m$, speed: $5m/s$

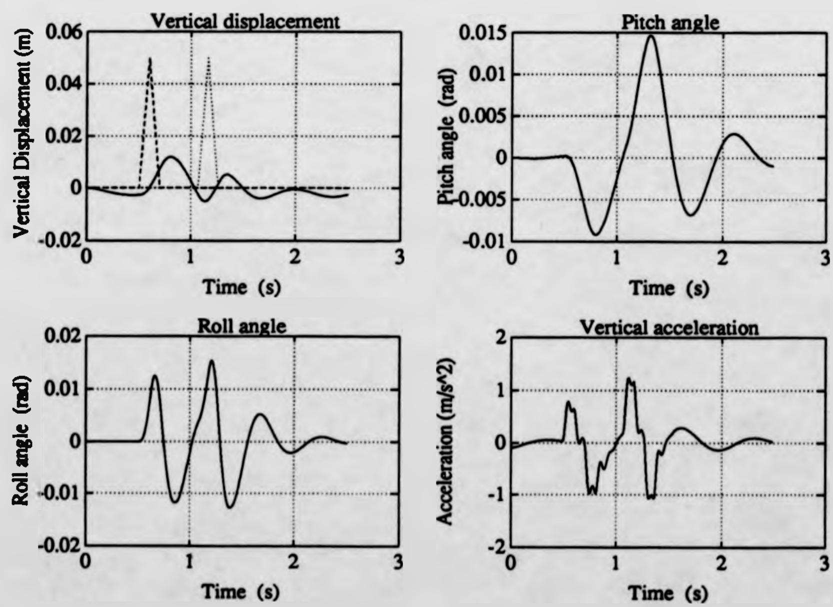


Figure C.42: Triangular bump, height: $0.05m$, speed: $5m/s$

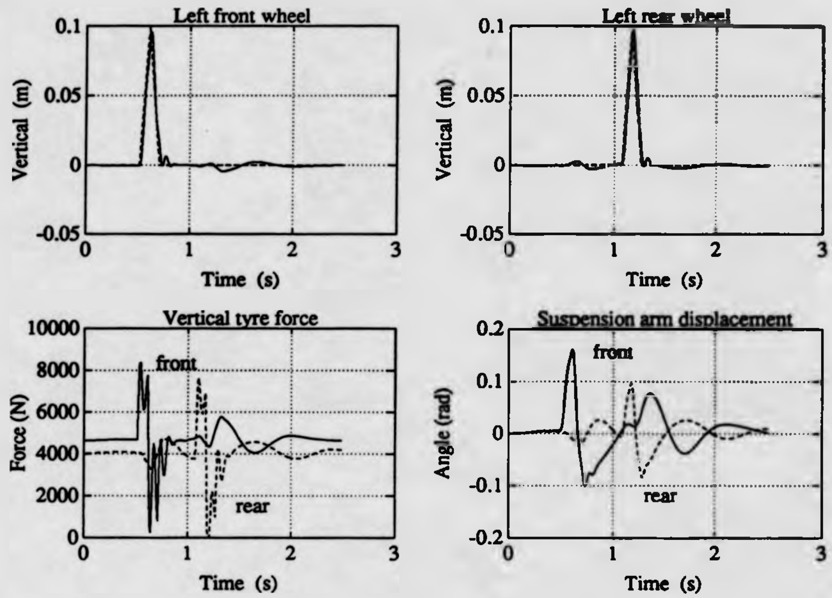


Figure C.43: Triangular bump, height: $0.1m$, speed: $5m/s$

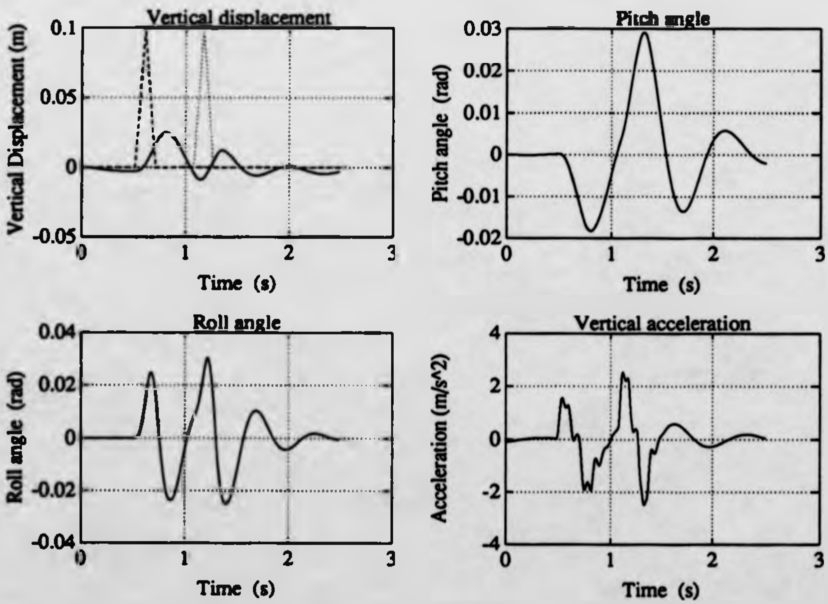


Figure C.44: Triangular bump, height: $0.1m$, speed: $5m/s$

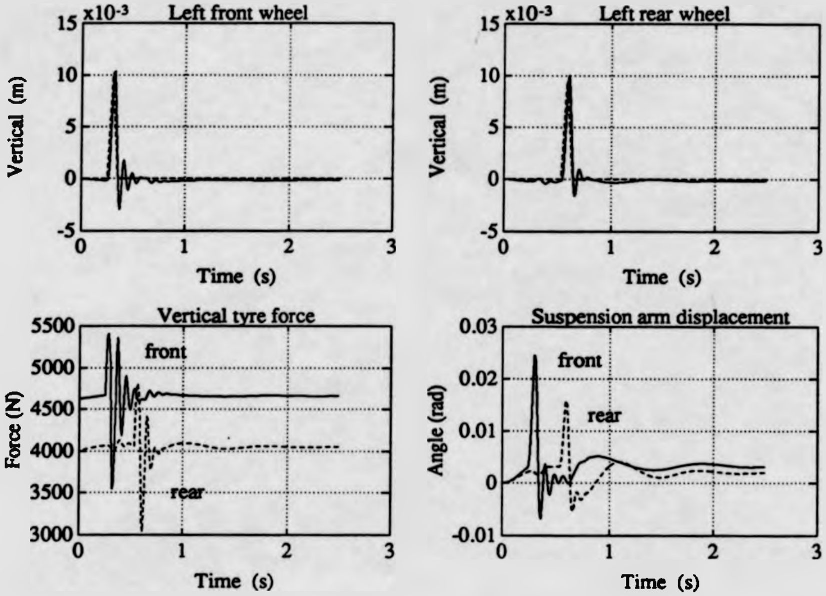


Figure C.45: Triangular bump, height: $0.01m$, speed: $10m/s$

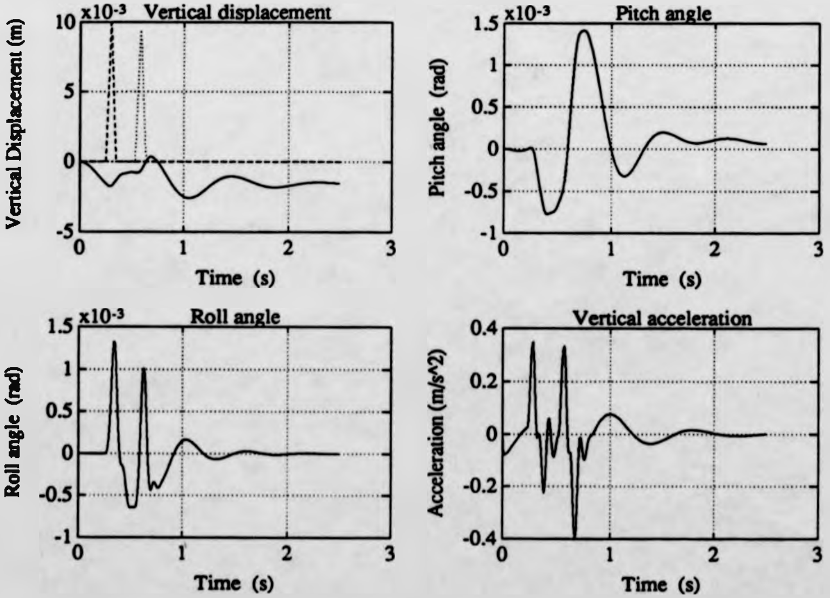


Figure C.46: Triangular bump, height: $0.01m$, speed: $10m/s$

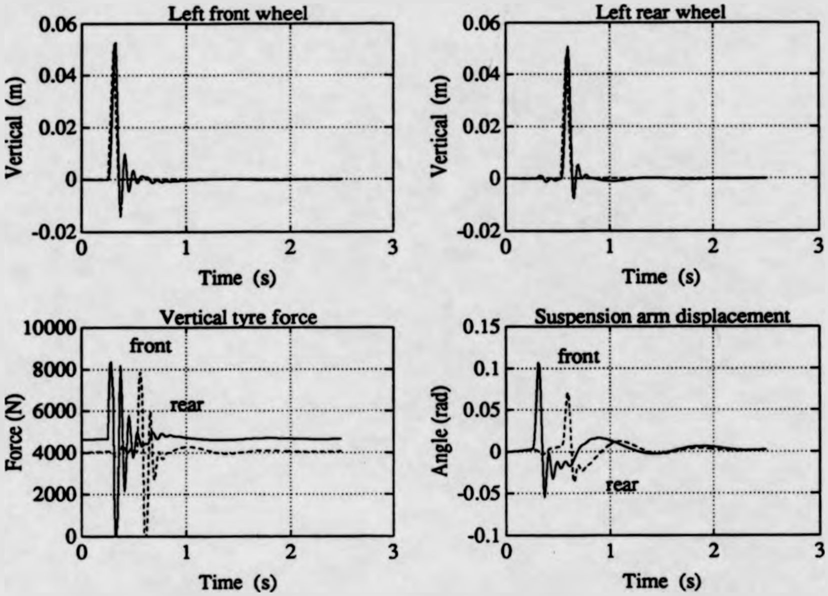


Figure C.47: Triangular bump, height: 0.05m, speed: 10m/s

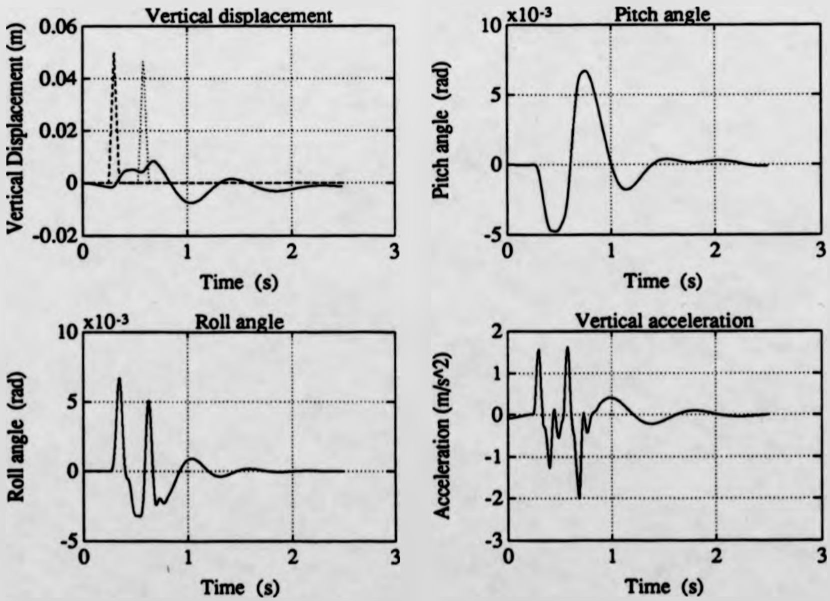


Figure C.48: Triangular bump, height: 0.05m, speed: 10m/s

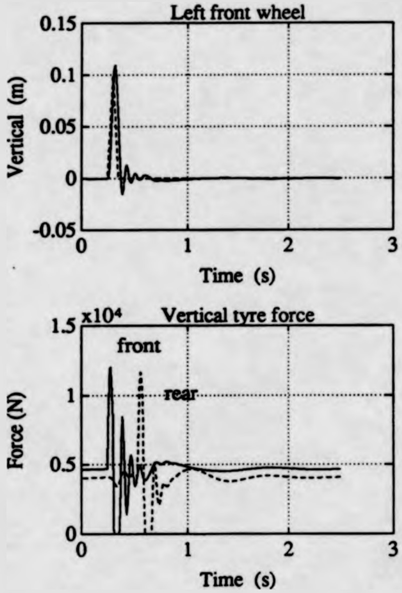


Figure C.49: Triangular bump, height: $0.1m$, speed: $10m/s$

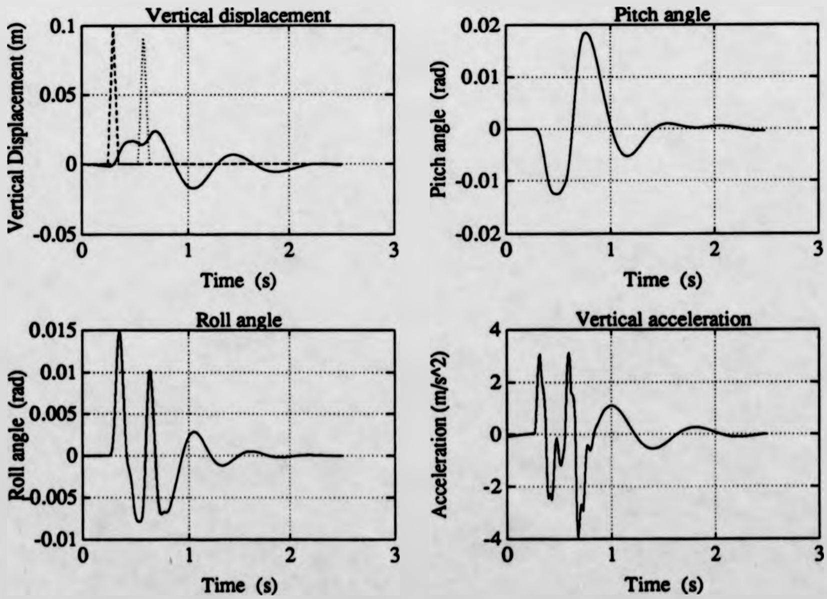


Figure C.50: Triangular bump, height: $0.1m$, speed: $10m/s$

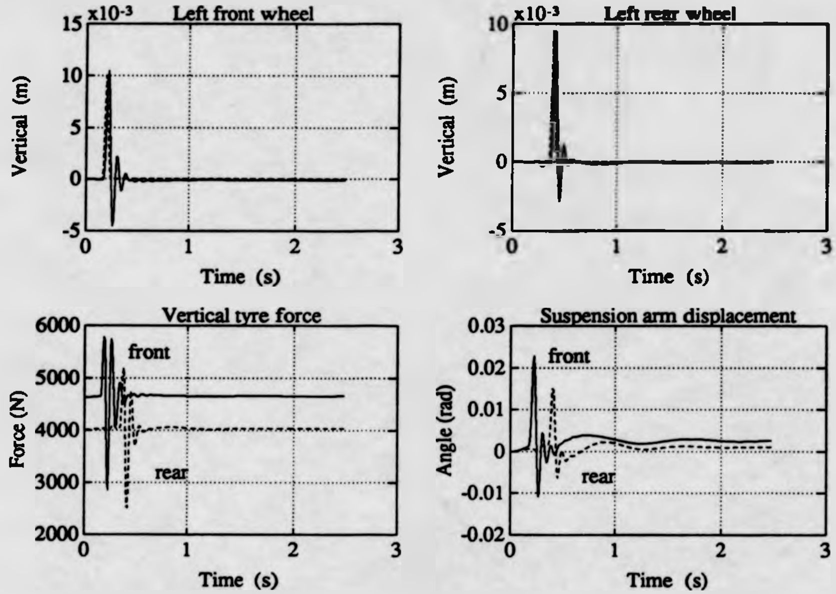


Figure C.51: Triangular bump, height: $0.01m$, speed: $15m/s$

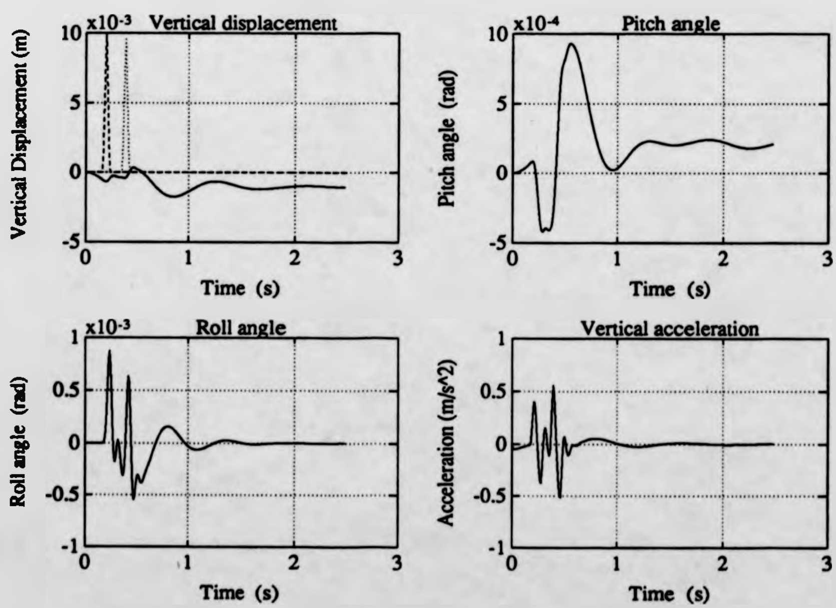


Figure C.52: Triangular bump, height: $0.01m$, speed: $15m/s$

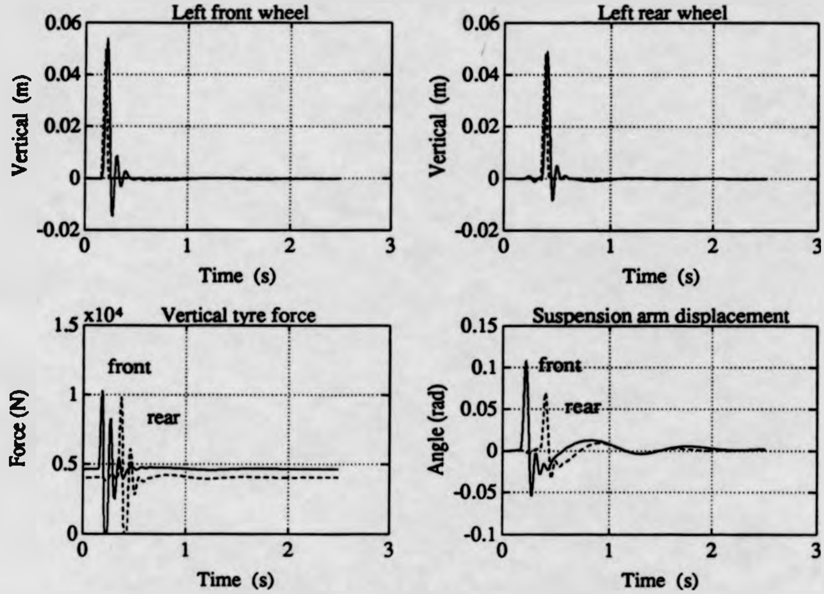


Figure C.53: Triangular bump, height: $0.05m$, speed: $15m/s$

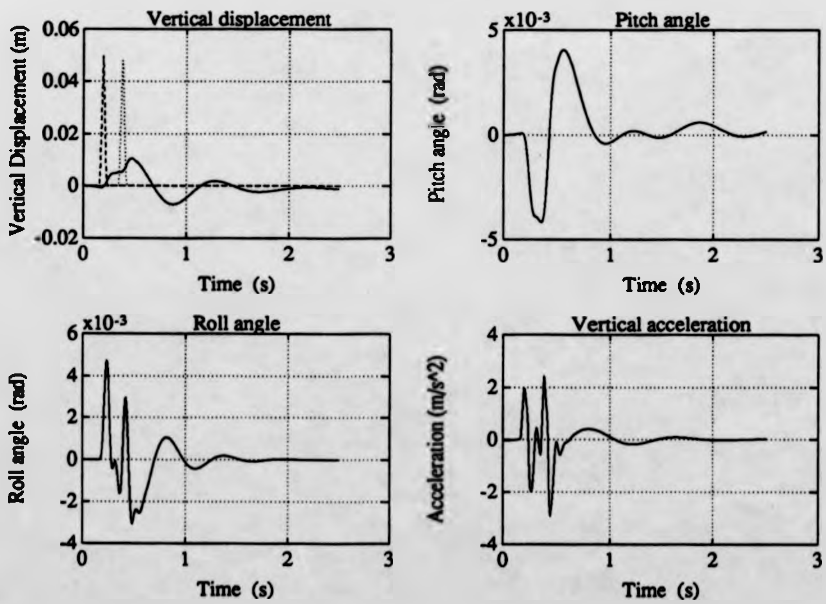


Figure C.54: Triangular bump, height: $0.05m$, speed: $15m/s$

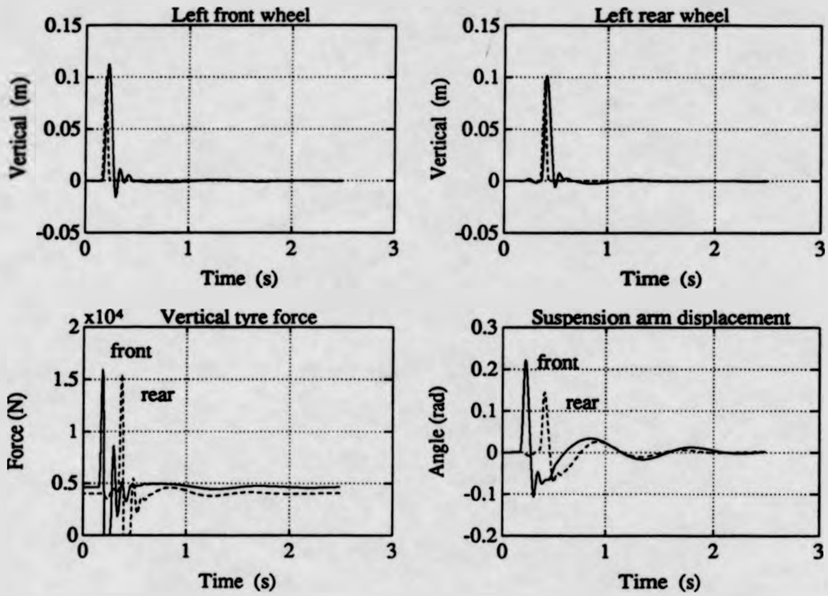


Figure C.55: Triangular bump, height: $0.1m$, speed: $15m/s$

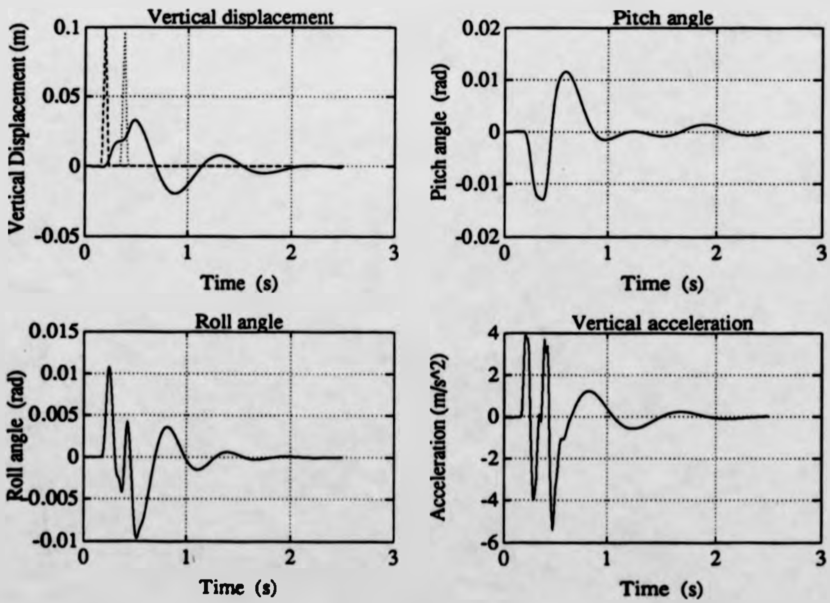


Figure C.56: Triangular bump, height: $0.1m$, speed: $15m/s$

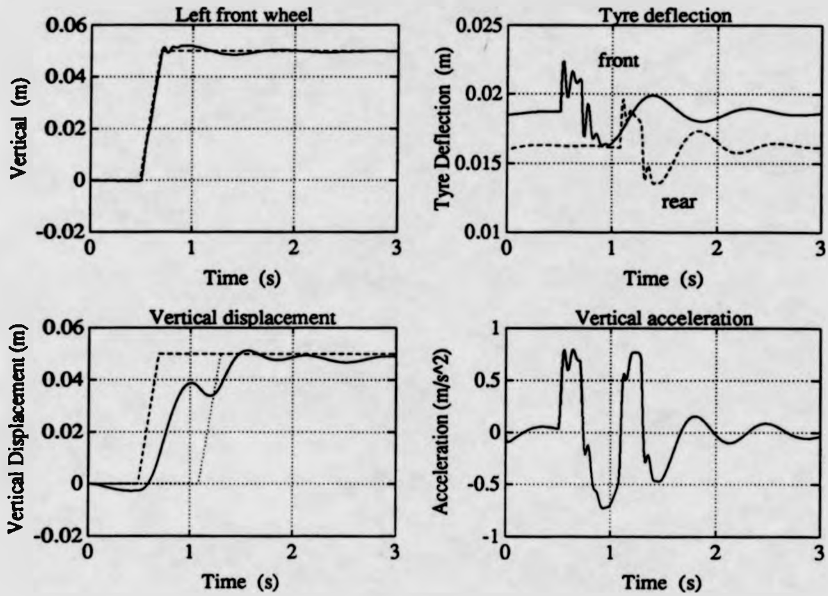


Figure C.57: Terminated ramp, height: $0.05m$, speed: $5m/s$

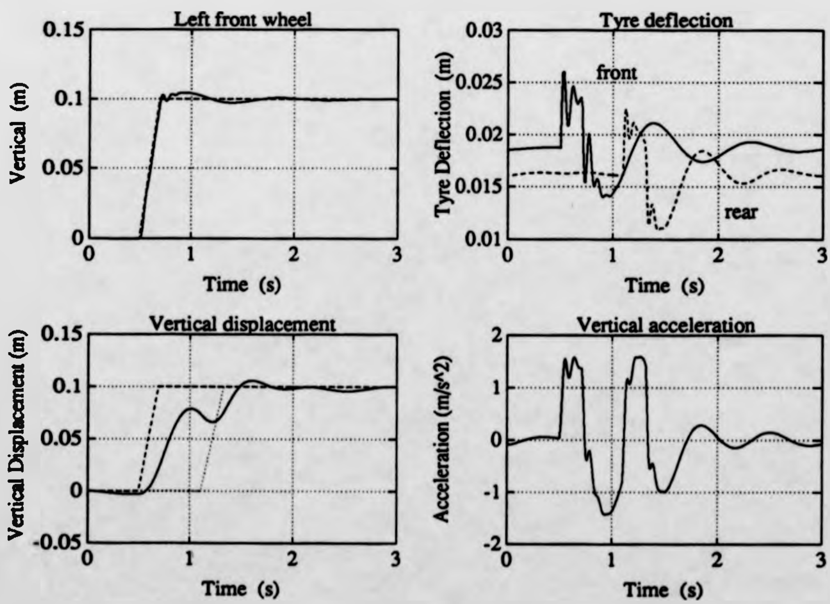


Figure C.58: Terminated ramp, height: $0.1m$, speed: $5m/s$

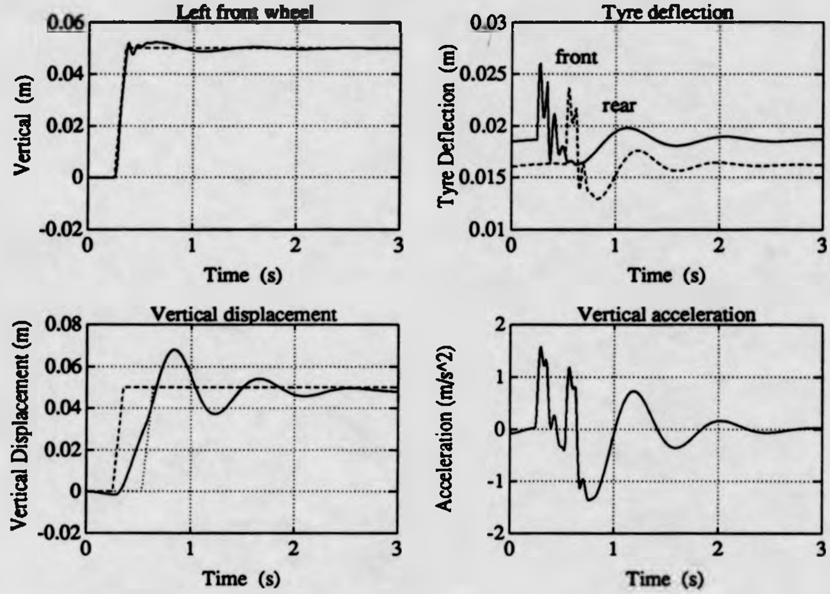


Figure C.59: Terminated ramp, height: $0.05m$, speed: $10m/s$

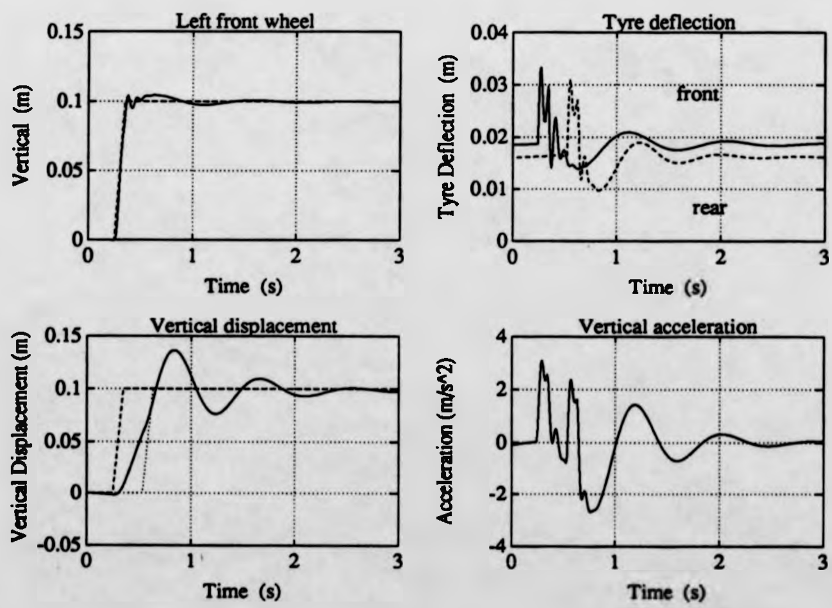


Figure C.60: Terminated ramp, height: $0.1m$, speed: $10m/s$

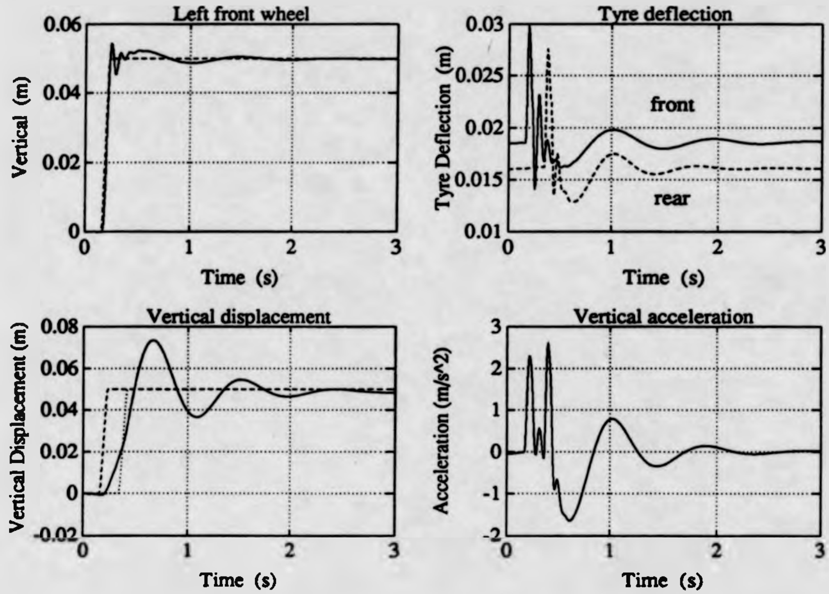


Figure C.61: Terminated ramp, height: $0.05m$, speed: $15m/s$

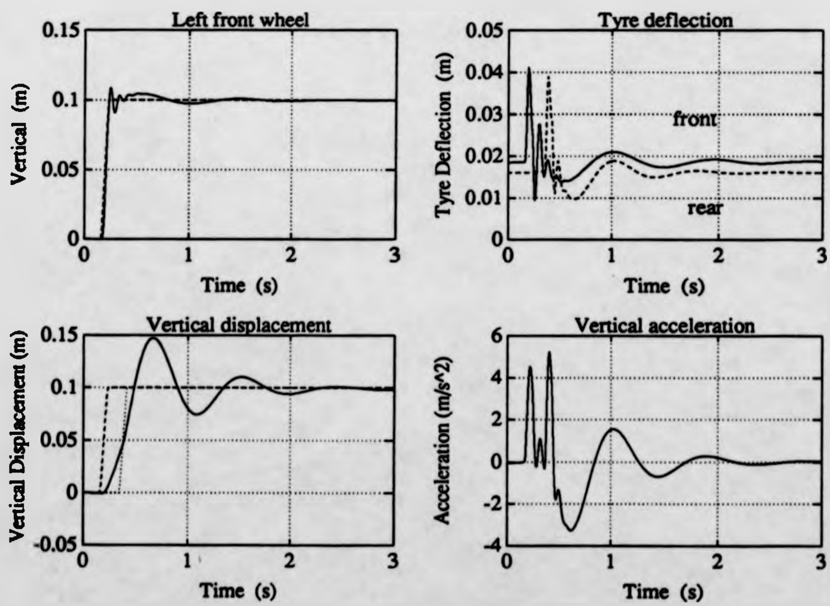


Figure C.62: Terminated ramp, height: $0.1m$, speed: $15m/s$

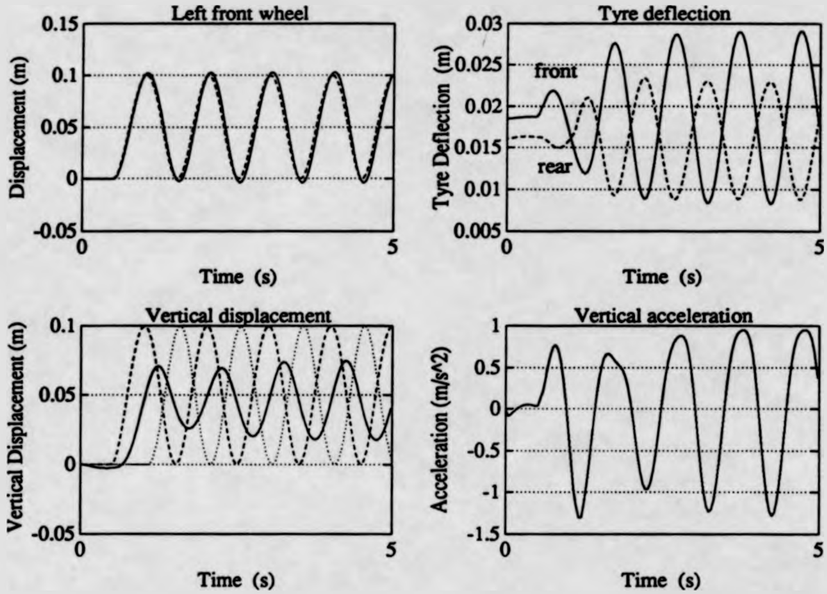


Figure C.63: Sinusoidal profile, amplitude: $0.05m$, speed: $5m/s$, length: $5m$

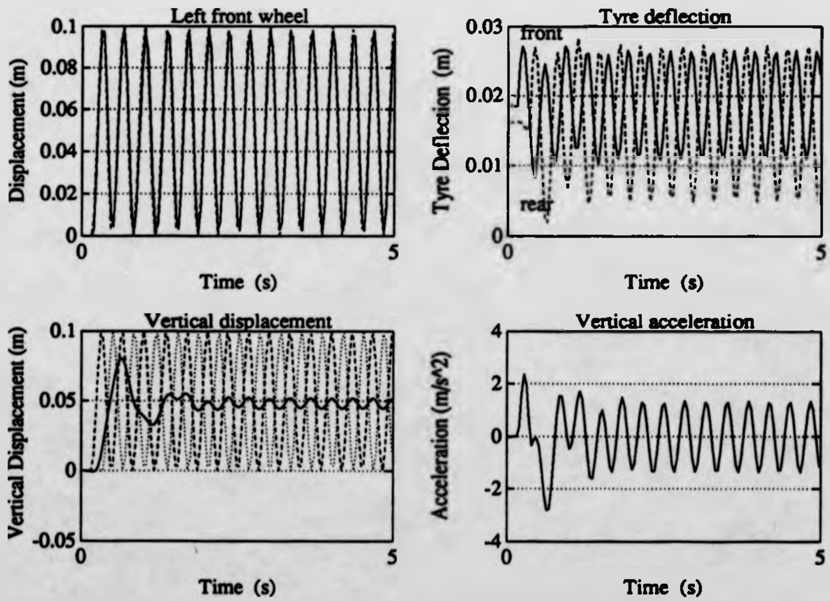


Figure C.64: Sinusoidal profile, amplitude: $0.05m$, speed: $15m/s$, length: $5m$

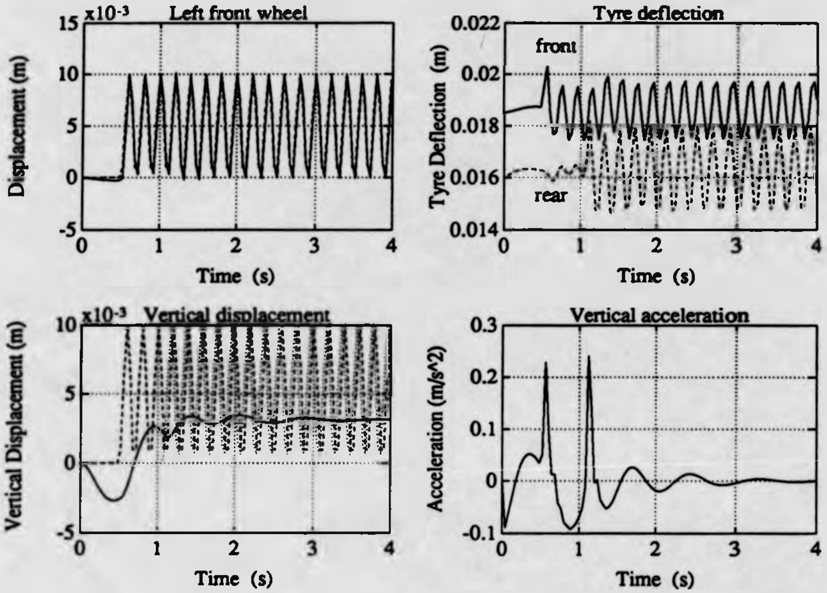


Figure C.65: Sinusoidal profile, amplitude: $0.005m$, speed: $5m/s$, length: $1m$

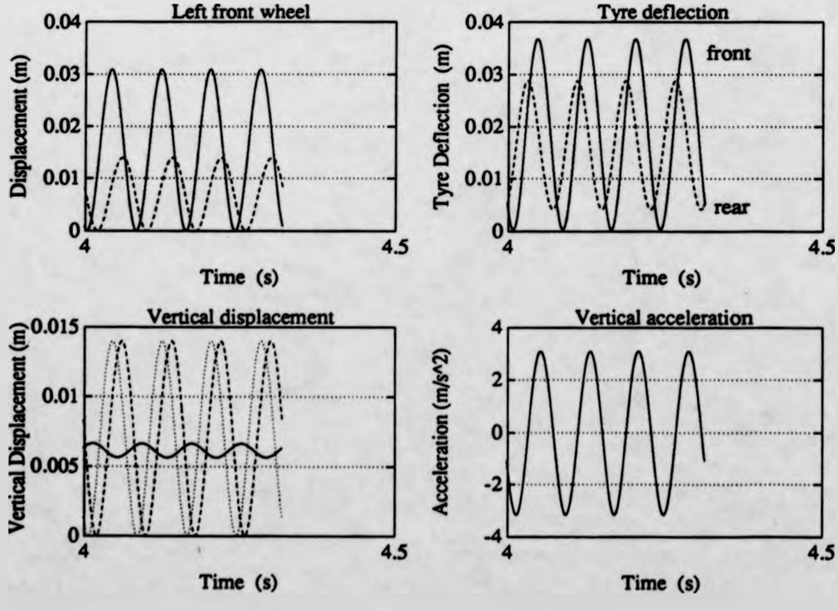


Figure C.66: Sinusoidal profile, amplitude: $0.007m$, speed: $12.5m/s$, length: $1m$

C.1.4 Aerodynamic Disturbances

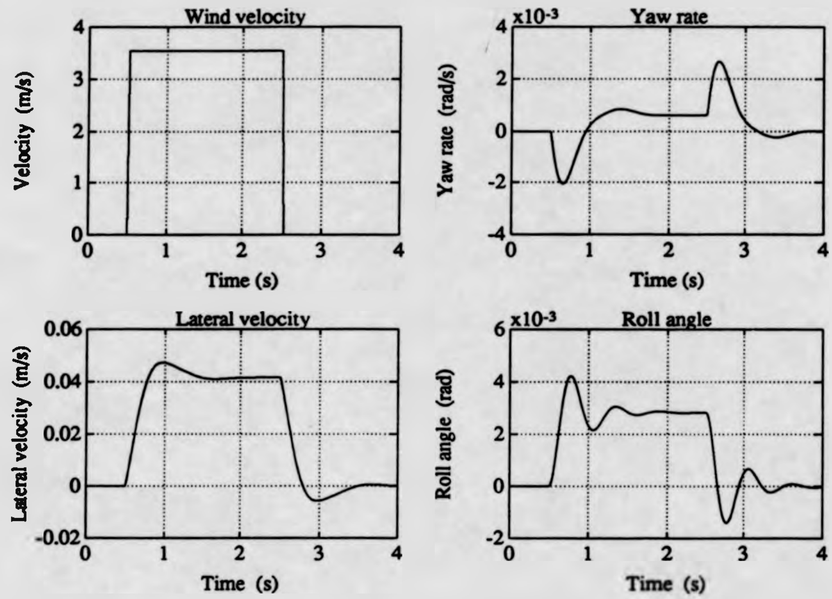


Figure C.67: Square pulse lateral wind, velocity: $3.54m/s$

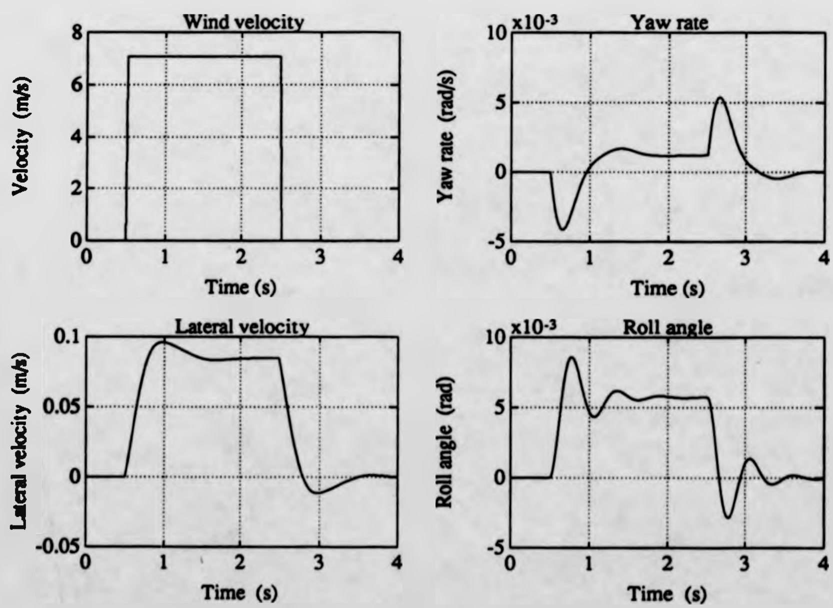


Figure C.68: Square pulse lateral wind, velocity: $7.08m/s$

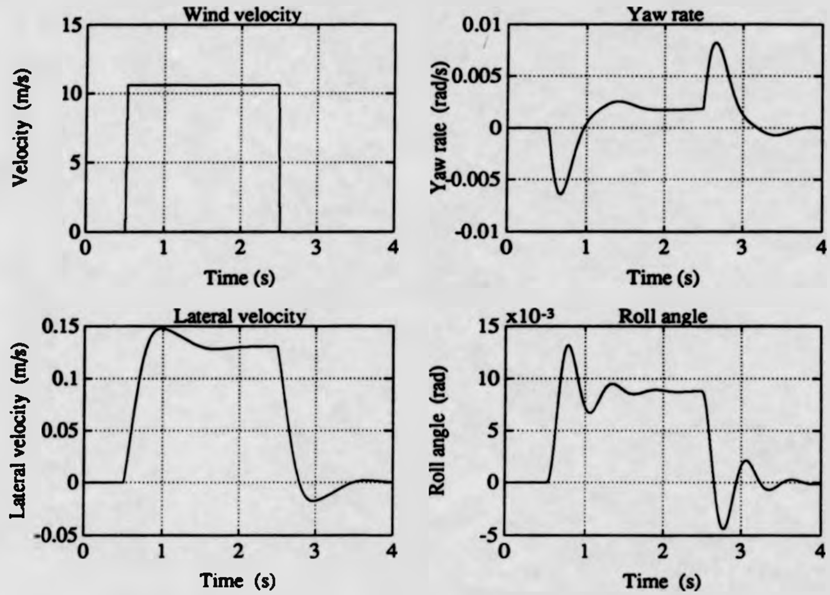


Figure C.69: Square pulse lateral wind, velocity: $10.62m/s$

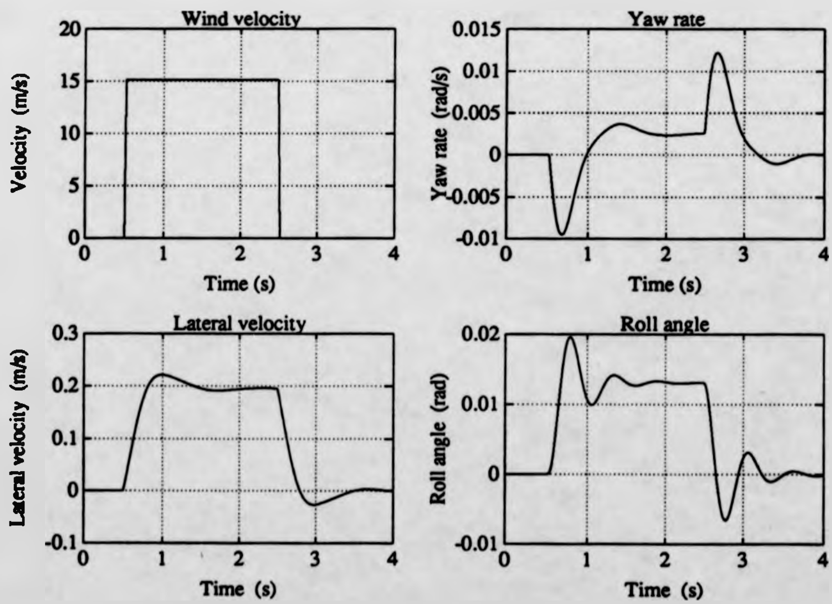


Figure C.70: Square pulse lateral wind, velocity: $15.16m/s$

C.2 Steady-State Manoeuvres

C.2.1 Cornering at Different speeds

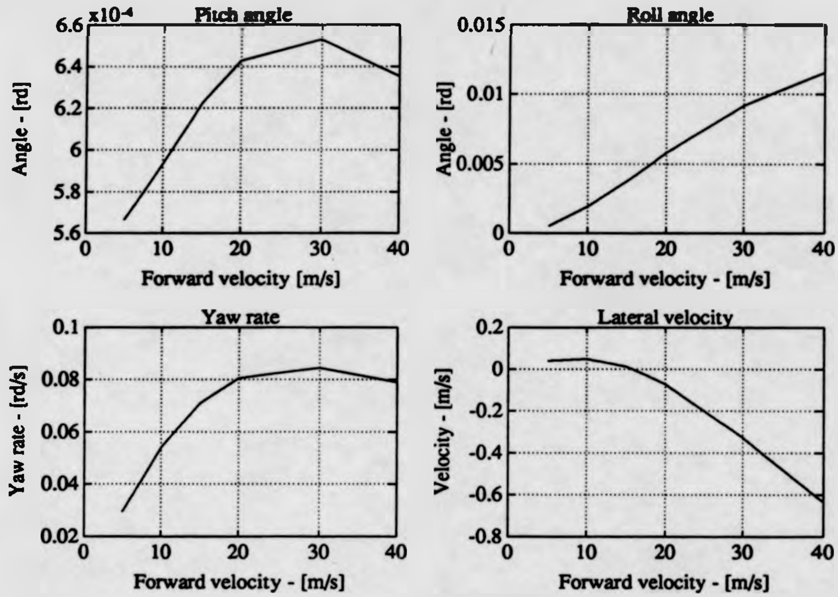


Figure C.71: Linear, without, steering: 20°

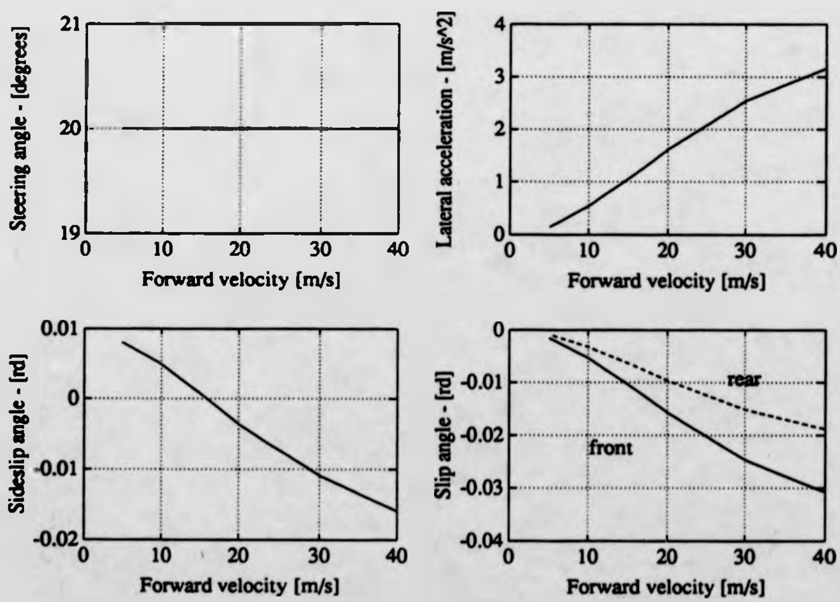


Figure C.72: Linear, without, steering: 20°

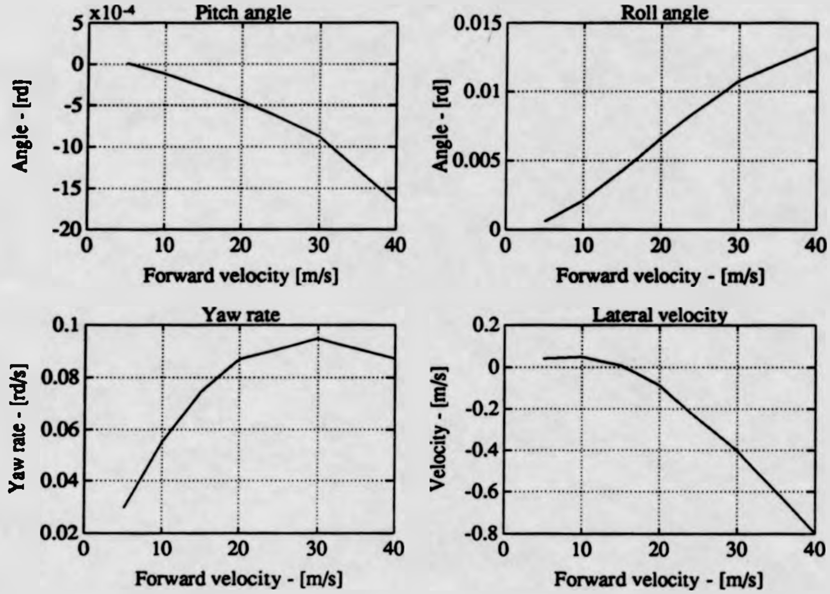


Figure C.73: Nonlinear, without, steering: 20°

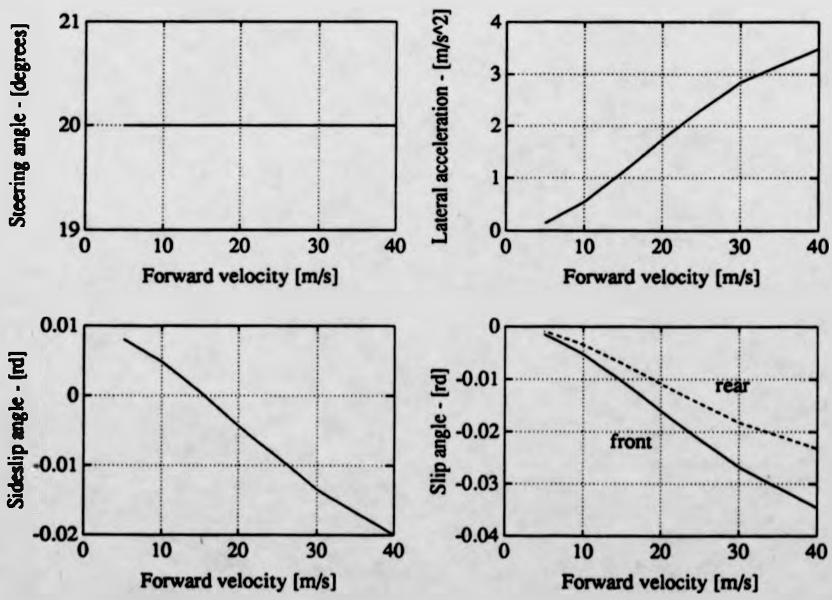


Figure C.74: Nonlinear, without, steering: 20°

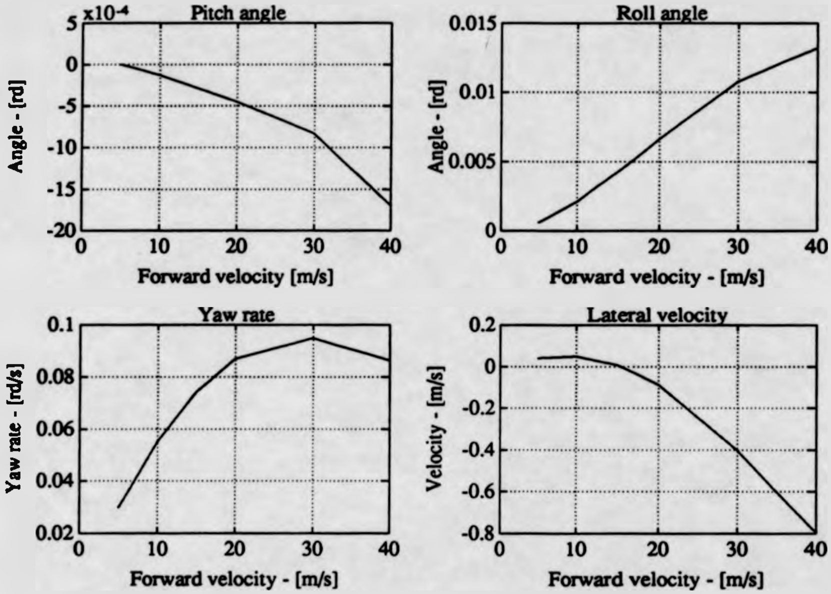


Figure C.75: Nonlinear, with, steering: 20°

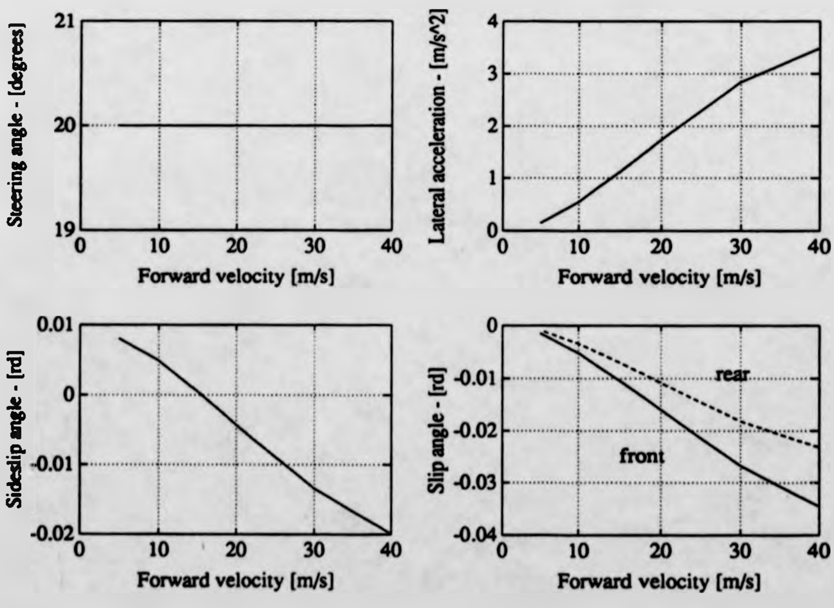


Figure C.76: Nonlinear, with, steering: 20°

C.2.2 Cornering with Different Steering Angles

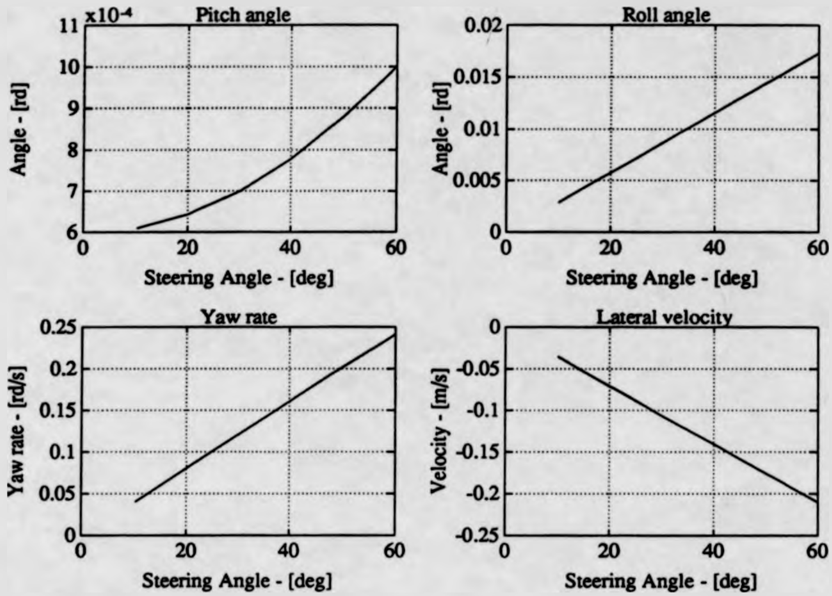


Figure C.77: Linear, without, speed: $20m/s$

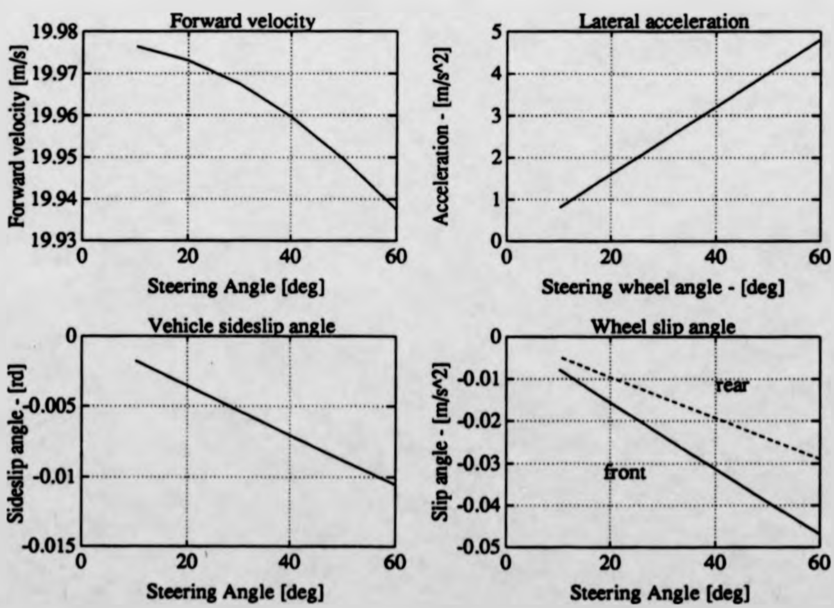


Figure C.78: Linear, without, speed: $20m/s$

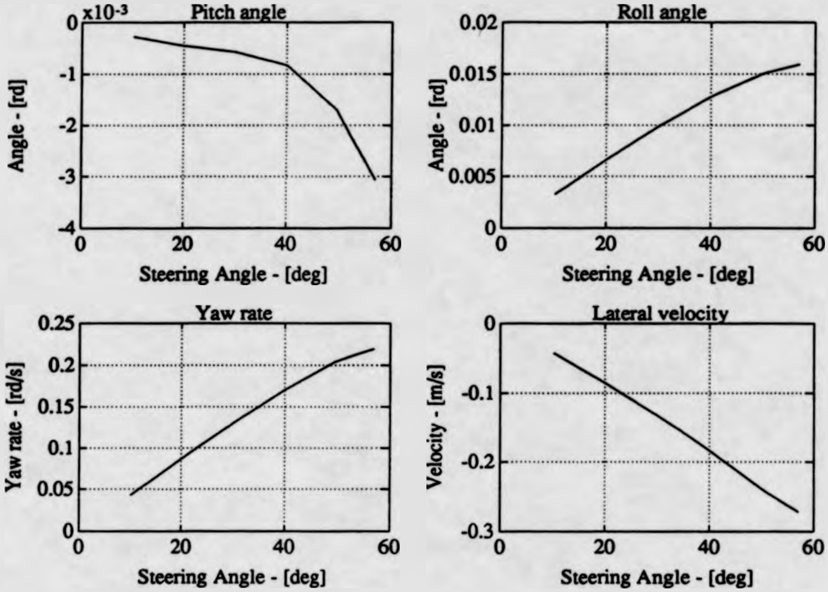


Figure C.79: Nonlinear, without, speed: $20m/s$

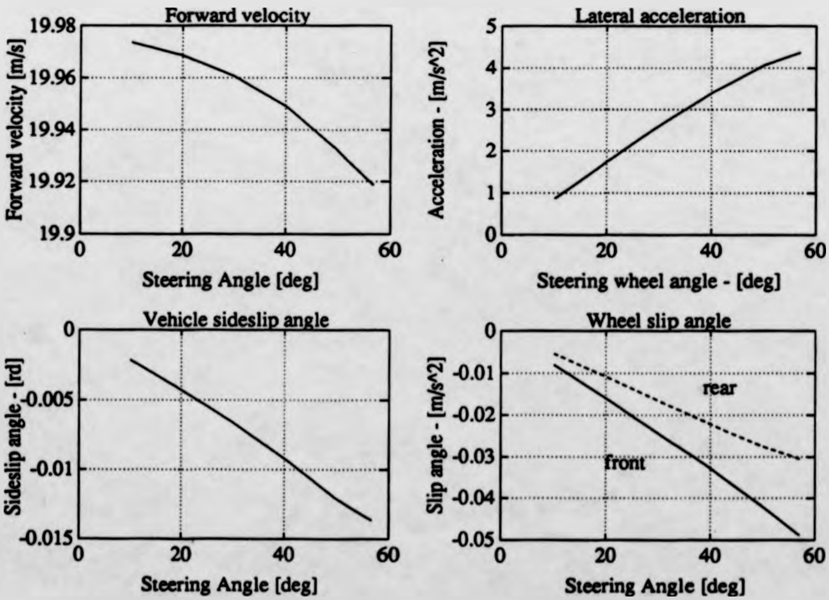


Figure C.80: Nonlinear, without, speed: $20m/s$

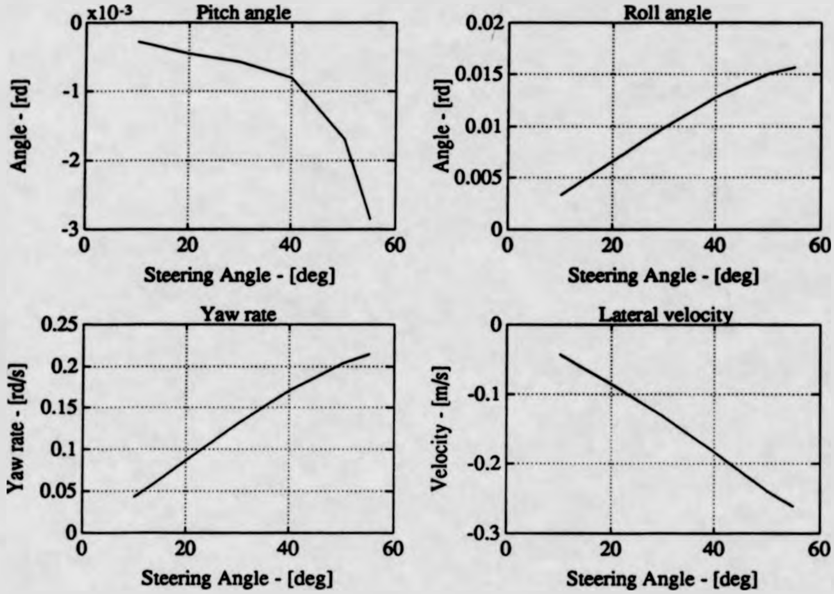


Figure C.81: Nonlinear, with, speed: $20m/s$

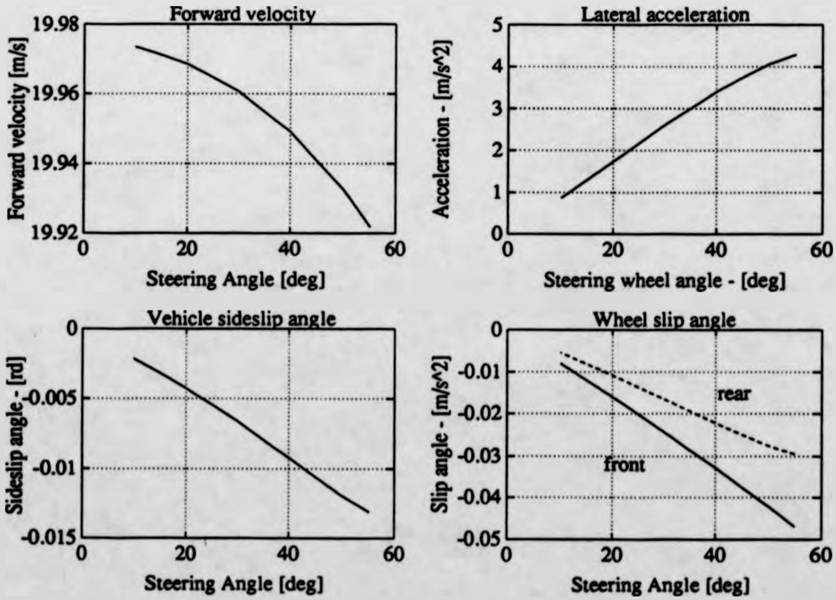


Figure C.82: Nonlinear, with, speed: $20m/s$

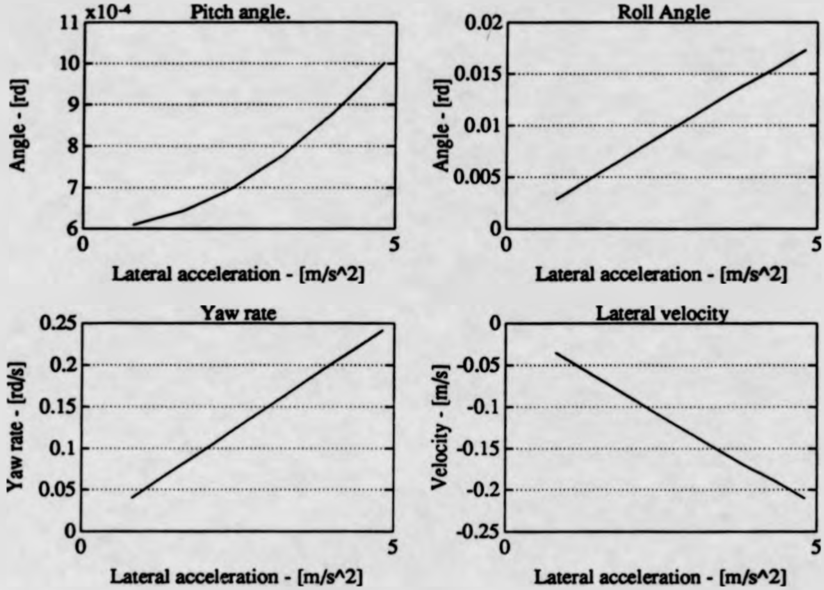


Figure C.83: Linear, without, speed: $20 m/s$

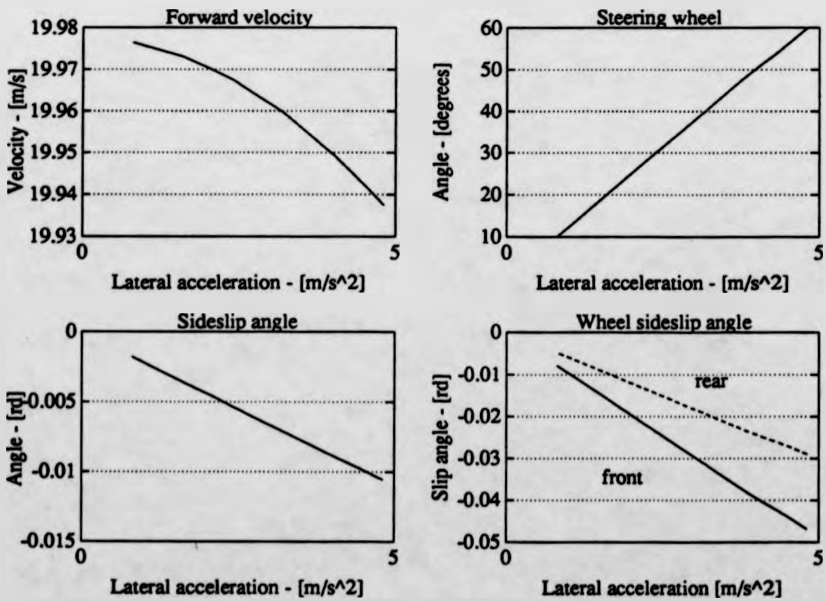


Figure C.84: Linear, without, speed: $20 m/s$

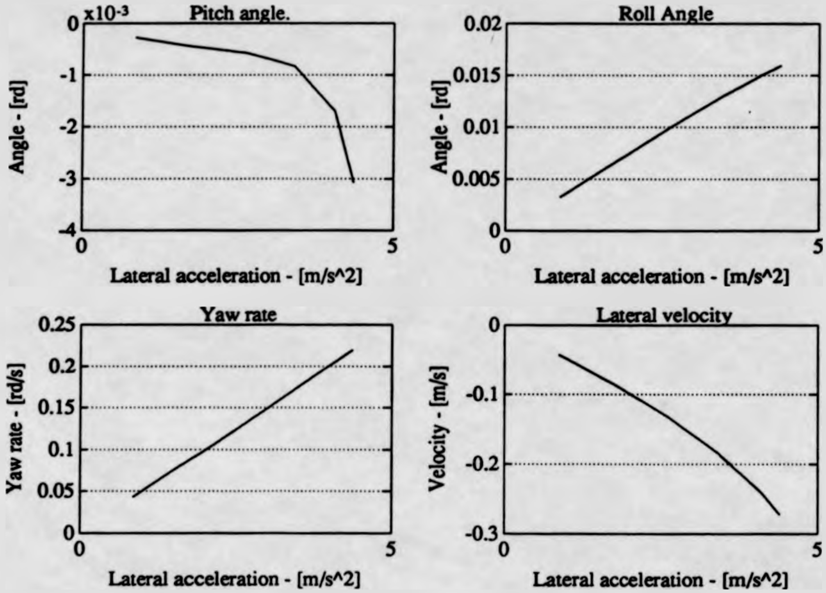


Figure C.85: Nonlinear, without, speed: $20m/s$

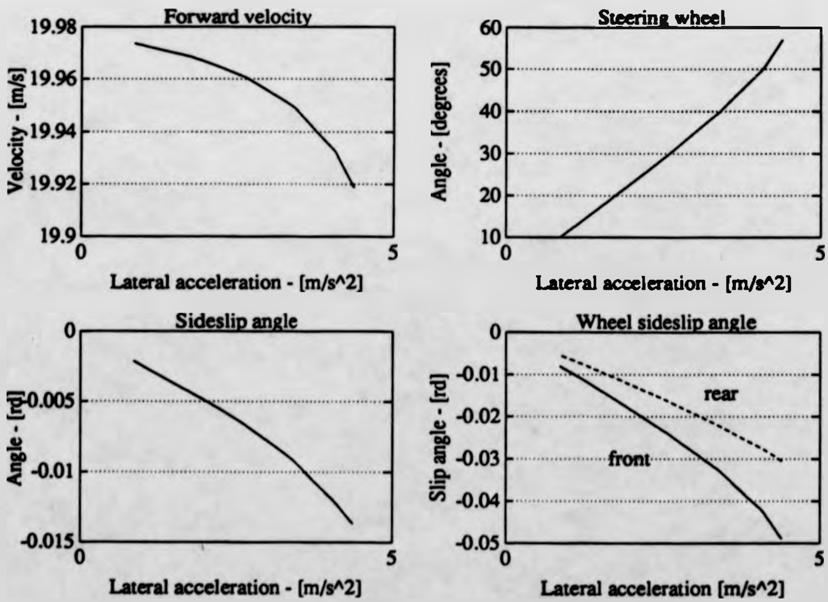


Figure C.86: Nonlinear, without, speed: $20m/s$

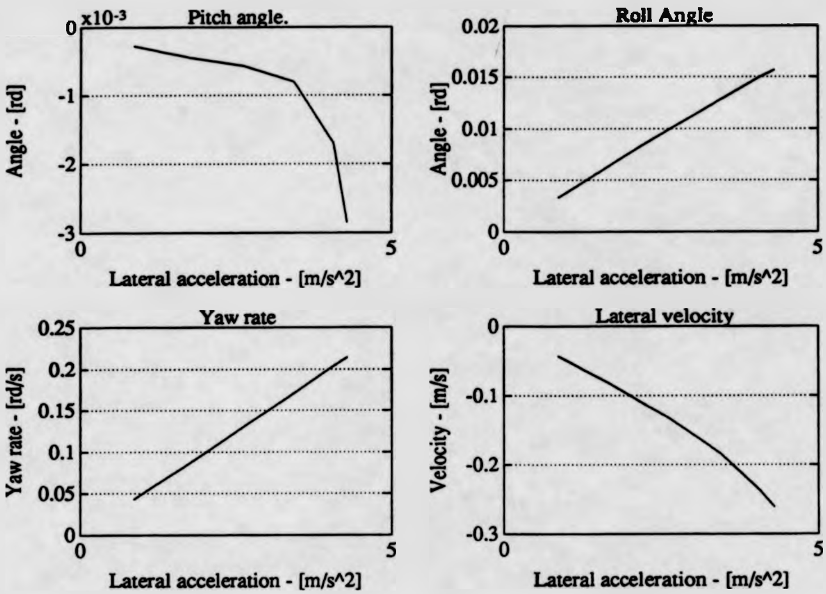


Figure C.87: Nonlinear, with, speed: $20m/s$

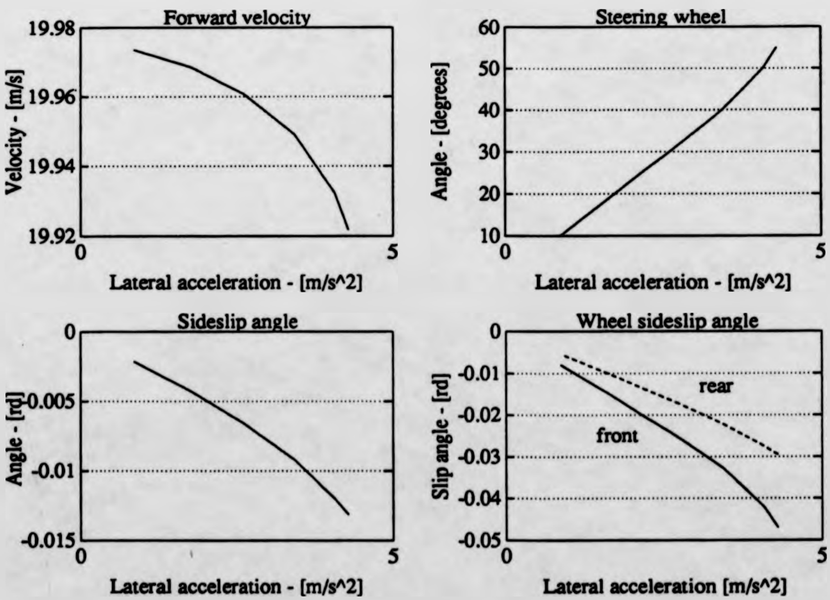


Figure C.88: Nonlinear, with, speed: $20m/s$

C.2.3 Crossing Obstacles of Different Heights

C.2.3.1 Left/Right In-phase Obstacles

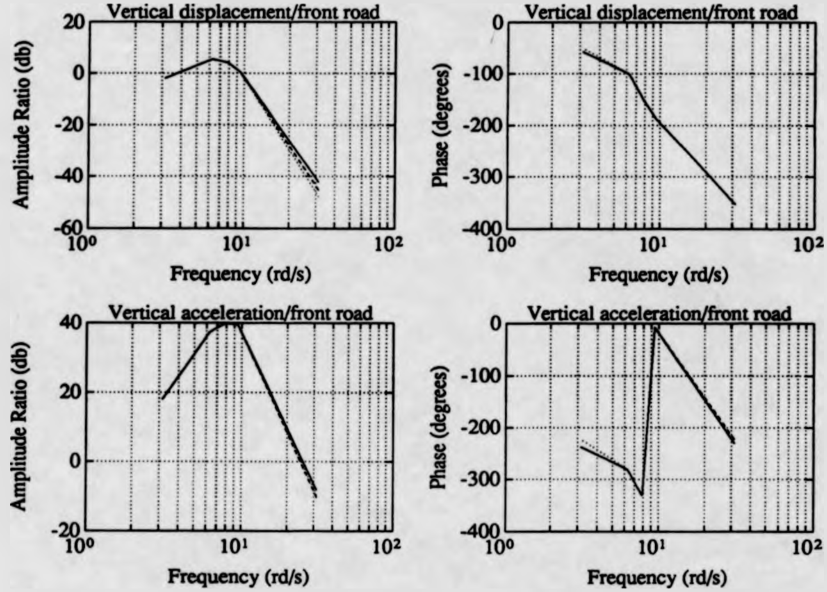


Figure C.89: In-phase, 0.5 - 5.0Hz, amplitudes: 0.005, 0.01 and 0.025m

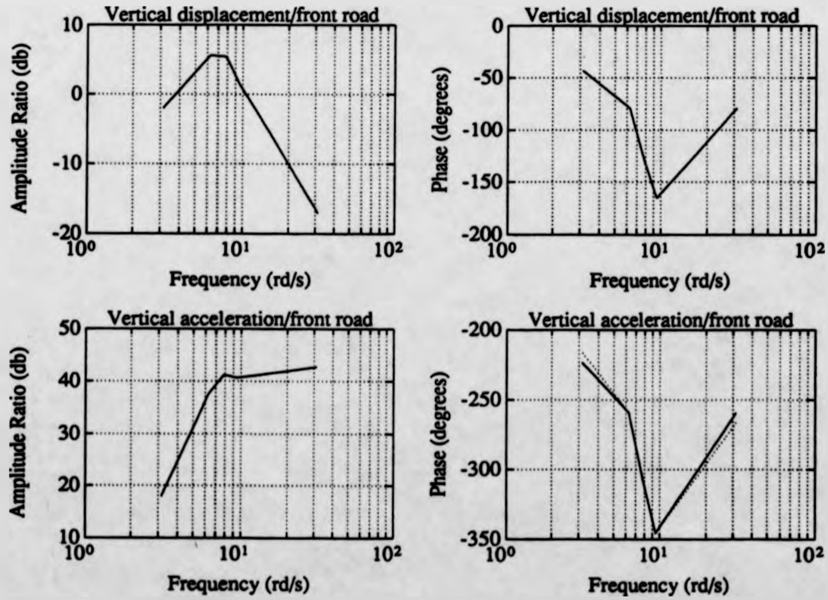


Figure C.90: In-phase, 0.5 - 30.0Hz, amplitude: 0.005m

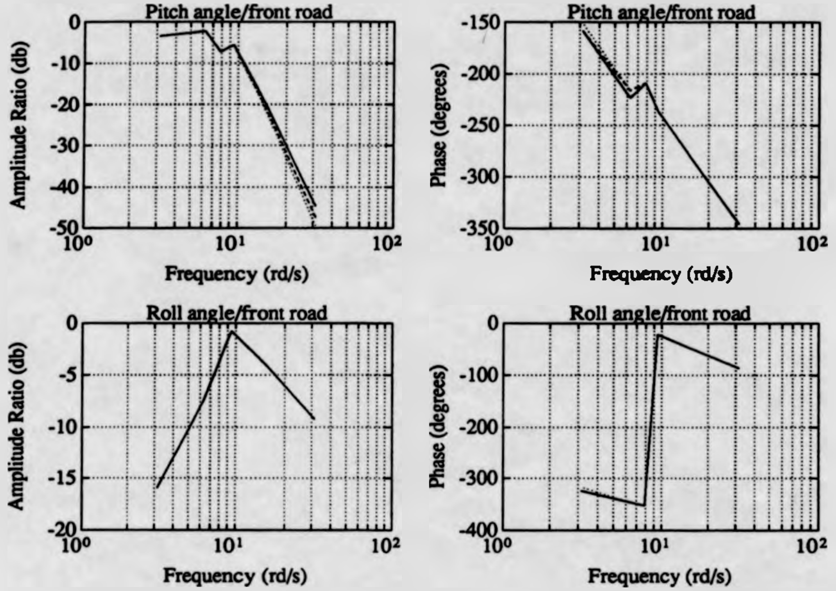


Figure C.91: In-phase, 0.5 - 5.0Hz, amplitudes: 0.005, 0.01 and 0.025m

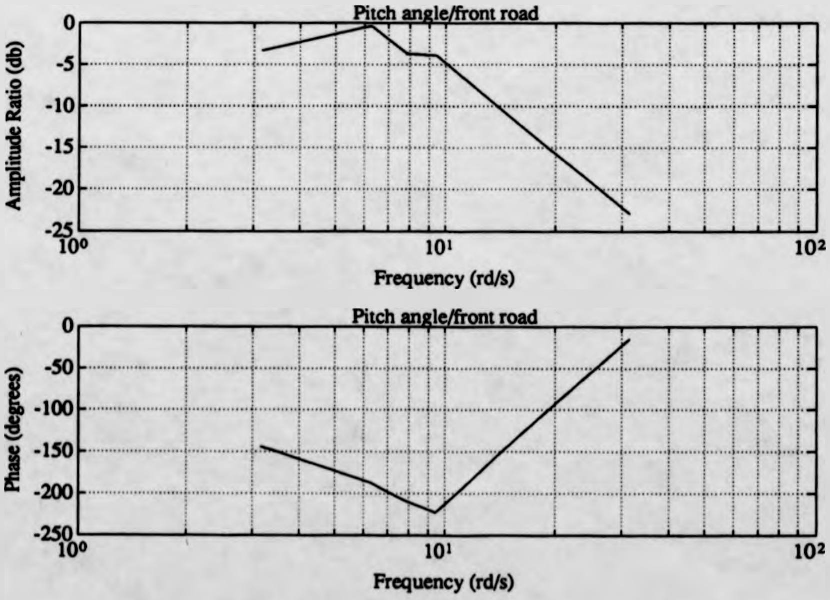


Figure C.92: In-phase, 0.5 - 30.0Hz, amplitude: 0.005m

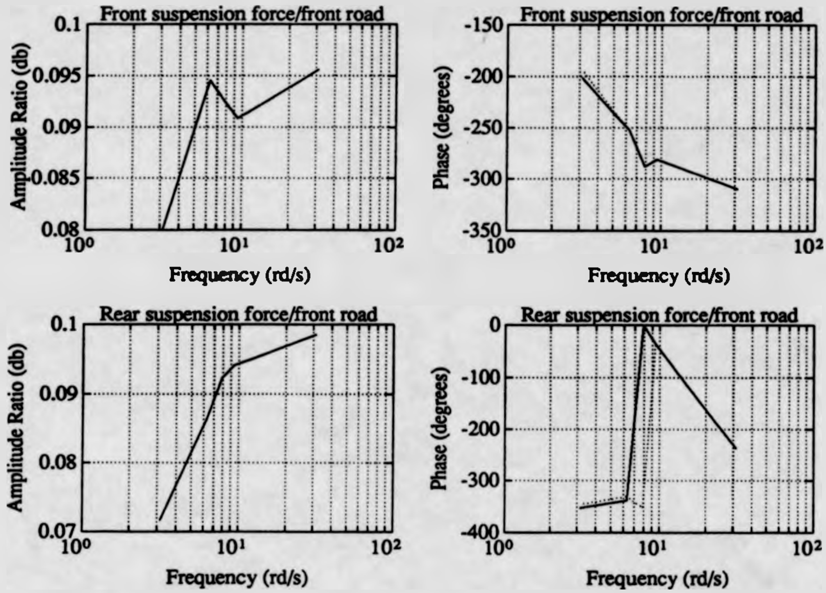


Figure C.93: In-phase, 0.5 - 5.0Hz, amplitudes: 0.005, 0.01 and 0.025m

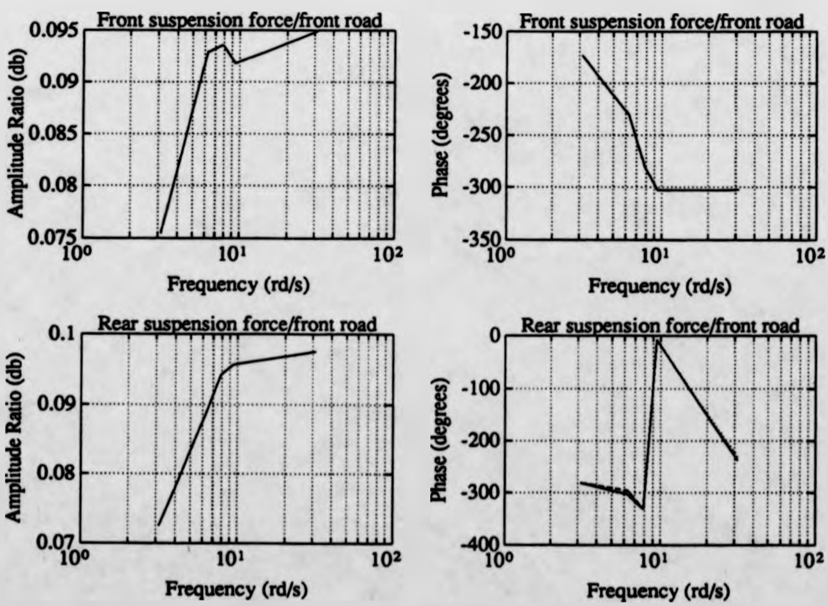


Figure C.94: In-phase, 0.5 - 30.0Hz, amplitude: 0.005m

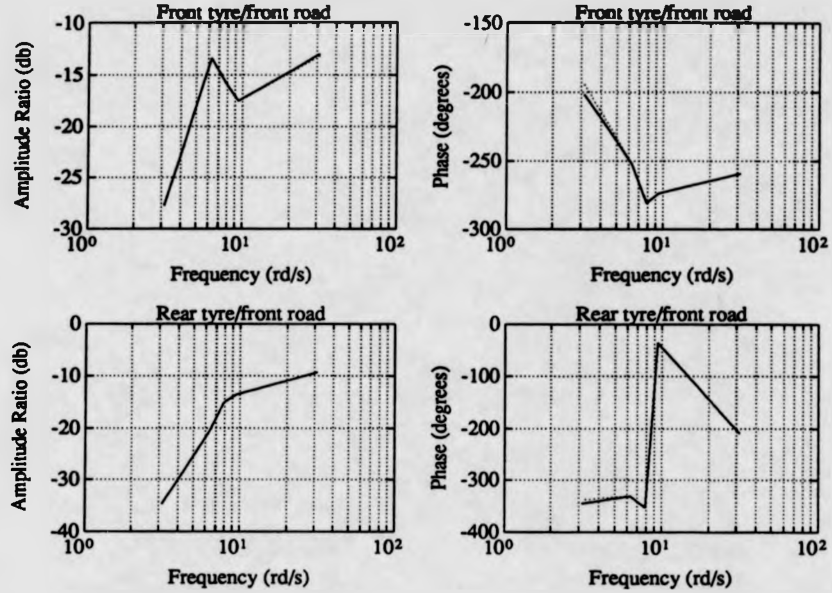


Figure C.95: In-phase, 0.5 - 5.0Hz, amplitudes: 0.005, 0.01 and 0.025m

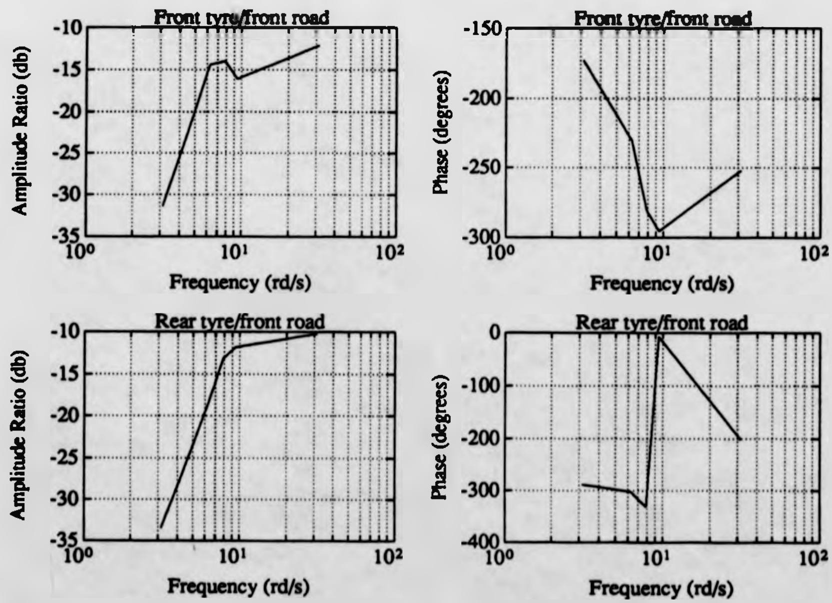


Figure C.96: In-phase, 0.5 - 30.0Hz, amplitude: 0.005m

C.2.3.2 Left/Right Out-of-phase Obstacles

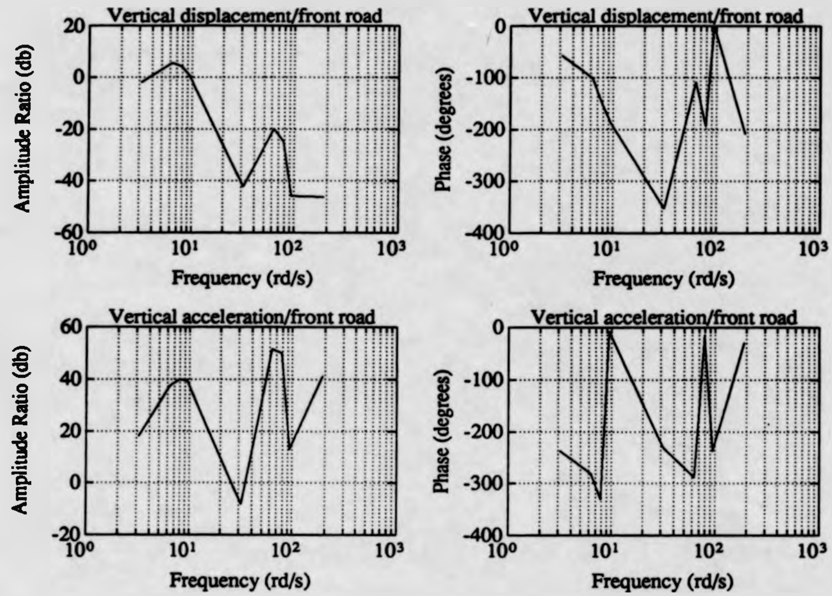


Figure C.97: Out-of-phase, 0.5 - 5.0Hz, amplitudes: 0.005, 0.01 and 0.025m

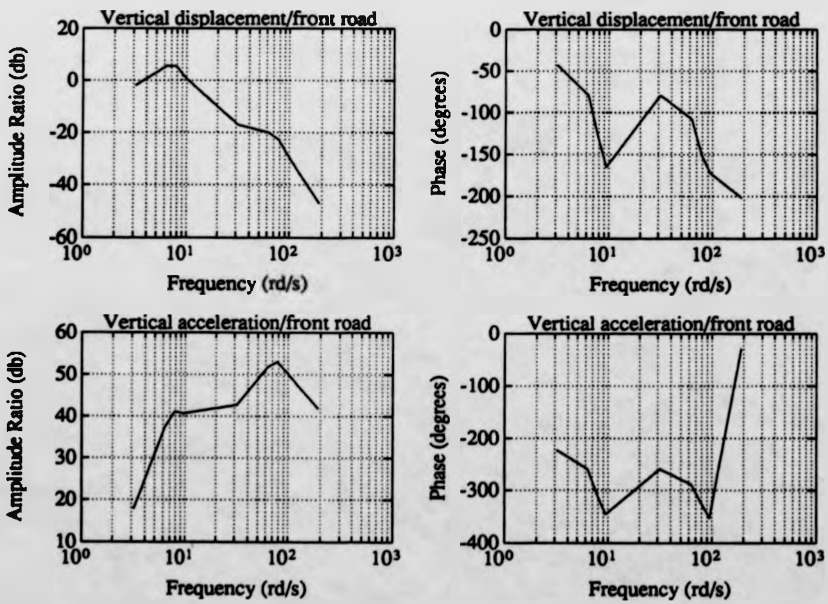


Figure C.98: Out-of-phase, 0.5 - 30.0Hz, amplitude: 0.005m

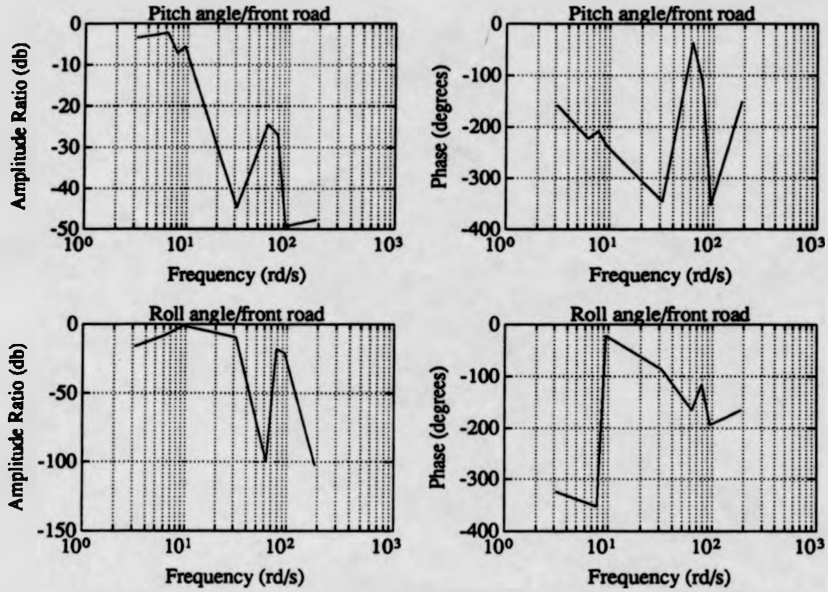


Figure C.99: Out-of-phase, 0.5 - 5.0Hz, amplitudes: 0.005, 0.01 and 0.025m

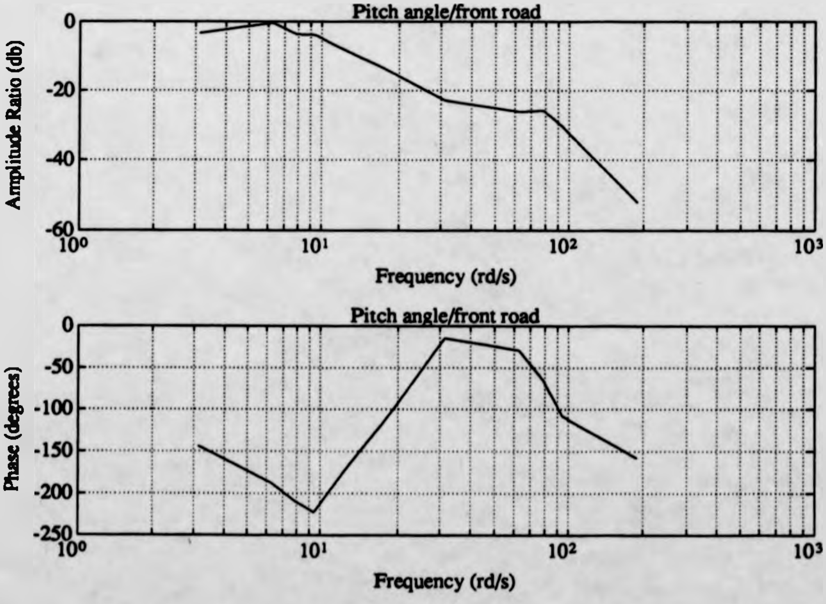


Figure C.100: Out-of-phase, 0.5 - 30.0Hz, amplitude: 0.005m

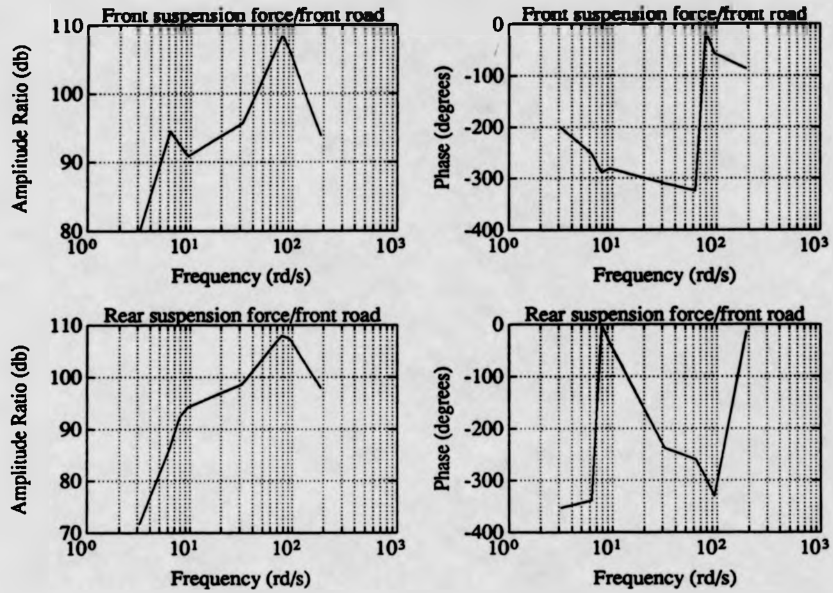


Figure C.101: Out-of-phase, 0.5 - 5.0Hz, amplitudes: 0.005, 0.01 and 0.025m

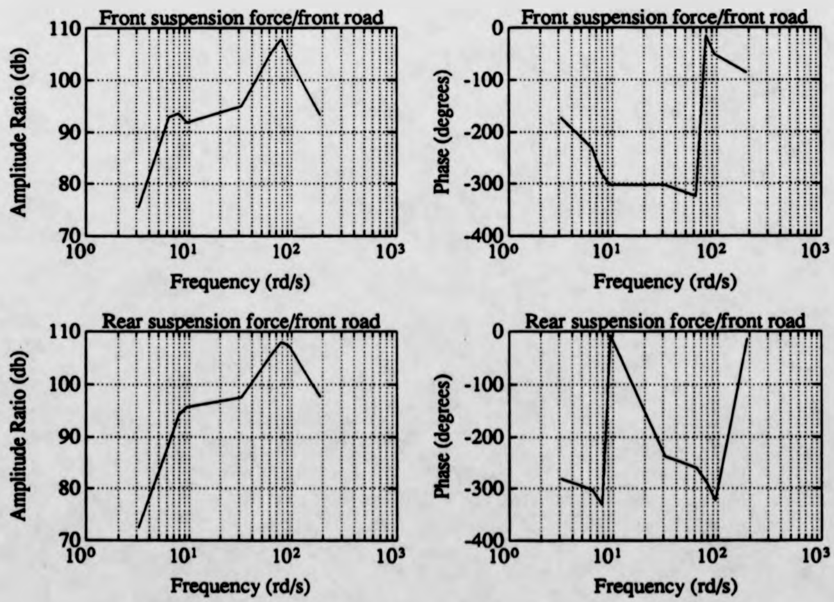


Figure C.102: Out-of-phase, 0.5 - 30.0Hz, amplitude: 0.005m

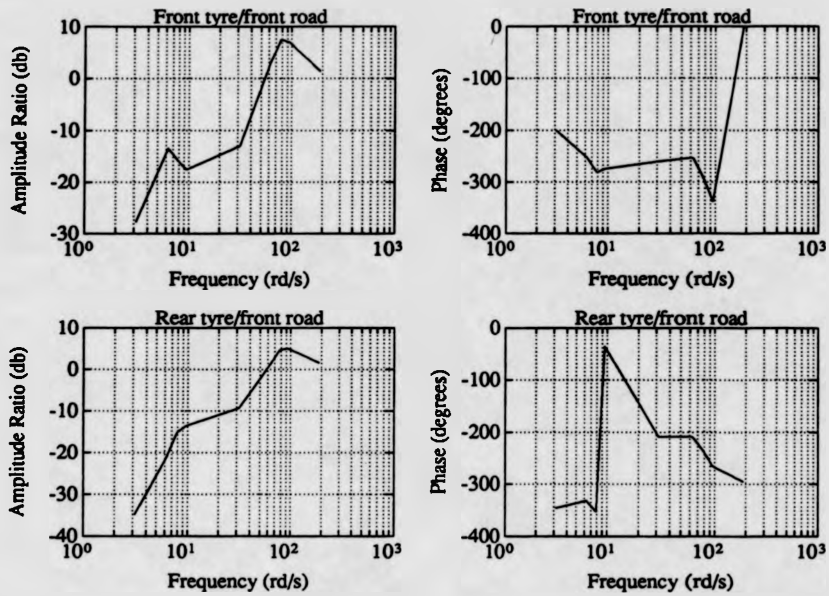


Figure C.103: Out-of-phase, 0.5 - 5.0Hz, amplitudes: 0.005, 0.01 and 0.025m

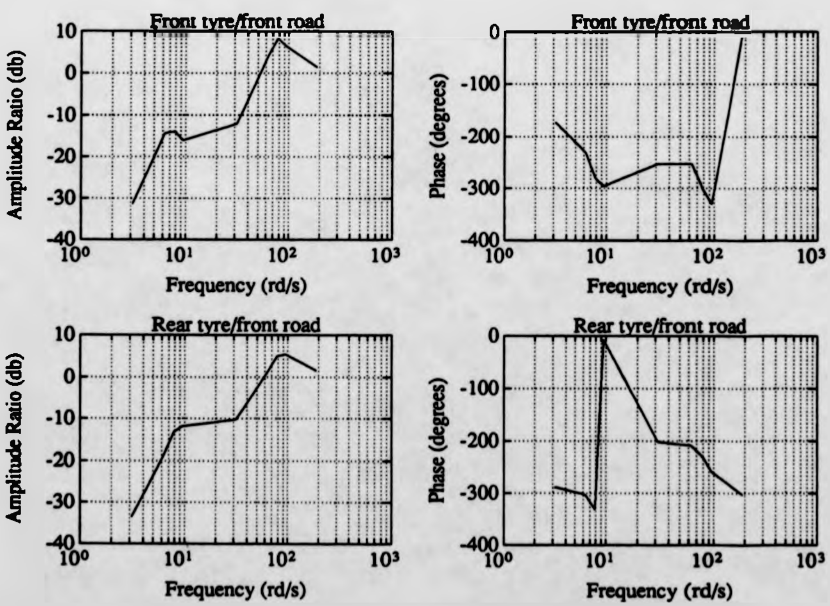


Figure C.104: Out-of-phase, 0.5 - 30.0Hz, amplitude: 0.005m

Appendix D

Linearised Model Matrices

D.1 Qualitative 3d Plots

D.1.1 Full Matrices

D.1.1.1 Straight Ahead Trim Condition

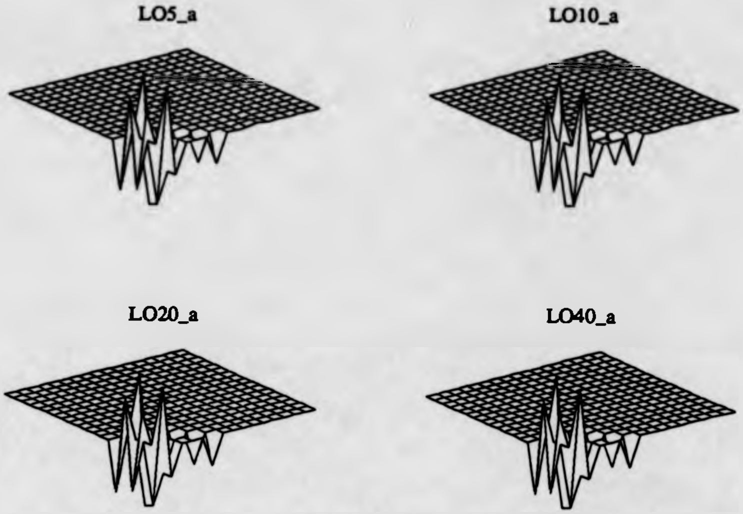


Figure D.1: Linear, without, speed: 5,10,20 and $40m/s$

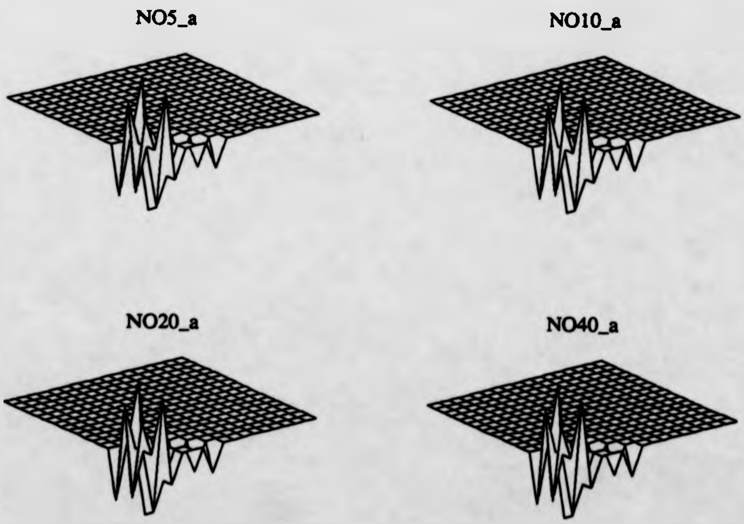


Figure D.2: Nonlinear, without, speed: 5,10,20 and $40m/s$

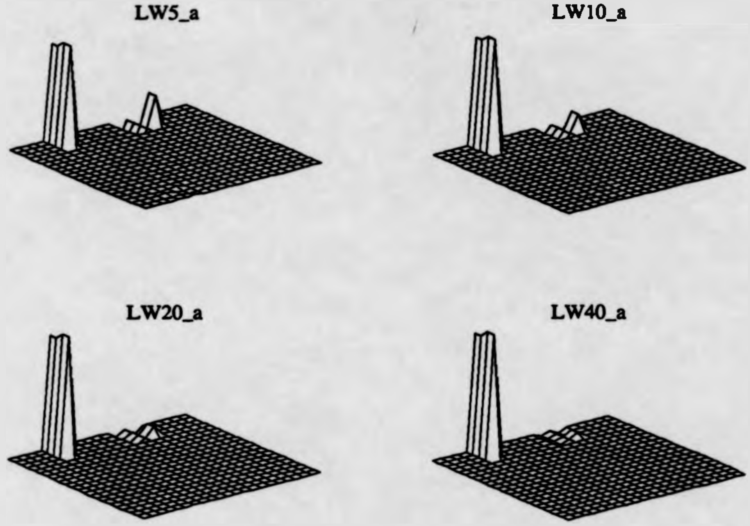


Figure D.3: Linear, with, speed: 5,10,20 and 40m/s

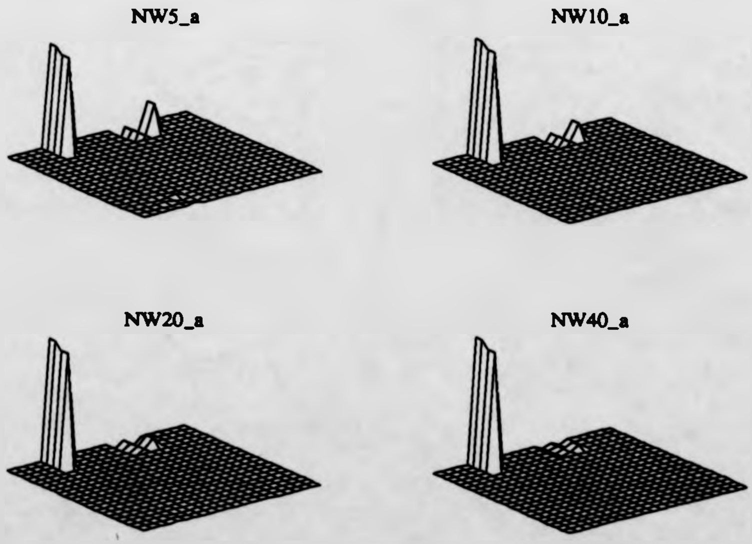


Figure D.4: Nonlinear, with, speed: 5,10,20 and 40m/s

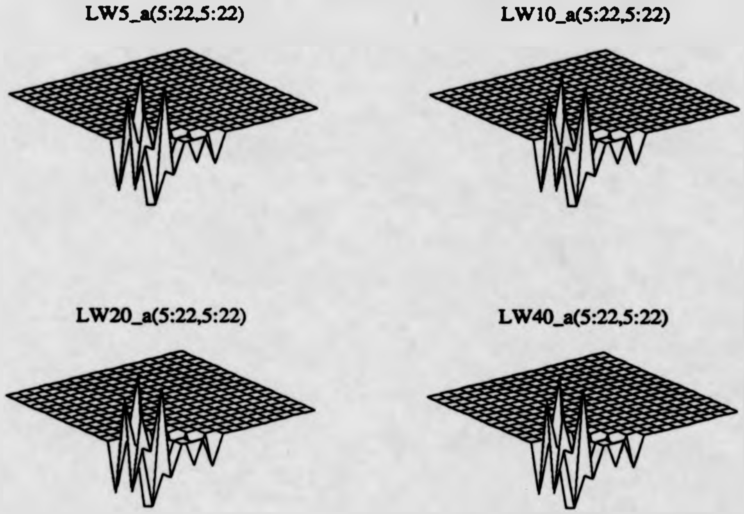


Figure D.5: Linear, with, speed: 5,10,20 and $40m/s$

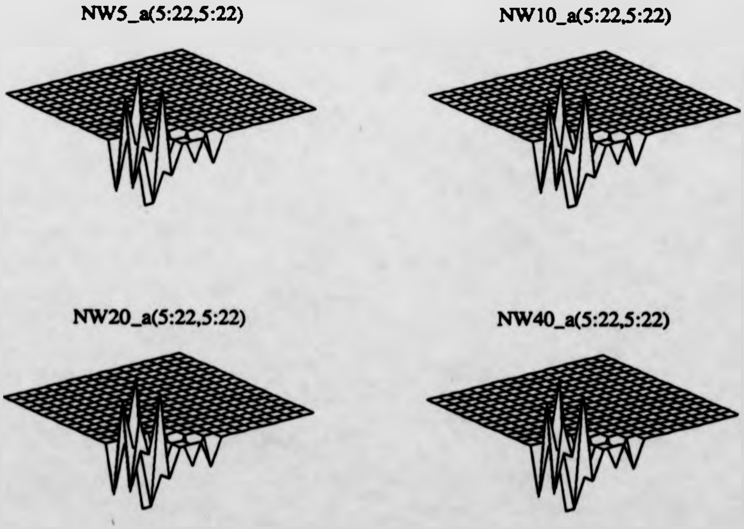


Figure D.6: Nonlinear, with, speed: 5,10,20 and $40m/s$

D.1.1.2 Cornering Trim Condition

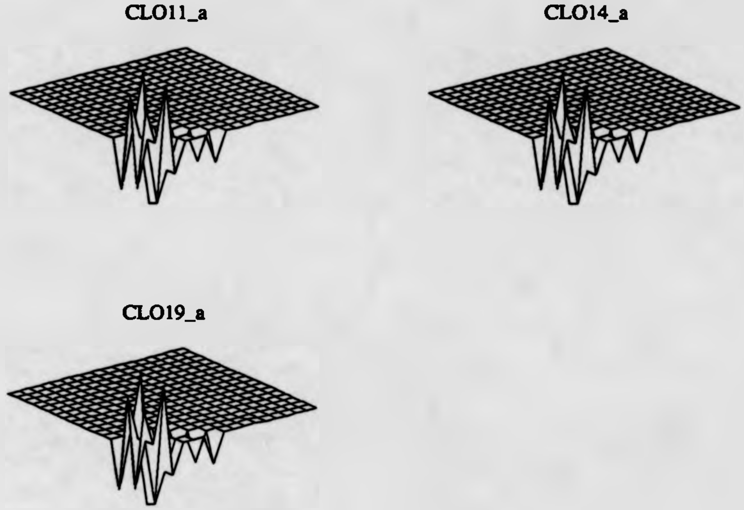


Figure D.7: Linear, without, speed: $10m/s$, steering: 10,45 and 90^o

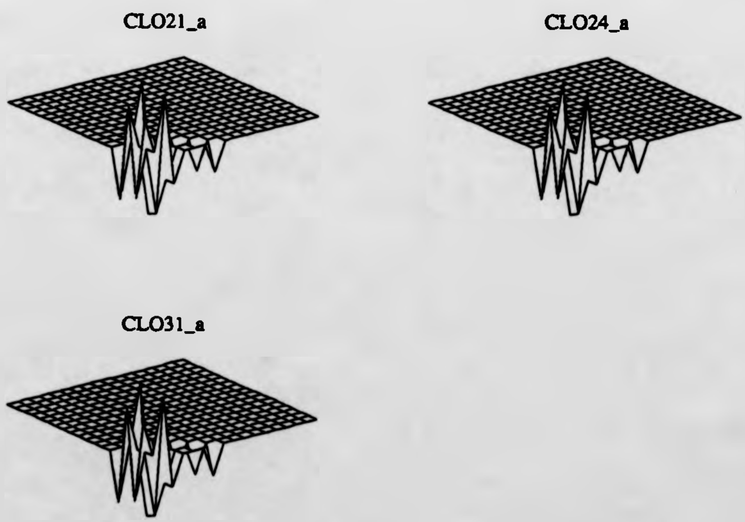


Figure D.8: Linear, without, speed: $20{,}30m/s$, steering: 10 and 45^o

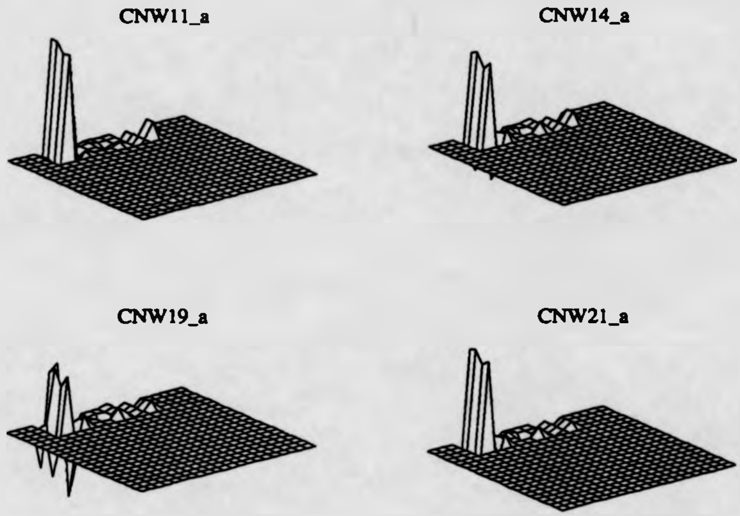


Figure D.9: Nonlinear, with, speed: 10,20m/s, steering: 10,45 and 90°

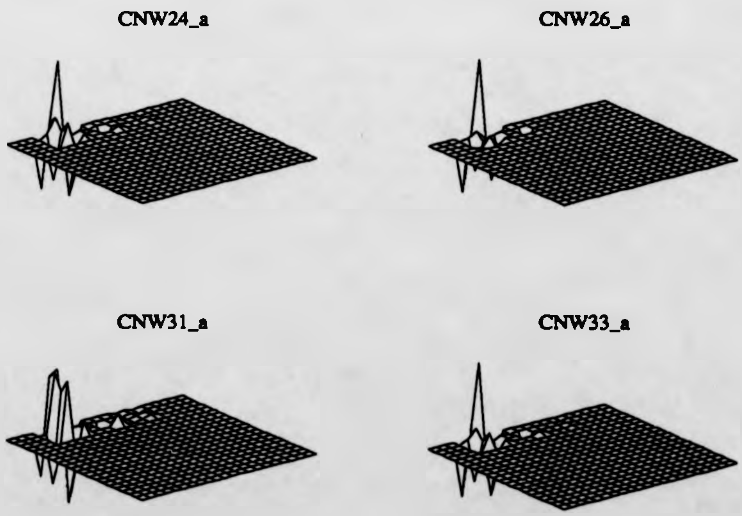


Figure D.10: Nonlinear, with, speed: 20,30m/s, steering: 45,60,10 and 30°

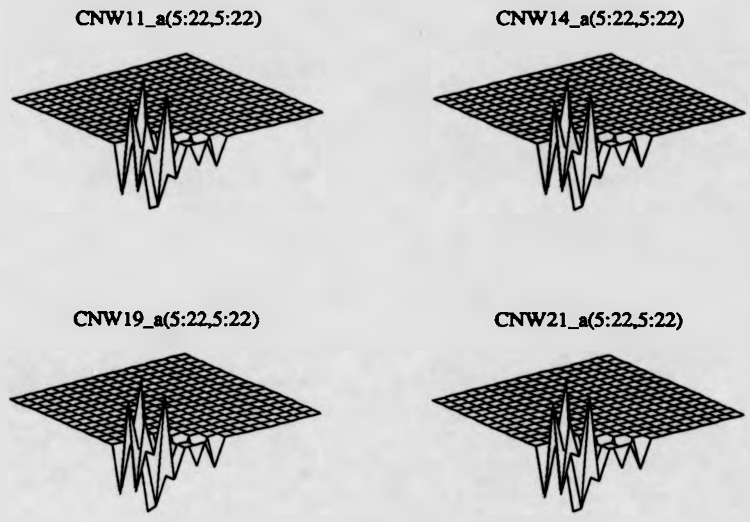


Figure D.11: Nonlinear, with, speed: 10,20m/s, steering: 10,45 and 90°

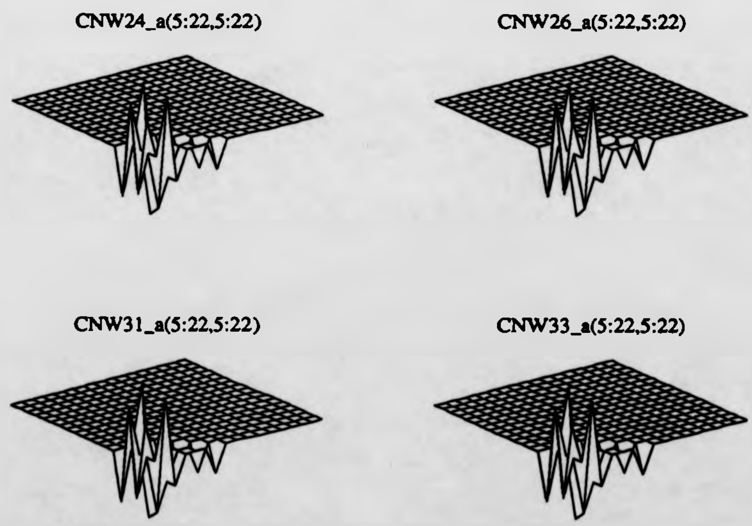


Figure D.12: Nonlinear, with, speed: 20,30m/s, steering: 45,60,10 and 30°

D.1.2 Sprung Mass Acceleration Coefficients

D.1.2.1 Straight Ahead Trim Condition

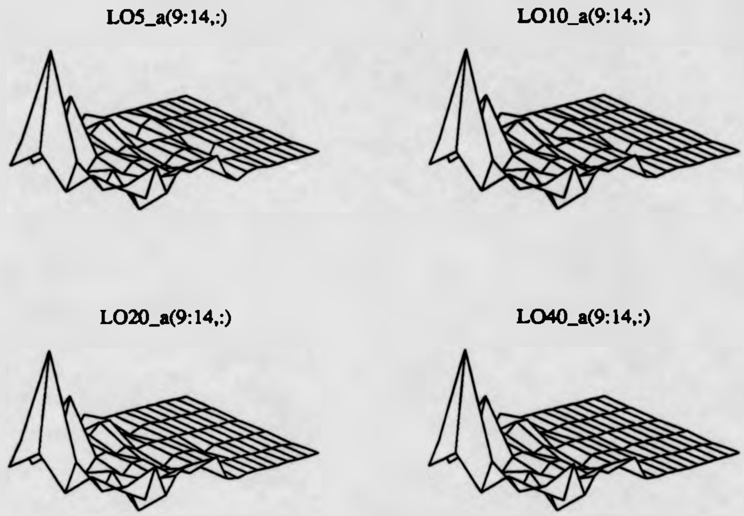


Figure D.13: Linear, without, speed: 5,10,20 and $40m/s$

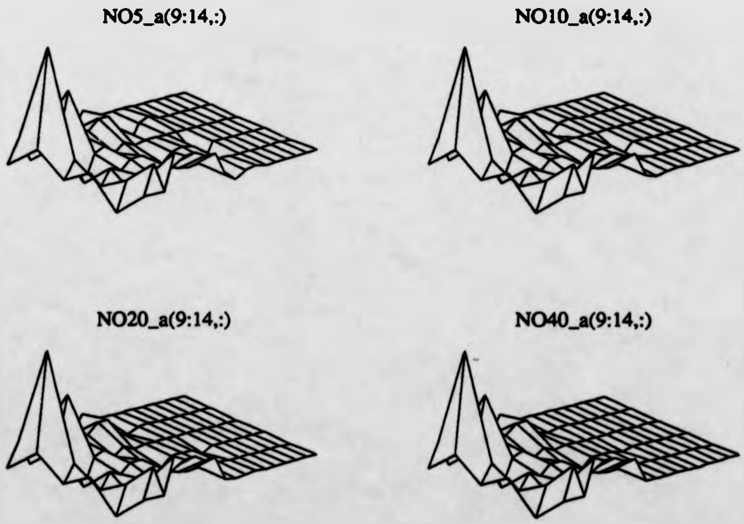


Figure D.14: Nonlinear, without, speed: 5,10,20 and $40m/s$

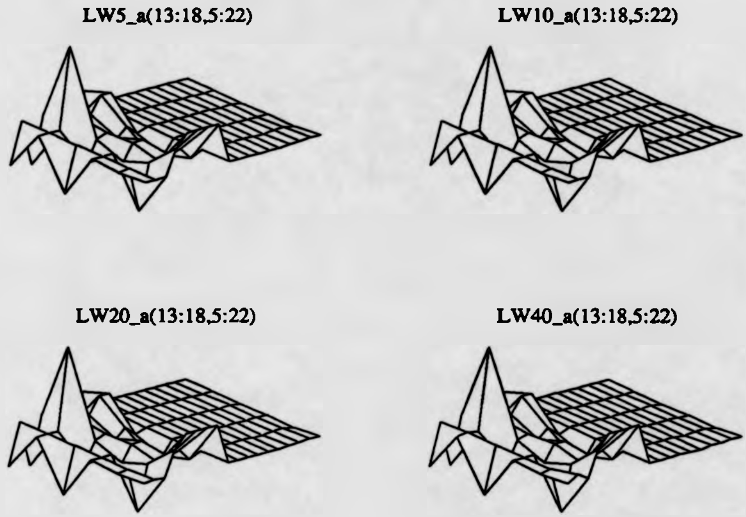


Figure D.15: Linear, with, speed: 5,10,20 and $40m/s$

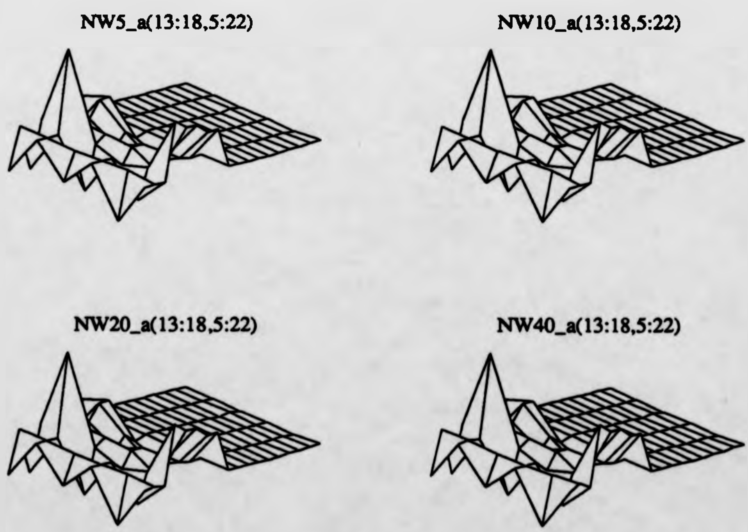
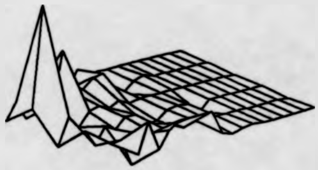


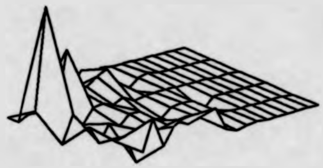
Figure D.16: Nonlinear, with, speed: 5,10,20 and $40m/s$

D.1.2.2 Cornering Trim Condition

CLO11_a(9:14,:)



CLO14_a(9:14,:)



CLO19_a(9:14,:)

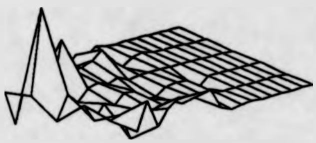
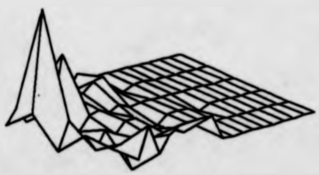
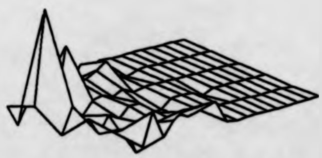


Figure D.17: Linear, without, speed: $10m/s$, steering: 10,45 and 90°

CLO21_a(9:14,:)



CLO24_a(9:14,:)



CLO31_a(9:14,:)

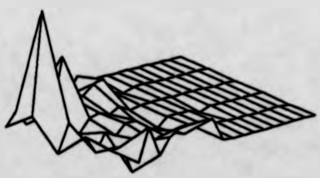


Figure D.18: Linear, without, speed: $20,30m/s$, steering: 10 and 45°

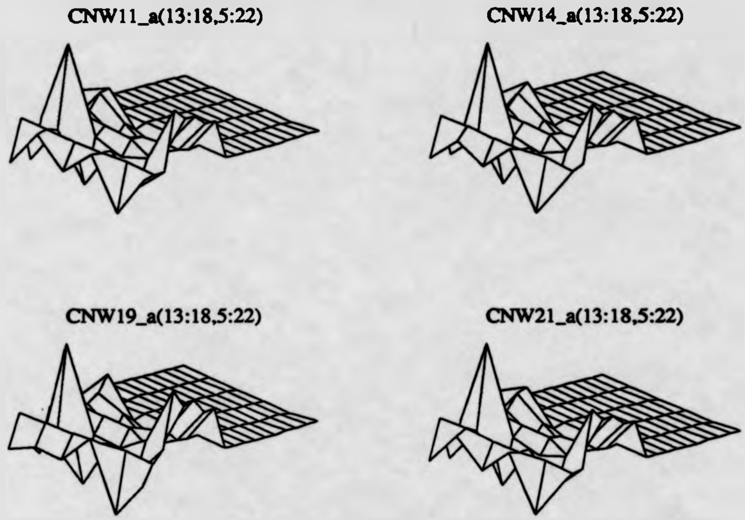


Figure D.19: Nonlinear, with, speed: 10,20m/s, steering: 10,45 and 90°

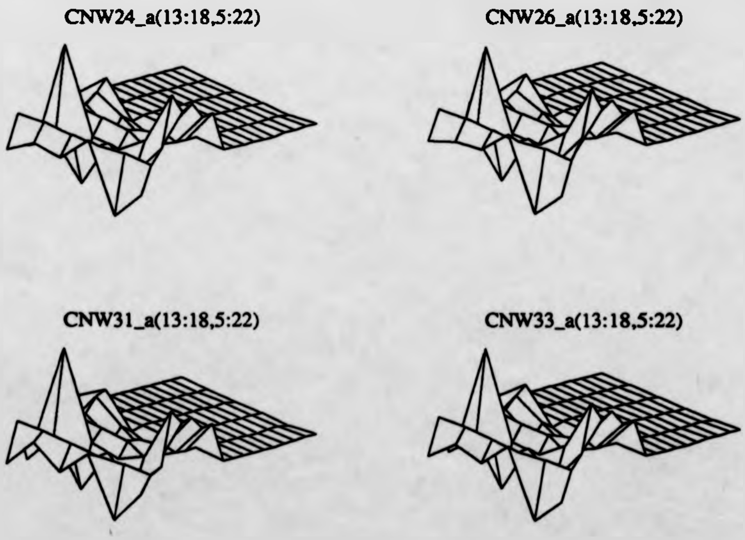


Figure D.20: Nonlinear, with, speed: 20,30m/s, steering: 45,60,10 and 30°

D.1.3 Sprung Mass Spring Coefficients

D.1.3.1 Straight Ahead Trim Condition

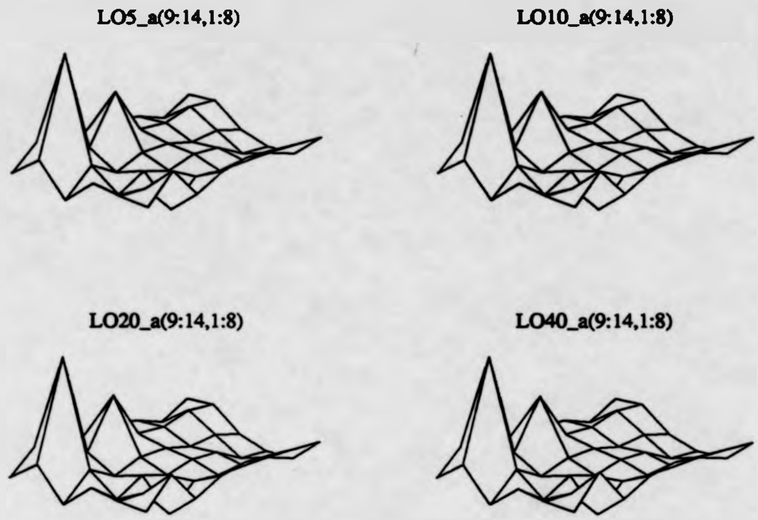


Figure D.21: Linear, without, speed: 5,10,20 and $40m/s$

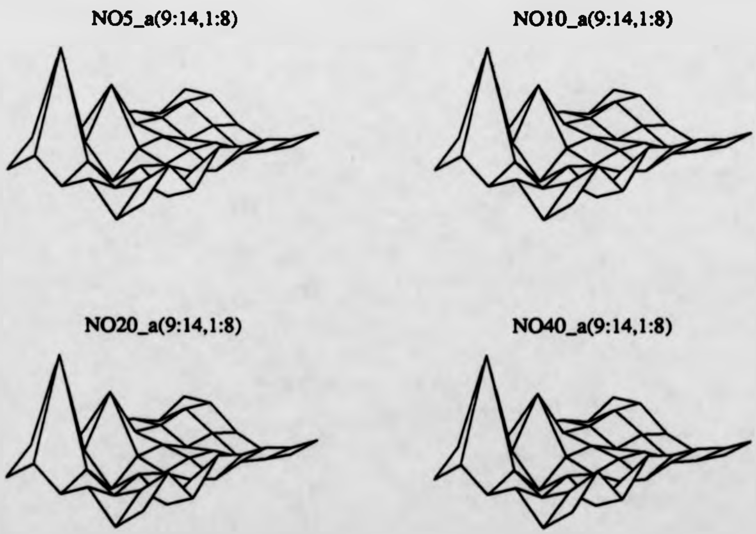


Figure D.22: Nonlinear, without, speed: 5,10,20 and $40m/s$

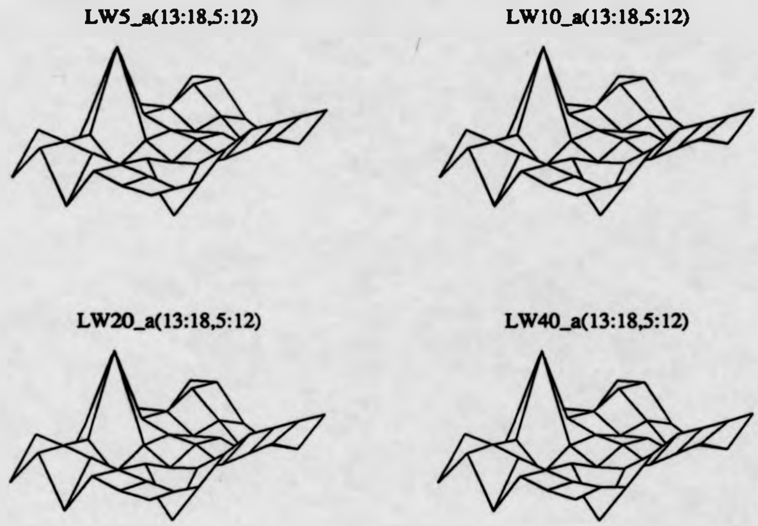


Figure D.23: Linear, with, speed: 5,10,20 and $40m/s$

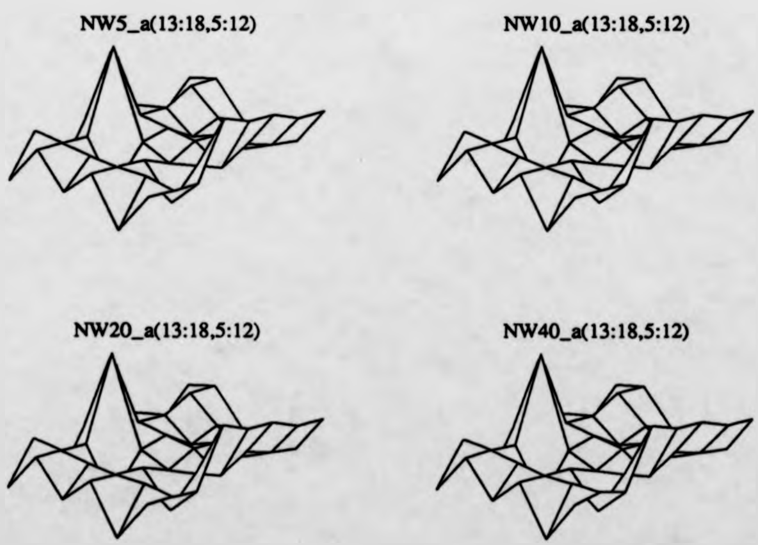


Figure D.24: Nonlinear, with, speed: 5,10,20 and $40m/s$

D.1.3.2 Cornering Trim Condition

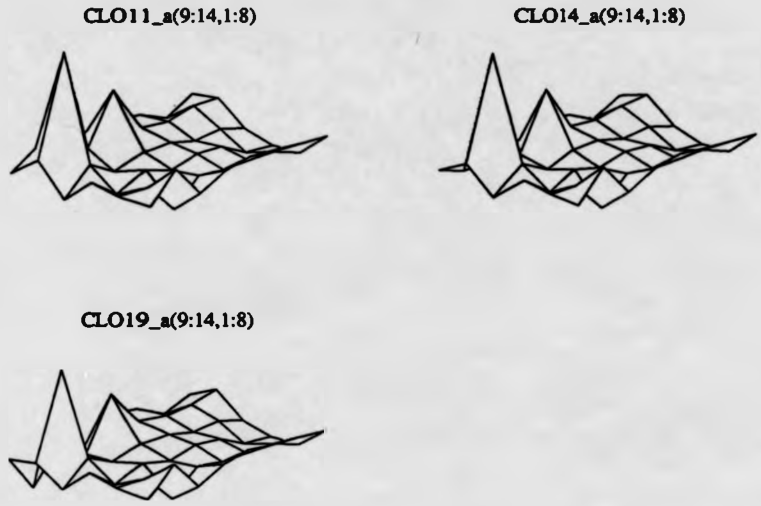


Figure D.25: Linear, without, speed: $10m/s$, steering: 10,45 and 90°

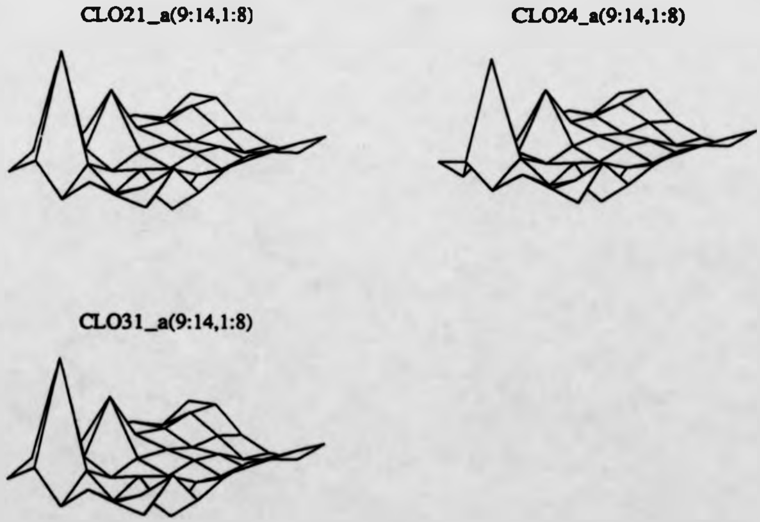


Figure D.26: Linear, without, speed: $20,30m/s$, steering: 10 and 45°

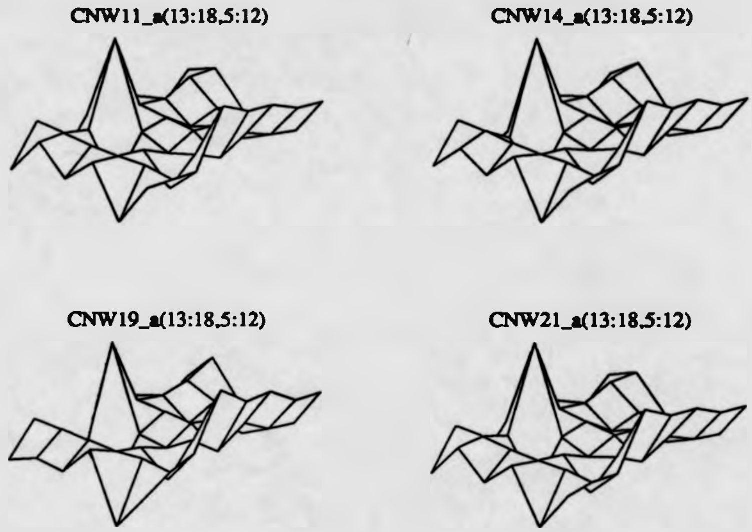


Figure D.27: Nonlinear, with, speed: 10,20m/s, steering: 10,45 and 90°

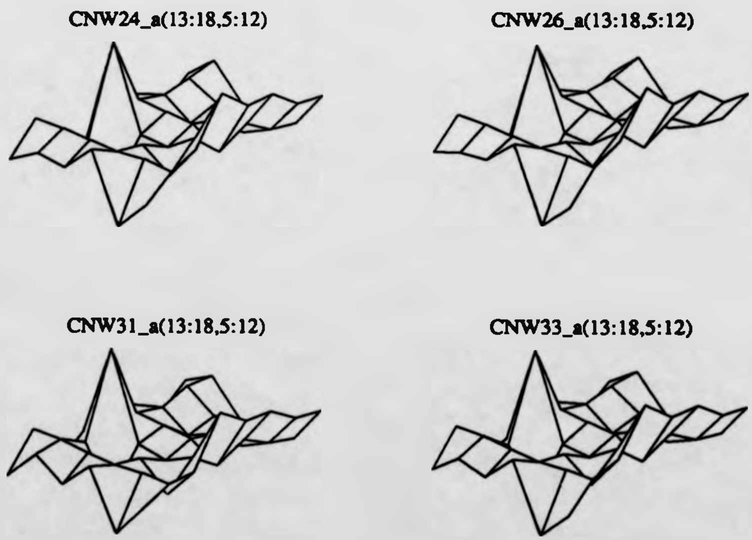


Figure D.28: Nonlinear, with, speed: 20,30m/s, steering: 45,60,10 and 30°

D.1.4 Sprung Mass Damping Coefficients

D.1.4.1 Straight Ahead Trim Condition

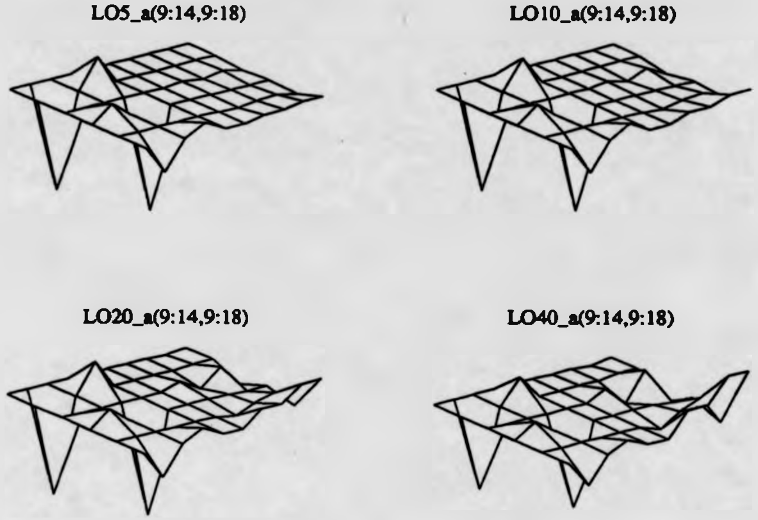


Figure D.29: Linear, without, speed: 5,10,20 and $40m/s$

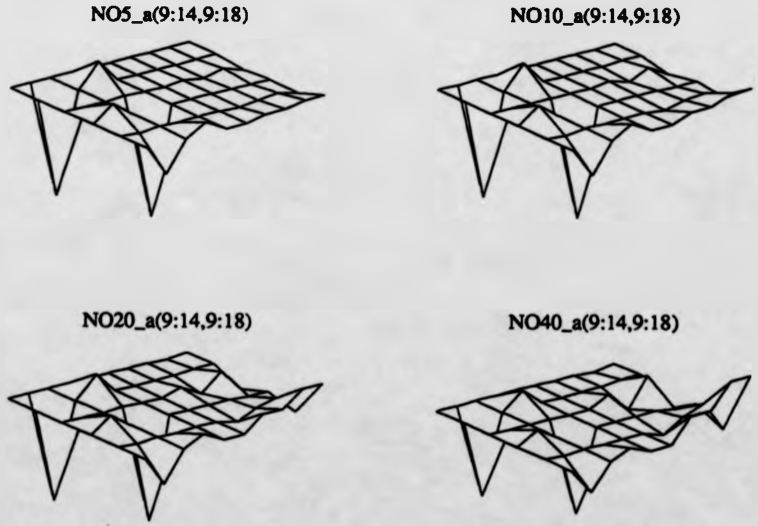


Figure D.30: Nonlinear, without, speed: 5,10,20 and $40m/s$

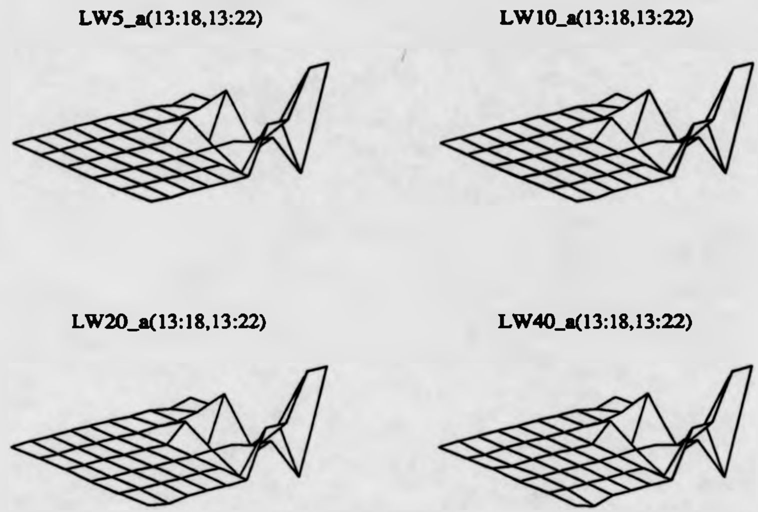


Figure D.31: Linear, with, speed: 5,10,20 and $40m/s$

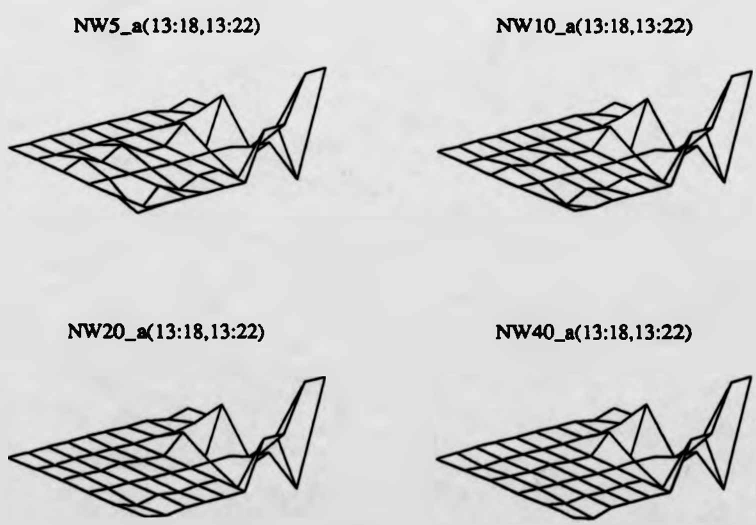


Figure D.32: Nonlinear, with, speed: 5,10,20 and $40m/s$

D.1.4.2 Cornering Trim Condition

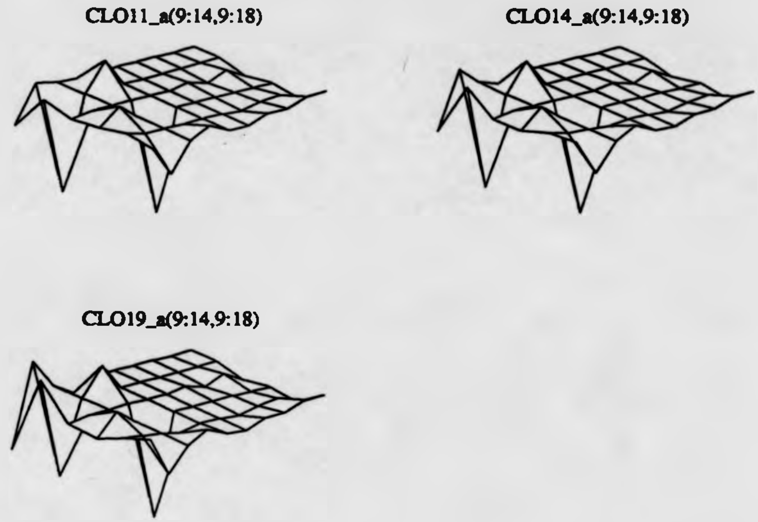


Figure D.33: Linear, without, speed: $10m/s$, steering: 10,45 and 90°

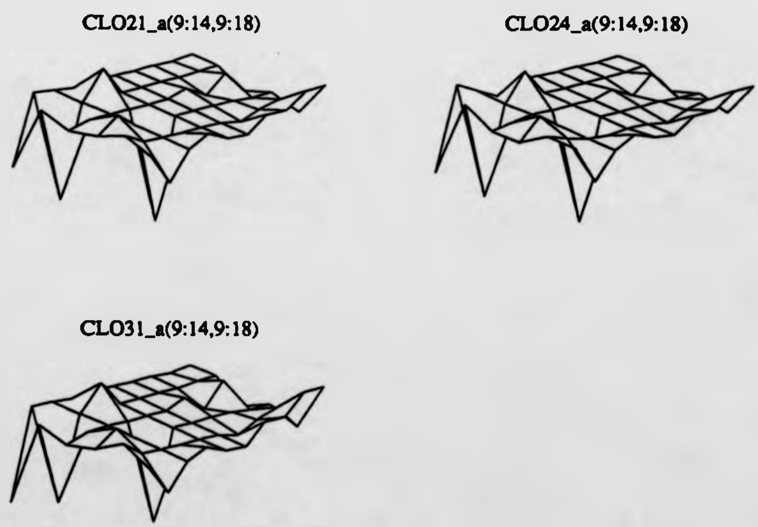


Figure D.34: Linear, without, speed: $20,30m/s$, steering: 10 and 45°

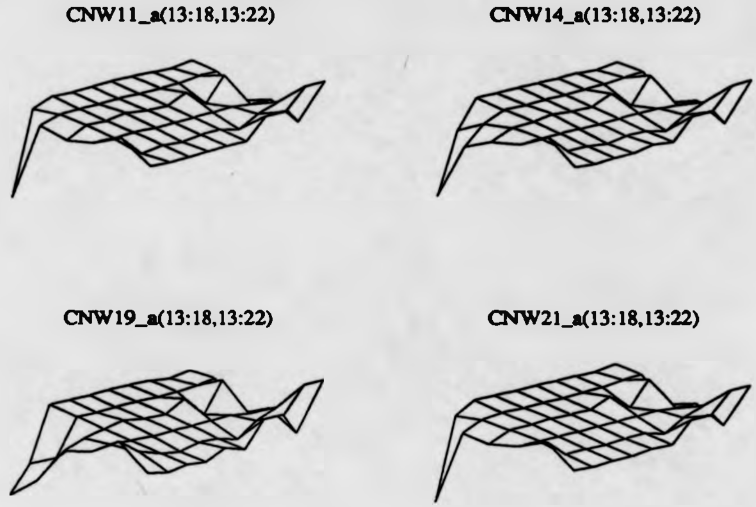


Figure D.35: Nonlinear, with, speed: 10,20m/s, steering: 10,45 and 90^o

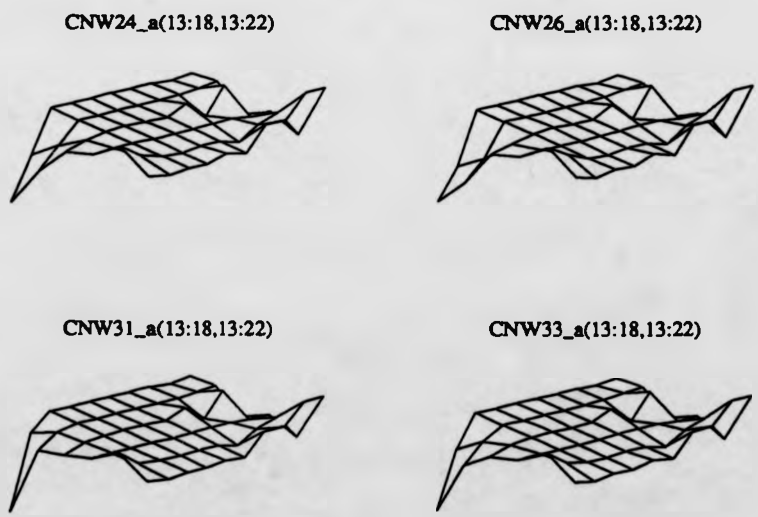


Figure D.36: Nonlinear, with, speed: 20,30m/s, steering: 45,60,10 and 30^o

D.2 Numerical Values

D.2.1 Straight Ahead Trim Conditions

```
>> L05_x

L05_x =

Q(3)
Q(4)
Q(5)
Q(6)
Q(7)
Q(8)
Q(9)
Q(10)
U(1)
U(2)
U(3)
U(4)
U(5)
U(6)
U(7)
U(8)
U(9)
U(10)

>> L05_a

L05_a =

   1.0e+04 *

  Columns 1 through 7

         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
   -0.0031   -0.0000   -0.0034   -0.0000    0.0000   -0.0000   -0.0011
         0    0.0122    0.0000    0.0060    0.0006    0.0006    0.0018
   -0.0035    0.0000   -0.0016    0.0000    0.0003   -0.0003    0.0003
   -0.0000   -0.0021    0.0000   -0.0008    0.0004    0.0004   -0.0010
   -0.0002    0.0000   -0.0033         0   -0.0002    0.0002    0.0004
    0.0000    0.0037    0.0000    0.0020    0.0029    0.0029    0.0030
   -1.1439    0.1068    1.5130   -0.8675   -0.6054   -0.0254   -0.0045
    1.1439    0.1068   -1.5130   -0.8675   -0.0254   -0.6054   -0.0055
   -0.9608    0.1287   -1.4391   -0.7291   -0.0036   -0.0035   -0.6957
    0.9608    0.1287    1.4391   -0.7291   -0.0035   -0.0036   -0.0029

  Columns 8 through 14

         0         0         0    0.0001         0         0         0
         0         0         0         0    0.0001         0         0
         0         0         0         0         0    0.0001         0
         0         0         0         0         0         0    0.0001
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
    0.0011   -0.0000    0.0000         0   -0.0000         0         0
    0.0018    0.0000   -0.0024         0    0.0008         0         0
   -0.0003    0.0000   -0.0000         0         0         0         0
   -0.0010    0.0000    0.0004         0   -0.0026         0         0
   -0.0004    0.0000         0         0         0         0         0
    0.0030    0.0000   -0.0007         0    0.0003         0         0
```



```
   -0.0055   -0.0000   -0.0214         0   -0.0302         0         0
   -0.0045    0.0000   -0.0214         0   -0.0302         0         0
   -0.0029   -0.0000   -0.0257         0    0.0398         0         0
   -0.6957    0.0000   -0.0257         0    0.0398         0         0

  Columns 15 through 18

         0         0         0         0
         0         0         0         0
         0         0         0         0
         0         0         0         0
    0.0001         0         0         0
         0    0.0001         0         0
         0         0    0.0001         0
         0         0         0    0.0001
   -0.0000    0.0000   -0.0000    0.0000
    0.0000    0.0000    0.0000    0.0000
    0.0000   -0.0000    0.0001   -0.0001
    0.0000    0.0000   -0.0000   -0.0000
   -0.0000    0.0000    0.0001   -0.0001
    0.0001    0.0001    0.0002    0.0002
   -0.0033   -0.0000   -0.0003   -0.0003
   -0.0000   -0.0033   -0.0003   -0.0003
   -0.0001   -0.0001   -0.0054    0.0000
   -0.0001   -0.0001    0.0000   -0.0054

>> L010_x

L010_x =

Q(3)
Q(4)
Q(5)
Q(6)
Q(7)
Q(8)
Q(9)
Q(10)
U(1)
U(2)
U(3)
U(4)
U(5)
U(6)
U(7)
U(8)
U(9)
U(10)

>> L010_a

L010_a =

   1.0e+04 *

  Columns 1 through 7

         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
   -0.0031   -0.0000   -0.0034   -0.0000    0.0000   -0.0000   -0.0011
```

```
      0.0000    0.0122   -0.0000    0.0060    0.0006    0.0006    0.0018
     -0.0035    0.0000   -0.0016    0.0000    0.0003   -0.0003    0.0003
     -0.0000   -0.0021   -0.0000   -0.0008    0.0004    0.0004   -0.0010
     -0.0002    0.0000   -0.0033    0.0000   -0.0002    0.0002    0.0004
      0.0000    0.0037   -0.0000    0.0020    0.0029    0.0029    0.0030
     -1.1439    0.1068    1.5130   -0.8675   -0.6054   -0.0254   -0.0045
      1.1439    0.1068   -1.5130   -0.8675   -0.0254   -0.6054   -0.0055
     -0.9608    0.1287   -1.4391   -0.7291   -0.0036   -0.0035   -0.6957
      0.9608    0.1287    1.4391   -0.7291   -0.0035   -0.0036   -0.0029

  Columns 8 through 14

           0         0         0    0.0001         0         0         0
           0         0         0         0    0.0001         0         0
           0         0         0         0         0    0.0001         0
           0         0         0         0         0         0    0.0001
           0         0         0         0         0         0         0
           0         0         0         0         0         0         0
           0         0         0         0         0         0         0
           0         0         0         0         0         0         0
      0.0011   -0.0000    0.0000         0         0         0         0
      0.0018         0   -0.0012         0    0.0004         0         0
     -0.0003    0.0000   -0.0000         0         0         0         0
     -0.0010    0.0000    0.0002         0   -0.0013         0         0
     -0.0004    0.0000   -0.0000         0   -0.0000         0         0
      0.0030   -0.0000   -0.0004         0    0.0001         0         0
     -0.0055   -0.0000   -0.0107         0   -0.0151         0         0
     -0.0045    0.0000   -0.0107         0   -0.0151         0         0
     -0.0029   -0.0000   -0.0129         0    0.0199         0         0
     -0.6957    0.0000   -0.0129         0    0.0199         0         0

  Columns 15 through 18

           0         0         0         0
           0         0         0         0
           0         0         0         0
           0         0         0         0
      0.0001         0         0         0
           0    0.0001         0         0
           0         0    0.0001         0
           0         0         0    0.0001
     -0.0000    0.0000   -0.0000    0.0000
      0.0000    0.0000    0.0000    0.0000
      0.0000   -0.0000    0.0001   -0.0001
      0.0000    0.0000   -0.0000   -0.0000
     -0.0000    0.0000    0.0001   -0.0001
      0.0001    0.0001    0.0002    0.0002
     -0.0033   -0.0000   -0.0003   -0.0003
     -0.0000   -0.0033   -0.0003   -0.0003
     -0.0001   -0.0001   -0.0054    0.0000
     -0.0001   -0.0001    0.0000   -0.0054

>> LO15_x

LO15_x =

Q(3)
Q(4)
Q(5)
Q(6)
Q(7)
Q(8)
Q(9)
Q(10)
U(1)
U(2)
```



```
U(3)
U(4)
U(5)
U(6)
U(7)
U(8)
U(9)
U(10)

>> LO15_a

LO15_a =

   1.0e+04 *

  Columns 1 through 7

           0         0         0         0         0         0         0
           0         0         0         0         0         0         0
           0         0         0         0         0         0         0
           0         0         0         0         0         0         0
           0         0         0         0         0         0         0
           0         0         0         0         0         0         0
           0         0         0         0         0         0         0
           0         0         0         0         0         0         0
     -0.0031   -0.0000   -0.0034    0.0000    0.0000   -0.0000   -0.0011
     -0.0000    0.0122   -0.0000    0.0060    0.0006    0.0006    0.0018
     -0.0035    0.0000   -0.0016   -0.0000    0.0003   -0.0003    0.0003
     -0.0000   -0.0021    0.0000   -0.0008    0.0004    0.0004   -0.0010
     -0.0002    0.0000   -0.0033   -0.0000   -0.0002    0.0002    0.0004
     -0.0000    0.0037   -0.0000    0.0020    0.0029    0.0029    0.0030
     -1.1439    0.1067    1.5130   -0.8675   -0.6054   -0.0254   -0.0045
      1.1439    0.1067   -1.5130   -0.8675   -0.0254   -0.6054   -0.0055
     -0.9608    0.1286   -1.4391   -0.7291   -0.0036   -0.0035   -0.6957
      0.9608    0.1286    1.4391   -0.7291   -0.0035   -0.0036   -0.0029

  Columns 8 through 14

           0         0         0    0.0001         0         0         0
           0         0         0         0    0.0001         0         0
           0         0         0         0         0    0.0001         0
           0         0         0         0         0         0    0.0001
           0         0         0         0         0         0         0
           0         0         0         0         0         0         0
           0         0         0         0         0         0         0
           0         0         0         0         0         0         0
      0.0011   -0.0000    0.0000         0         0         0         0
      0.0018   -0.0000   -0.0008         0    0.0003         0         0
     -0.0003    0.0000   -0.0000         0         0         0         0
     -0.0010    0.0000    0.0001         0   -0.0009         0         0
     -0.0004    0.0000   -0.0000         0    0.0000         0         0
      0.0030   -0.0000   -0.0002         0    0.0001         0         0
     -0.0055   -0.0000   -0.0071         0   -0.0101         0         0
     -0.0045    0.0000   -0.0071         0   -0.0101         0         0
     -0.0029   -0.0000   -0.0086         0    0.0133         0         0
     -0.6957    0.0000   -0.0086         0    0.0133         0         0

  Columns 15 through 18

           0         0         0         0
           0         0         0         0
           0         0         0         0
           0         0         0         0
      0.0001         0         0         0
           0    0.0001         0         0
           0         0    0.0001         0
```

```
         0         0         0    0.0001
   -0.0000    0.0000   -0.0000    0.0000
    0.0000    0.0000    0.0000    0.0000
    0.0000   -0.0000    0.0001   -0.0001
    0.0000    0.0000   -0.0000   -0.0000
   -0.0000    0.0000    0.0001   -0.0001
    0.0001    0.0001    0.0002    0.0002
   -0.0033   -0.0000   -0.0003   -0.0003
   -0.0000   -0.0033   -0.0003   -0.0003
   -0.0001   -0.0001   -0.0054    0.0000
   -0.0001   -0.0001    0.0000   -0.0054

>> LO20_x

LO20_x =

Q(3)
Q(4)
Q(5)
Q(6)
Q(7)
Q(8)
Q(9)
Q(10)
U(1)
U(2)
U(3)
U(4)
U(5)
U(6)
U(7)
U(8)
U(9)
U(10)

>> LO20_a

LO20_a =

   1.0e+04 *

  Columns 1 through 7

         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
   -0.0031   -0.0000   -0.0034         0    0.0000   -0.0000   -0.0011
    0.0000    0.0122   -0.0000    0.0060    0.0006    0.0006    0.0018
   -0.0035    0.0000   -0.0016   -0.0000    0.0003   -0.0003    0.0003
   -0.0000   -0.0021    0.0000   -0.0008    0.0004    0.0004   -0.0010
   -0.0002    0.0000   -0.0033         0   -0.0002    0.0002    0.0004
    0.0000    0.0038   -0.0000    0.0020    0.0029    0.0029    0.0030
   -1.1439    0.1066    1.5130   -0.8675   -0.6054   -0.0254   -0.0045
    1.1439    0.1067   -1.5130   -0.8675   -0.0254   -0.6054   -0.0055
   -0.9608    0.1285   -1.4391   -0.7291   -0.0036   -0.0035   -0.6957
    0.9608    0.1286    1.4391   -0.7291   -0.0035   -0.0036   -0.0029

  Columns 8 through 14

         0         0         0    0.0001         0         0         0
         0         0         0         0    0.0001         0         0
```



```
         0         0         0         0         0    0.0001         0
         0         0         0         0         0         0    0.0001
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
    0.0011   -0.0000    0.0000         0         0         0         0
    0.0018   -0.0000   -0.0006         0    0.0002         0         0
   -0.0003    0.0000   -0.0000         0         0         0         0
   -0.0010    0.0000    0.0001         0   -0.0007         0         0
   -0.0004    0.0000   -0.0000         0   -0.0000         0         0
    0.0030   -0.0000   -0.0002         0    0.0001         0         0
   -0.0055   -0.0000   -0.0053         0   -0.0076         0         0
   -0.0045    0.0000   -0.0053         0   -0.0076         0         0
   -0.0029   -0.0000   -0.0064         0    0.0100         0         0
   -0.6957    0.0000   -0.0064         0    0.0100         0         0

  Columns 15 through 18

         0         0         0         0
         0         0         0         0
         0         0         0         0
         0         0         0         0
    0.0001         0         0         0
         0    0.0001         0         0
         0         0    0.0001         0
         0         0         0    0.0001
   -0.0000    0.0000   -0.0000    0.0000
    0.0000    0.0000    0.0000    0.0000
    0.0000   -0.0000    0.0001   -0.0001
    0.0000    0.0000   -0.0000   -0.0000
   -0.0000    0.0000    0.0001   -0.0001
    0.0001    0.0001    0.0002    0.0002
   -0.0033   -0.0000   -0.0003   -0.0003
   -0.0000   -0.0033   -0.0003   -0.0003
   -0.0001   -0.0001   -0.0054    0.0000
   -0.0001   -0.0001    0.0000   -0.0054

>> LO40_x

LO40_x =

Q(3)
Q(4)
Q(5)
Q(6)
Q(7)
Q(8)
Q(9)
Q(10)
U(1)
U(2)
U(3)
U(4)
U(5)
U(6)
U(7)
U(8)
U(9)
U(10)

>> LO40_a

LO40_a =

   1.0e+04 *
```

```

 Columns 1 through 7

         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
   -0.0031   -0.0000   -0.0034         0    0.0000   -0.0000   -0.0011
   -0.0000    0.0123   -0.0000    0.0060    0.0006    0.0006    0.0018
   -0.0035    0.0001   -0.0016   -0.0000    0.0003   -0.0003    0.0003
   -0.0000   -0.0022   -0.0000   -0.0008    0.0004    0.0004   -0.0010
   -0.0002    0.0000   -0.0033         0   -0.0002    0.0002    0.0004
    0.0000    0.0041   -0.0000    0.0020    0.0029    0.0029    0.0030
   -1.1439    0.1058    1.5130   -0.8675   -0.6054   -0.0254   -0.0045
    1.1439    0.1062   -1.5130   -0.8675   -0.0254   -0.6054   -0.0055
   -0.9608    0.1279   -1.4391   -0.7291   -0.0036   -0.0035   -0.6958
    0.9608    0.1282    1.4391   -0.7291   -0.0035   -0.0036   -0.0029

 Columns 8 through 14

         0         0         0    0.0001         0         0         0
         0         0         0         0    0.0001         0         0
         0         0         0         0         0    0.0001         0
         0         0         0         0         0         0    0.0001
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
    0.0011   -0.0000         0         0         0         0         0
    0.0018   -0.0000   -0.0003         0    0.0001         0         0
   -0.0003    0.0000   -0.0000         0         0         0         0
   -0.0010   -0.0000    0.0001         0   -0.0003         0         0
   -0.0004    0.0000   -0.0000         0         0         0         0
    0.0030    0.0000   -0.0001         0    0.0000         0         0
   -0.0055   -0.0000   -0.0026         0   -0.0038         0         0
   -0.0045    0.0000   -0.0026         0   -0.0038         0         0
   -0.0029   -0.0000   -0.0032         0    0.0050         0         0
   -0.6958    0.0000   -0.0032         0    0.0050         0         0

 Columns 15 through 18

         0         0         0         0
         0         0         0         0
         0         0         0         0
         0         0         0         0
    0.0001         0         0         0
         0    0.0001         0         0
         0         0    0.0001         0
         0         0         0    0.0001
   -0.0000    0.0000   -0.0000    0.0000
    0.0000    0.0000    0.0000    0.0000
    0.0000   -0.0000    0.0001   -0.0001
    0.0000    0.0000   -0.0000   -0.0000
   -0.0000    0.0000    0.0001   -0.0001
    0.0001    0.0001    0.0002    0.0002
   -0.0033   -0.0000   -0.0003   -0.0003
   -0.0000   -0.0033   -0.0003   -0.0003
   -0.0001   -0.0001   -0.0054    0.0000
   -0.0001   -0.0001    0.0000   -0.0054

>> who

```



```
Your variables are:

LO10_a    LO15_a    LO20_a    LO40_a    LO5_a
LO10_x    LO15_x    LO20_x    LO40_x    LO5_x

>> load LW5
>> load LW10
>> load LW15
>> load LW20
>> load LW40
>> who

Your variables are:

LO10_a    LO20_a    LO5_a     LW15_a    LW40_a
LO10_x    LO20_x    LO5_x     LW15_x    LW40_x
LO15_a    LO40_a    LW10_a    LW20_a    LW5_a
LO15_x    LO40_x    LW10_x    LW20_x    LW5_x

>> LW5_x

LW5_x =

FTL(1)
FTL(2)
FTL(3)
FTL(4)
Q(3)
Q(4)
Q(5)
Q(6)
Q(7)
Q(8)
Q(9)
Q(10)
U(1)
U(2)
U(3)
U(4)
U(5)
U(6)
U(7)
U(8)
U(9)
U(10)

>> LW5_a


LW5_a =

  Columns 1 through 6

  -1.6667e+01            0            0            0            0   1.1000e+06
            0  -1.6667e+01            0            0            0   1.1000e+06
            0            0  -1.6667e+01            0            0   1.1667e+06
            0            0            0  -1.6667e+01            0   1.1667e+06
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0  -3.0650e+01  -5.5879e-05
```

```
  4.1910e-04   4.1910e-04   5.6694e-04   5.6694e-04            0   1.8968e-02
           0            0            0            0  -3.4514e+01   1.1995e-02
  2.8408e-04   2.8408e-04  -4.3540e-04  -4.3540e-04  -2.5687e-06  -4.9977e-03
           0            0  -9.3132e-06  -9.3132e-06  -1.8130e+00   2.7940e-05
 -3.7244e-05  -3.7247e-05  -9.3157e-06  -9.3157e-06   3.7198e-05   7.5344e-02
           0            0  -5.9605e-04  -5.9605e-04  -1.1439e+04  -1.5497e-01
           0            0  -2.9802e-04  -2.9802e-04   1.1439e+04  -1.0461e-01
           0            0            0            0  -9.6075e+03  -1.1861e-01
           0            0            0            0   9.6075e+03  -8.3148e-02

  Columns 7 through 12

           0            0            0            0            0            0
           0            0            0            0            0            0
           0            0            0            0            0            0
           0            0            0            0            0            0
           0            0            0            0            0            0
           0            0            0            0            0            0
           0            0            0            0            0            0
           0            0            0            0            0            0
           0            0            0            0            0            0
           0            0            0            0            0            0
           0            0            0            0            0            0
           0            0            0            0            0            0
 -3.4007e+01  -1.3970e-05   3.4010e-01  -3.4010e-01  -1.0508e+01   1.0508e+01
  1.1642e-06   6.0285e+01   6.3465e+00   6.3465e+00   1.8332e+01   1.8332e+01
 -1.6016e+01   3.0175e-03   2.8512e+00  -2.8497e+00   3.4659e+00  -3.4658e+00
  1.7473e-06  -7.6622e+00   4.4477e+00   4.4477e+00  -1.0485e+01  -1.0485e+01
 -3.3433e+01            0  -1.5894e+00   1.5894e+00   4.4000e+00  -4.4000e+00
  1.8647e-05   1.9681e+01   2.9176e+01   2.9176e+01   2.9567e+01   2.9567e+01
  1.5130e+04  -8.6747e+03  -6.0540e+03  -2.5429e+02  -4.5373e+01  -5.4658e+01
 -1.5130e+04  -8.6747e+03  -2.5428e+02  -6.0540e+03  -5.4659e+01  -4.5373e+01
 -1.4391e+04  -7.2914e+03  -3.5886e+01  -3.4806e+01  -6.9574e+03  -2.8628e+01
  1.4391e+04  -7.2914e+03  -3.4803e+01  -3.5885e+01  -2.8628e+01  -6.9574e+03

  Columns 13 through 18

           0  -2.2000e+05            0  -2.9040e+05            0            0
           0  -2.2000e+05            0  -2.9040e+05            0            0
           0  -2.3333e+05            0   3.5000e+05            0            0
           0  -2.3333e+05            0   3.5000e+05            0            0
           0            0   1.0000e+00            0            0            0
           0            0            0   1.0000e+00            0            0
           0            0            0            0   1.0000e+00            0
           0            0            0            0            0   1.0000e+00
           0            0            0            0            0            0
           0            0            0            0            0            0
           0            0            0            0            0            0
           0            0            0            0            0            0
 -2.7055e-03   1.3970e-05            0            0            0            0
  2.3283e-07  -5.4471e-03            0            0            0            0
  2.4140e-03  -3.0175e-03            0            0            0            0
  1.1857e-07   9.9820e-04            0            0            0            0
  9.2573e-04  -9.3132e-06            0            0            0            0
  5.5879e-06  -1.4110e-02            0            0            0            0
 -2.5630e-03   3.0398e-02            0            0            0            0
  2.4438e-03   1.7881e-02            0            0            0            0
 -6.0797e-03   2.3842e-02            0            0            0            0
  6.0201e-03   1.4603e-02            0            0            0            0

  Columns 19 through 22

           0            0            0            0
           0            0            0            0
           0            0            0            0
           0            0            0            0
```



```
           0            0            0            0
           0            0            0            0
           0            0            0            0
           0            0            0            0
  1.0000e+00            0            0            0
           0   1.0000e+00            0            0
           0            0   1.0000e+00            0
           0            0            0   1.0000e+00
 -9.2667e-03   9.2620e-03  -6.5654e-02   6.5654e-02
  2.8012e-02   2.8012e-02   1.1801e-01   1.1801e-01
  3.7573e-01  -3.7577e-01   7.3854e-01  -7.3858e-01
  2.9224e-02   2.9224e-02  -9.1057e-02  -9.1055e-02
 -2.6416e-01   2.6416e-01   5.9737e-01  -5.9738e-01
  9.7559e-01   9.7560e-01   1.9293e+00   1.9292e+00
 -3.2975e+01  -1.3739e-01  -3.1009e+00  -3.2750e+00
 -1.3769e-01  -3.2975e+01  -3.2747e+00  -3.1009e+00
 -1.1241e+00  -1.2067e+00  -5.4258e+01   2.8908e-02
 -1.2067e+00  -1.1244e+00   2.8014e-02  -5.4257e+01

>> LW10_x

LW10_x =

FTL(1)
FTL(2)
FTL(3)
FTL(4)
Q(3)
Q(4)
Q(5)
Q(6)
Q(7)
Q(8)
Q(9)
Q(10)
U(1)
U(2)
U(3)
U(4)
U(5)
U(6)
U(7)
U(8)
U(9)
U(10)

>> LW10_a

LW10_a =

  Columns 1 through 6

 -3.3333e+01            0            0            0            0   2.2000e+06
           0  -3.3333e+01            0            0            0   2.2000e+06
           0            0  -3.3333e+01            0            0   2.3333e+06
           0            0            0  -3.3333e+01            0   2.3333e+06
           0            0            0            0            0            0
           0            0            0            0            0            0
           0            0            0            0            0            0
           0            0            0            0            0            0
           0            0            0            0            0            0
           0            0            0            0            0            0
           0            0            0            0            0            0
           0            0            0            0            0            0
           0            0            0            0  -3.0650e+01  -2.1420e-04
  4.1910e-04   4.1910e-04   5.6811e-04   5.6811e-04   2.9104e-07   8.0232e-02
```

```
             0            0            0            0  -3.4514e+01   5.7183e-02
    2.8523e-04   2.8524e-04  -4.3657e-04  -4.3657e-04  -4.7244e-07  -1.9984e-02
             0            0            0            0  -1.8130e+00   1.7323e-03
   -3.7247e-05  -3.7244e-05  -1.8635e-05  -1.8635e-05   4.6266e-06   2.9655e-01
             0            0  -2.9802e-04  -2.9802e-04  -1.1439e+04  -6.2555e-01
             0            0  -2.9802e-04  -2.9802e-04   1.1439e+04  -3.9786e-01
             0            0            0            0  -9.6075e+03  -4.8459e-01
             0            0            0            0   9.6075e+03  -3.1054e-01

  Columns 7 through 12

             0            0            0            0            0            0
             0            0            0            0            0            0
             0            0            0            0            0            0
             0            0            0            0            0            0
             0            0            0            0            0            0
             0            0            0            0            0            0
             0            0            0            0            0            0
             0            0            0            0            0            0
             0            0            0            0            0            0
             0            0            0            0            0            0
             0            0            0            0            0            0
             0            0            0            0            0            0
   -3.3988e+01  -1.8626e-05   3.4008e-01  -3.4010e-01  -1.0508e+01   1.0508e+01
   -1.1642e-06   6.0266e+01   6.3463e+00   6.3463e+00   1.8332e+01   1.8332e+01
   -1.5994e+01   3.0547e-03   2.8512e+00  -2.8497e+00   3.4660e+00  -3.4661e+00
   -3.3528e-06  -7.6553e+00   4.4475e+00   4.4475e+00  -1.0485e+01  -1.0485e+01
   -3.3433e+01   9.3132e-06  -1.5892e+00   1.5892e+00   4.4003e+00  -4.4004e+00
   -3.7166e-05   1.9681e+01   2.9176e+01   2.9176e+01   2.9567e+01   2.9567e+01
    1.5130e+04  -8.6747e+03  -6.0540e+03  -2.5429e+02  -4.5373e+01  -5.4659e+01
   -1.5130e+04  -8.6747e+03  -2.5428e+02  -6.0540e+03  -5.4658e+01  -4.5372e+01
   -1.4391e+04  -7.2914e+03  -3.5887e+01  -3.4805e+01  -6.9574e+03  -2.8626e+01
    1.4391e+04  -7.2914e+03  -3.4802e+01  -3.5885e+01  -2.8626e+01  -6.9574e+03

  Columns 13 through 18

             0  -2.2000e+05            0  -2.9040e+05            0            0
             0  -2.2000e+05            0  -2.9040e+05            0            0
             0  -2.3333e+05            0   3.5000e+05            0            0
             0  -2.3333e+05            0   3.5000e+05            0            0
             0            0   1.0000e+00            0            0            0
             0            0            0   1.0000e+00            0            0
             0            0            0            0   1.0000e+00            0
             0            0            0            0            0   1.0000e+00
             0            0            0            0            0            0
             0            0            0            0            0            0
             0            0            0            0            0            0
             0            0            0            0            0            0
   -5.4119e-03   1.8626e-05            0            0            0            0
             0  -1.0894e-02            0            0            0            0
    5.1372e-03  -3.0175e-03            0            0            0            0
    3.6271e-09   1.9953e-03            0            0            0            0
    1.8496e-03            0            0            0            0            0
   -9.3037e-07  -2.8210e-02            0            0            0            0
   -5.6326e-03   5.5134e-02            0            0            0            0
    5.6624e-03   4.2617e-02            0            0            0            0
   -1.2428e-02   4.2617e-02            0            0            0            0
    1.2428e-02   3.3081e-02            0            0            0            0

  Columns 19 through 22

             0            0            0            0
             0            0            0            0
             0            0            0            0
             0            0            0            0
             0            0            0            0
```



```
             0            0            0            0
             0            0            0            0
             0            0            0            0
    1.0000e+00            0            0            0
             0   1.0000e+00            0            0
             0            0   1.0000e+00            0
             0            0            0   1.0000e+00
   -9.2667e-03   9.2667e-03  -6.5658e-02   6.5658e-02
    2.8011e-02   2.8011e-02   1.1801e-01   1.1801e-01
    3.7570e-01  -3.7570e-01   7.3858e-01  -7.3861e-01
    2.9224e-02   2.9224e-02  -9.1055e-02  -9.1056e-02
   -2.6417e-01   2.6416e-01   5.9740e-01  -5.9739e-01
    9.7555e-01   9.7557e-01   1.9292e+00   1.9293e+00
   -3.2975e+01  -1.3739e-01  -3.1012e+00  -3.2747e+00
   -1.3739e-01  -3.2975e+01  -3.2750e+00  -3.1015e+00
   -1.1235e+00  -1.2070e+00  -5.4258e+01   2.9206e-02
   -1.2070e+00  -1.1238e+00   2.9206e-02  -5.4258e+01

>> LW15_x

LW15_x =

FTL(1)
FTL(2)
FTL(3)
FTL(4)
Q(3)
Q(4)
Q(5)
Q(6)
Q(7)
Q(8)
Q(9)
Q(10)
U(1)
U(2)
U(3)
U(4)
U(5)
U(6)
U(7)
U(8)
U(9)
U(10)

>> LW15_a

LW15_a =

  Columns 1 through 6

   -5.0000e+01            0            0            0            0   3.3000e+06
             0  -5.0000e+01            0            0            0   3.3000e+06
             0            0  -5.0000e+01            0            0   3.5000e+06
             0            0            0  -5.0000e+01            0   3.5000e+06
             0            0            0            0            0            0
             0            0            0            0            0            0
             0            0            0            0            0            0
             0            0            0            0            0            0
             0            0            0            0            0            0
             0            0            0            0            0            0
             0            0            0            0            0            0
             0            0            0            0            0            0
             0            0            0            0  -3.0650e+01  -4.0978e-04
    4.1910e-04   4.1910e-04   5.6694e-04   5.6694e-04  -1.1642e-06   1.8229e-01
             0            0   3.7253e-05   3.7253e-05  -3.4511e+01   1.2333e-01
```

```
     2.8468e-04   2.8468e-04  -4.3596e-04  -4.3596e-04  -7.9649e-07  -4.4968e-02
              0            0   1.8626e-05   1.8626e-05  -1.8146e+00   6.6683e-03
    -3.7249e-05  -3.7245e-05  -2.7940e-05  -2.7940e-05  -1.8694e-05   6.6859e-01
              0            0  -2.9802e-04  -2.9802e-04  -1.1439e+04  -1.3918e+00
              0            0  -2.9802e-04  -2.9802e-04   1.1439e+04  -9.1791e-01
              0            0            0            0  -9.6075e+03  -1.0905e+00
              0            0            0            0   9.6075e+03  -7.0244e-01

  Columns 7 through 12

              0            0            0            0            0            0
              0            0            0            0            0            0
              0            0            0            0            0            0
              0            0            0            0            0            0
              0            0            0            0            0            0
              0            0            0            0            0            0
              0            0            0            0            0            0
              0            0            0            0            0            0
              0            0            0            0            0            0
              0            0            0            0            0            0
              0            0            0            0            0            0
              0            0            0            0            0            0
    -3.3957e+01   1.8626e-05   3.4003e-01  -3.4003e-01  -1.0508e+01   1.0508e+01
    -1.1642e-06   6.0234e+01   6.3459e+00   6.3459e+00   1.8331e+01   1.8331e+01
    -1.5961e+01  -3.0361e-03   2.8511e+00  -2.8525e+00   3.4636e+00  -3.4636e+00
     2.3234e-07  -7.6439e+00   4.4472e+00   4.4472e+00  -1.0484e+01  -1.0484e+01
    -3.3434e+01  -1.8626e-05  -1.5904e+00   1.5904e+00   4.4026e+00  -4.4026e+00
    -1.8565e-05   1.9681e+01   2.9175e+01   2.9175e+01   2.9566e+01   2.9566e+01
     1.5130e+04  -8.6747e+03  -6.0539e+03  -2.5427e+02  -4.5361e+01  -5.4670e+01
    -1.5130e+04  -8.6747e+03  -2.5428e+02  -6.0540e+03  -5.4670e+01  -4.5361e+01
    -1.4391e+04  -7.2914e+03  -3.5883e+01  -3.4803e+01  -6.9574e+03  -2.8623e+01
     1.4391e+04  -7.2914e+03  -3.4804e+01  -3.5886e+01  -2.8624e+01  -6.9574e+03

  Columns 13 through 18

              0  -2.2000e+05            0  -2.9040e+05            0            0
              0  -2.2000e+05            0  -2.9040e+05            0            0
              0  -2.3333e+05            0   3.5000e+05            0            0
              0  -2.3333e+05            0   3.5000e+05            0            0
              0            0   1.0000e+00            0            0            0
              0            0            0   1.0000e+00            0            0
              0            0            0            0   1.0000e+00            0
              0            0            0            0            0   1.0000e+00
              0            0            0            0            0            0
              0            0            0            0            0            0
              0            0            0            0            0            0
              0            0            0            0            0            0
    -8.1162e-03   3.7253e-05            0            0            0            0
    -2.3283e-07  -1.6346e-02            0            0            0            0
     7.6716e-03  -9.0152e-03            0            0            0            0
     3.9492e-08   2.9954e-03            0            0            0            0
     2.6685e-03  -1.8626e-05            0            0            0            0
    -2.4830e-06  -4.2357e-02            0            0            0            0
    -8.7221e-03   9.1791e-02            0            0            0            0
     8.7420e-03   5.4538e-02            0            0            0            0
    -1.8318e-02   7.0930e-02            0            0            0            0
     1.8378e-02   4.4405e-02            0            0            0            0

  Columns 19 through 22

              0            0            0            0
              0            0            0            0
              0            0            0            0
              0            0            0            0
              0            0            0            0
              0            0            0            0
```



```
              0            0            0            0
              0            0            0            0
     1.0000e+00            0            0            0
              0   1.0000e+00            0            0
              0            0   1.0000e+00            0
              0            0            0   1.0000e+00
    -9.2387e-03   9.2387e-03  -6.5640e-02   6.5640e-02
     2.8011e-02   2.8012e-02   1.1801e-01   1.1801e-01
     3.8177e-01  -3.8181e-01   7.3558e-01  -7.3558e-01
     2.9225e-02   2.9224e-02  -9.1057e-02  -9.1057e-02
    -2.6256e-01   2.6256e-01   5.9735e-01  -5.9737e-01
     9.7555e-01   9.7557e-01   1.9293e+00   1.9293e+00
    -3.2983e+01  -1.2934e-01  -3.0950e+00  -3.2809e+00
    -1.2934e-01  -3.2984e+01  -3.2809e+00  -3.0950e+00
    -1.1364e+00  -1.1939e+00  -5.4253e+01   2.4438e-02
    -1.1939e+00  -1.1370e+00   2.4140e-02  -5.4253e+01

>> LW20_x

LW20_x =

FTL(1)
FTL(2)
FTL(3)
FTL(4)
Q(3)
Q(4)
Q(5)
Q(6)
Q(7)
Q(8)
Q(9)
Q(10)
U(1)
U(2)
U(3)
U(4)
U(5)
U(6)
U(7)
U(8)
U(9)
U(10)

>> LW20_a

LW20_a =

  Columns 1 through 6

    -6.6667e+01            0            0            0            0   4.4000e+06
              0  -6.6667e+01            0            0            0   4.4000e+06
              0            0  -6.6667e+01            0            0   4.6667e+06
              0            0            0  -6.6667e+01            0   4.6667e+06
              0            0            0            0            0            0
              0            0            0            0            0            0
              0            0            0            0            0            0
              0            0            0            0            0            0
              0            0            0            0            0            0
              0            0            0            0            0            0
              0            0            0            0            0            0
              0            0            0            0            0            0
              0            0            0            0  -3.0650e+01  -7.8231e-04
     4.1910e-04   4.1910e-04   5.6927e-04   5.6927e-04   1.4552e-07   3.2512e-01
              0            0   9.3132e-06   9.3132e-06  -3.4511e+01   2.1954e-01
     2.8523e-04   2.8522e-04  -4.3598e-04  -4.3598e-04  -3.7446e-07  -7.9945e-02
```

```
           0             0    1.8626e-05    1.8626e-05   -1.8130e+00    1.1623e-02
 -3.7249e-05   -3.7251e-05   -6.3949e-09   -6.3949e-09    1.6637e-06    1.1912e+00
           0             0   -2.9802e-04   -2.9802e-04   -1.1439e+04   -2.4799e+00
           0             0   -5.9605e-04   -5.9605e-04    1.1439e+04   -1.6335e+00
           0             0   -5.9605e-04   -5.9605e-04   -9.6075e+03   -1.9398e+00
           0             0             0             0    9.6075e+03   -1.2517e+00

 Columns 7 through 12

           0             0             0             0             0             0
           0             0             0             0             0             0
           0             0             0             0             0             0
           0             0             0             0             0             0
           0             0             0             0             0             0
           0             0             0             0             0             0
           0             0             0             0             0             0
           0             0             0             0             0             0
           0             0             0             0             0             0
           0             0             0             0             0             0
           0             0             0             0             0             0
           0             0             0             0             0             0
 -3.3912e+01             0    3.4001e-01   -3.4001e-01   -1.0508e+01    1.0508e+01
 -4.6566e-06    6.0190e+01    6.3453e+00    6.3453e+00    1.8329e+01    1.8329e+01
 -1.5912e+01   -2.9942e-03    2.8509e+00   -2.8524e+00    3.4643e+00   -3.4643e+00
  1.6734e-06   -7.6278e+00    4.4467e+00    4.4467e+00   -1.0483e+01   -1.0483e+01
 -3.3433e+01             0   -1.5884e+00    1.5884e+00    4.4020e+00   -4.4020e+00
 -9.3090e-05    1.9681e+01    2.9174e+01    2.9174e+01    2.9566e+01    2.9566e+01
  1.5130e+04   -8.6747e+03   -6.0539e+03   -2.5428e+02   -4.5364e+01   -5.4666e+01
 -1.5130e+04   -8.6747e+03   -2.5428e+02   -6.0539e+03   -5.4666e+01   -4.5363e+01
 -1.4391e+04   -7.2914e+03   -3.5886e+01   -3.4796e+01   -6.9574e+03   -2.8622e+01
  1.4391e+04   -7.2914e+03   -3.4799e+01   -3.5888e+01   -2.8622e+01   -6.9574e+03

 Columns 13 through 18

           0   -2.2000e+05             0   -2.9040e+05             0             0
           0   -2.2000e+05             0   -2.9040e+05             0             0
           0   -2.3333e+05             0    3.5000e+05             0             0
           0   -2.3333e+05             0    3.5000e+05             0             0
           0             0    1.0000e+00             0             0             0
           0             0             0    1.0000e+00             0             0
           0             0             0             0    1.0000e+00             0
           0             0             0             0             0    1.0000e+00
           0             0             0             0             0             0
           0             0             0             0             0             0
           0             0             0             0             0             0
           0             0             0             0             0             0
 -1.0820e-02    7.4506e-05             0             0             0             0
 -1.1642e-07   -2.1707e-02             0             0             0             0
  1.0143e-02   -1.2047e-02             0             0             0             0
  8.6408e-08    3.9995e-03             0             0             0             0
  3.6974e-03   -1.8626e-05             0             0             0             0
 -4.6564e-07   -6.1262e-02             0             0             0             0
 -1.1101e-02    1.3083e-01             0             0             0             0
  1.1086e-02    8.0764e-02             0             0             0             0
 -2.4647e-02    9.9540e-02             0             0             0             0
  2.4676e-02    6.3777e-02             0             0             0             0

 Columns 19 through 22

           0             0             0             0
           0             0             0             0
           0             0             0             0
           0             0             0             0
           0             0             0             0
           0             0             0             0
           0             0             0             0
```



```
           0             0             0             0
  1.0000e+00             0             0             0
           0    1.0000e+00             0             0
           0             0    1.0000e+00             0
           0             0             0    1.0000e+00
 -9.2387e-03    9.2760e-03   -6.5640e-02    6.5640e-02
  2.8011e-02    2.8010e-02    1.1801e-01    1.1801e-01
  3.8176e-01   -3.8179e-01    7.3558e-01   -7.3558e-01
  2.9223e-02    2.9225e-02   -9.1055e-02   -9.1056e-02
 -2.6414e-01    2.6414e-01    5.9739e-01   -5.9739e-01
  9.7560e-01    9.7558e-01    1.9293e+00    1.9293e+00
 -3.2987e+01   -1.2517e-01   -3.0953e+00   -3.2809e+00
 -1.2517e-01   -3.2988e+01   -3.2812e+00   -3.0950e+00
 -1.1331e+00   -1.1975e+00   -5.4254e+01    2.4438e-02
 -1.1984e+00   -1.1331e+00    2.4438e-02   -5.4254e+01

>> LW40_x

LW40_x =

FTL(1)
FTL(2)
FTL(3)
FTL(4)
Q(3)
Q(4)
Q(5)
Q(6)
Q(7)
Q(8)
Q(9)
Q(10)
U(1)
U(2)
U(3)
U(4)
U(5)
U(6)
U(7)
U(8)
U(9)
U(10)

>> LW40_a

LW40_a =

  Columns 1 through 6

 -1.3333e+02             0             0             0             0    8.8000e+06
           0   -1.3333e+02             0             0             0    8.8000e+06
           0             0   -1.3333e+02             0             0    9.3333e+06
           0             0             0   -1.3333e+02             0    9.3333e+06
           0             0             0             0             0             0
           0             0             0             0             0             0
           0             0             0             0             0             0
           0             0             0             0             0             0
           0             0             0             0             0             0
           0             0             0             0             0             0
           0             0             0             0             0             0
           0             0             0             0             0             0
           0             0             0             0   -3.0651e+01   -3.1292e-03
  4.1910e-04    4.1910e-04    5.6578e-04    5.6578e-04   -5.8208e-06    1.3049e+00
           0             0             0             0   -3.4511e+01    8.7827e-01
  2.8409e-04    2.8409e-04   -4.3655e-04   -4.3655e-04   -1.9220e-06   -3.1976e-01
           0             0             0             0   -1.8147e+00    4.3362e-02
```

```
  -3.7246e-05  -3.7247e-05  -3.7260e-05  -3.7260e-05   6.5141e-05   4.7551e+00
            0            0            0            0  -1.1439e+04  -9.9117e+00
            0            0            0            0   1.1439e+04  -6.5096e+00
            0            0            0            0  -9.6075e+03  -7.7409e+00
            0            0            0            0   9.6075e+03  -5.0032e+00

  Columns 7 through 12

            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
  -3.3608e+01            0   3.3945e-01  -3.3945e-01  -1.0510e+01   1.0510e+01
  -1.1642e-06   5.9886e+01   6.3414e+00   6.3414e+00   1.8320e+01   1.8320e+01
  -1.5592e+01  -2.9802e-03   2.8466e+00  -2.8481e+00   3.4691e+00  -3.4691e+00
  -6.1340e-07  -7.5178e+00   4.4438e+00   4.4438e+00  -1.0475e+01  -1.0475e+01
  -3.3433e+01            0  -1.5866e+00   1.5865e+00   4.4085e+00  -4.4086e+00
  -1.6269e-05   1.9681e+01   2.9165e+01   2.9165e+01   2.9563e+01   2.9563e+01
   1.5130e+04  -8.6747e+03  -6.0537e+03  -2.5428e+02  -4.5351e+01  -5.4668e+01
  -1.5130e+04  -8.6747e+03  -2.5428e+02  -6.0537e+03  -5.4668e+01  -4.5350e+01
  -1.4391e+04  -7.2914e+03  -3.5872e+01  -3.4789e+01  -6.9576e+03  -2.8591e+01
   1.4391e+04  -7.2914e+03  -3.4792e+01  -3.5875e+01  -2.8591e+01  -6.9576e+03

  Columns 13 through 18

            0  -2.2000e+05            0  -2.9040e+05            0            0
            0  -2.2000e+05            0  -2.9040e+05            0            0
            0  -2.3333e+05            0   3.5000e+05            0            0
            0  -2.3333e+05            0   3.5000e+05            0            0
            0            0   1.0000e+00            0            0            0
            0            0            0   1.0000e+00            0            0
            0            0            0            0   1.0000e+00            0
            0            0            0            0            0   1.0000e+00
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
  -2.1651e-02   1.4901e-04            0            0            0            0
  -5.8208e-08  -4.3500e-02            0            0            0            0
   2.0288e-02  -2.1011e-02            0            0            0            0
  -1.0759e-10   7.9932e-03            0            0            0            0
   7.3165e-03  -1.6391e-03            0            0            0            0
   8.8596e-11  -1.1764e-01            0            0            0            0
  -2.2389e-02   2.4259e-01            0            0            0            0
   2.2387e-02   1.6391e-01            0            0            0            0
  -4.9189e-02   1.9133e-01            0            0            0            0
   4.9189e-02   1.2457e-01            0            0            0            0

  Columns 19 through 22

            0            0            0            0
            0            0            0            0
            0            0            0            0
            0            0            0            0
            0            0            0            0
            0            0            0            0
            0            0            0            0
            0            0            0            0
```



```
   1.0000e+00            0            0            0
            0   1.0000e+00            0            0
            0            0   1.0000e+00            0
            0            0            0   1.0000e+00
  -9.3877e-03   9.3877e-03  -6.5714e-02   6.5714e-02
   2.8007e-02   2.8007e-02   1.1801e-01   1.1801e-01
   3.7864e-01  -3.7864e-01   7.3537e-01  -7.3552e-01
   2.9226e-02   2.9224e-02  -9.1056e-02  -9.1056e-02
  -2.6256e-01   2.6256e-01   5.9739e-01  -5.9739e-01
   9.7552e-01   9.7554e-01   1.9293e+00   1.9293e+00
  -3.2977e+01  -1.3538e-01  -3.0951e+00  -3.2809e+00
  -1.3545e-01  -3.2977e+01  -3.2811e+00  -3.0951e+00
  -1.1307e+00  -1.1981e+00  -5.4253e+01   2.4438e-02
  -1.1986e+00  -1.1319e+00   2.3842e-02  -5.4253e+01

>> diary off
>> NO5_x

NO5_x =

Q(3)
Q(4)
Q(5)
Q(6)
Q(7)
Q(8)
Q(9)
Q(10)
U(1)
U(2)
U(3)
U(4)
U(5)
U(6)
U(7)
U(8)
U(9)
U(10)

>> diary off
>> NO5_a

NO5_a =

   1.0e+04 *

  Columns 1 through 7

         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
   -0.0031    0.0000   -0.0034   -0.0000    0.0000   -0.0000   -0.0011
    0.0000    0.0120   -0.0000    0.0061    0.0007    0.0007    0.0018
   -0.0022   -0.0001   -0.0032    0.0000    0.0006   -0.0006    0.0003
    0.0000   -0.0018   -0.0000   -0.0007    0.0005    0.0005   -0.0010
   -0.0033    0.0002   -0.0022         0   -0.0006    0.0006    0.0001
   -0.0000    0.0037   -0.0000    0.0043    0.0037    0.0037    0.0030
   -1.2031    0.1172    1.5831   -0.9076   -0.6308   -0.0248   -0.0055
    1.2031    0.1083   -1.5831   -0.9076   -0.0248   -0.6308   -0.0045
   -0.9555    0.1179   -1.4393   -0.7320   -0.0038   -0.0050   -0.6950
    0.9555    0.1187    1.4393   -0.7320   -0.0050   -0.0038   -0.0036
```

```

  Columns 8 through 14

         0         0         0    0.0001         0         0         0
         0         0         0         0    0.0001         0         0
         0         0         0         0         0    0.0001         0
         0         0         0         0         0         0    0.0001
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
    0.0011   -0.0000   -0.0000         0   -0.0000         0         0
    0.0018         0   -0.0024         0    0.0006         0         0
   -0.0003    0.0000    0.0000         0    0.0000         0         0
   -0.0010    0.0000    0.0004         0   -0.0026         0         0
   -0.0001    0.0000   -0.0000         0   -0.0000         0         0
    0.0030    0.0000   -0.0007         0    0.0003         0         0
   -0.0045   -0.0000   -0.0234         0   -0.0328         0         0
   -0.0055    0.0000   -0.0217         0   -0.0306         0         0
   -0.0036   -0.0000   -0.0236         0    0.0368         0         0
   -0.6950    0.0000   -0.0237         0    0.0367         0         0

  Columns 15 through 18

         0         0         0         0
         0         0         0         0
         0         0         0         0
         0         0         0         0
    0.0001         0         0         0
         0    0.0001         0         0
         0         0    0.0001         0
         0         0         0    0.0001
   -0.0000    0.0000   -0.0000    0.0000
    0.0000    0.0000    0.0000    0.0000
    0.0000   -0.0000    0.0001   -0.0001
    0.0000    0.0000   -0.0000   -0.0000
   -0.0000    0.0000    0.0001   -0.0001
    0.0001    0.0001    0.0002    0.0002
   -0.0033   -0.0000   -0.0003   -0.0003
   -0.0000   -0.0033   -0.0003   -0.0003
   -0.0001   -0.0001   -0.0054    0.0000
   -0.0001   -0.0001    0.0000   -0.0054

>> NO10_x

NO10_x =

Q(3)
Q(4)
Q(5)
Q(6)
Q(7)
Q(8)
Q(9)
Q(10)
U(1)
U(2)
U(3)
U(4)
U(5)
U(6)
U(7)
U(8)
U(9)
U(10)

```



```
>> NO10_a

NO10_a =

   1.0e+04 *

  Columns 1 through 7

         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
   -0.0031    0.0000   -0.0034   -0.0000    0.0000   -0.0000   -0.0011
   -0.0000    0.0120    0.0000    0.0061    0.0007    0.0007    0.0018
   -0.0022   -0.0001   -0.0032    0.0000    0.0006   -0.0006    0.0003
   -0.0000   -0.0018    0.0000   -0.0007    0.0005    0.0005   -0.0010
   -0.0033    0.0002   -0.0022   -0.0000   -0.0006    0.0006    0.0001
   -0.0000    0.0037    0.0000    0.0043    0.0037    0.0037    0.0030
   -1.2031    0.1171    1.5831   -0.9076   -0.6308   -0.0248   -0.0055
    1.2031    0.1083   -1.5831   -0.9076   -0.0248   -0.6308   -0.0045
   -0.9555    0.1179   -1.4393   -0.7320   -0.0038   -0.0050   -0.6950
    0.9555    0.1187    1.4393   -0.7320   -0.0050   -0.0038   -0.0036

  Columns 8 through 14

         0         0         0    0.0001         0         0         0
         0         0         0         0    0.0001         0         0
         0         0         0         0         0    0.0001         0
         0         0         0         0         0         0    0.0001
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
    0.0011   -0.0000   -0.0000         0   -0.0000         0         0
    0.0018   -0.0000   -0.0012         0    0.0003         0         0
   -0.0003    0.0000    0.0000         0    0.0000         0         0
   -0.0010   -0.0000    0.0002         0   -0.0013         0         0
   -0.0001    0.0000   -0.0000         0   -0.0000         0         0
    0.0030   -0.0000   -0.0004         0    0.0001         0         0
   -0.0045   -0.0000   -0.0117         0   -0.0164         0         0
   -0.0055    0.0000   -0.0108         0   -0.0153         0         0
   -0.0036   -0.0000   -0.0118         0    0.0184         0         0
   -0.6950    0.0000   -0.0119         0    0.0184         0         0

  Columns 15 through 18

         0         0         0         0
         0         0         0         0
         0         0         0         0
         0         0         0         0
    0.0001         0         0         0
         0    0.0001         0         0
         0         0    0.0001         0
         0         0         0    0.0001
   -0.0000    0.0000   -0.0000    0.0000
    0.0000    0.0000    0.0000    0.0000
    0.0000   -0.0000    0.0001   -0.0001
    0.0000    0.0000   -0.0000   -0.0000
   -0.0000    0.0000    0.0001   -0.0001
    0.0001    0.0001    0.0002    0.0002
   -0.0033   -0.0000   -0.0003   -0.0003
   -0.0000   -0.0033   -0.0003   -0.0003
```

```
    -0.0001   -0.0001   -0.0054    0.0000
    -0.0001   -0.0001    0.0000   -0.0054

>> NO15_x

NO15_x =

Q(3)
Q(4)
Q(5)
Q(6)
Q(7)
Q(8)
Q(9)
Q(10)
U(1)
U(2)
U(3)
U(4)
U(5)
U(6)
U(7)
U(8)
U(9)
U(10)

>> NO15_a

NO15_a =

   1.0e+04 *

  Columns 1 through 7

         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
   -0.0031    0.0000   -0.0034    0.0000    0.0000   -0.0000   -0.0011
    0.0000    0.0120   -0.0000    0.0061    0.0007    0.0007    0.0018
   -0.0022   -0.0001   -0.0032   -0.0000    0.0006   -0.0006    0.0003
   -0.0000   -0.0018    0.0000   -0.0007    0.0005    0.0005   -0.0010
   -0.0033    0.0002   -0.0022         0   -0.0006    0.0006    0.0001
    0.0000    0.0037   -0.0000    0.0043    0.0037    0.0037    0.0030
   -1.2031    0.1171    1.5831   -0.9076   -0.6308   -0.0248   -0.0055
    1.2031    0.1083   -1.5831   -0.9076   -0.0248   -0.6308   -0.0045
   -0.9555    0.1178   -1.4393   -0.7320   -0.0038   -0.0050   -0.6950
    0.9555    0.1186    1.4393   -0.7320   -0.0050   -0.0038   -0.0036

  Columns 8 through 14

         0         0         0    0.0001         0         0         0
         0         0         0         0    0.0001         0         0
         0         0         0         0         0    0.0001         0
         0         0         0         0         0         0    0.0001
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
    0.0011   -0.0000   -0.0000         0   -0.0000         0         0
    0.0018   -0.0000   -0.0008         0    0.0002         0         0
   -0.0003    0.0000    0.0000         0    0.0000         0         0
```



```
   -0.0010   -0.0000    0.0001         0   -0.0009         0         0
   -0.0001    0.0000   -0.0000         0   -0.0000         0         0
    0.0030   -0.0000   -0.0002         0    0.0001         0         0
   -0.0045   -0.0000   -0.0078         0   -0.0109         0         0
   -0.0055    0.0000   -0.0072         0   -0.0102         0         0
   -0.0036   -0.0000   -0.0079         0    0.0122         0         0
   -0.6950    0.0000   -0.0079         0    0.0122         0         0

  Columns 15 through 18

         0         0         0         0
         0         0         0         0
         0         0         0         0
         0         0         0         0
    0.0001         0         0         0
         0    0.0001         0         0
         0         0    0.0001         0
         0         0         0    0.0001
   -0.0000    0.0000   -0.0000    0.0000
    0.0000    0.0000    0.0000    0.0000
    0.0000   -0.0000    0.0001   -0.0001
    0.0000    0.0000   -0.0000   -0.0000
   -0.0000    0.0000    0.0001   -0.0001
    0.0001    0.0001    0.0002    0.0002
   -0.0033   -0.0000   -0.0003   -0.0003
   -0.0000   -0.0033   -0.0003   -0.0003
   -0.0001   -0.0001   -0.0054    0.0000
   -0.0001   -0.0001    0.0000   -0.0054

>> NO20_x

NO20_x =

Q(3)
Q(4)
Q(5)
Q(6)
Q(7)
Q(8)
Q(9)
Q(10)
U(1)
U(2)
U(3)
U(4)
U(5)
U(6)
U(7)
U(8)
U(9)
U(10)

>> NO20_a

NO20_a =

   1.0e+04 *

  Columns 1 through 7

         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
```

```
          0         0         0         0         0         0         0
          0         0         0         0         0         0         0
    -0.0031    0.0000   -0.0034         0    0.0000   -0.0000   -0.0011
          0    0.0120    0.0000    0.0061    0.0007    0.0007    0.0018
    -0.0022   -0.0001   -0.0032   -0.0000    0.0006   -0.0006    0.0003
    -0.0000   -0.0018    0.0000   -0.0007    0.0005    0.0005   -0.0010
    -0.0033    0.0002   -0.0022   -0.0000   -0.0006    0.0006    0.0001
     0.0600    0.0038    0.0000    0.0043    0.0037    0.0037    0.0030
    -1.2031    0.1170    1.5831   -0.9076   -0.6308   -0.0248   -0.0055
     1.2031    0.1082   -1.5831   -0.9076   -0.0248   -0.6308   -0.0045
    -0.9555    0.1177   -1.4393   -0.7320   -0.0038   -0.0050   -0.6950
     0.9555    0.1186    1.4393   -0.7320   -0.0050   -0.0038   -0.0036

  Columns 8 through 14

          0         0         0    0.0001         0         0         0
          0         0         0         0    0.0001         0         0
          0         0         0         0         0    0.0001         0
          0         0         0         0         0         0    0.0001
          0         0         0         0         0         0         0
          0         0         0         0         0         0         0
          0         0         0         0         0         0         0
          0         0         0         0         0         0         0
     0.0011   -0.0000   -0.0000         0   -0.0000         0         0
     0.0018   -0.0000   -0.0006         0    0.0001         0         0
    -0.0003    0.0000    0.0000         0    0.0000         0         0
    -0.0010   -0.0000    0.0001         0   -0.0006         0         0
    -0.0001    0.0000   -0.0000         0   -0.0000         0         0
     0.0030   -0.0000   -0.0002         0    0.0001         0         0
    -0.0045   -0.0000   -0.0059         0   -0.0082         0         0
    -0.0055    0.0000   -0.0054         0   -0.0077         0         0
    -0.0036   -0.0000   -0.0059         0    0.0092         0         0
    -0.6950    0.0000   -0.0059         0    0.0092         0         0

  Columns 15 through 18

          0         0         0         0
          0         0         0         0
          0         0         0         0
          0         0         0         0
     0.0001         0         0         0
          0    0.0001         0         0
          0         0    0.0001         0
          0         0         0    0.0001
    -0.0000    0.0000   -0.0000    0.0000
     0.0000    0.0000    0.0000    0.0000
     0.0000   -0.0000    0.0001   -0.0001
     0.0000    0.0000   -0.0000   -0.0000
    -0.0000    0.0000    0.0001   -0.0001
     0.0001    0.0001    0.0002    0.0002
    -0.0033   -0.0000   -0.0003   -0.0003
    -0.0000   -0.0033   -0.0003   -0.0003
    -0.0001   -0.0001   -0.0054    0.0000
    -0.0001   -0.0001    0.0000   -0.0054

>> NO40_x

NO40_x =

Q(3)
Q(4)
Q(5)
Q(6)
Q(7)
Q(8)
Q(9)
```



```
Q(10)
U(1)
U(2)
U(3)
U(4)
U(5)
U(6)
U(7)
U(8)
U(9)
U(10)

>> NO40_a

NO40_a =

   1.0e+04 *

  Columns 1 through 7

          0         0         0         0         0         0         0
          0         0         0         0         0         0         0
          0         0         0         0         0         0         0
          0         0         0         0         0         0         0
          0         0         0         0         0         0         0
          0         0         0         0         0         0         0
          0         0         0         0         0         0         0
          0         0         0         0         0         0         0
    -0.0031    0.0000   -0.0033         0    0.0000   -0.0000   -0.0011
     0.0000    0.0121         0    0.0061    0.0007    0.0007    0.0018
    -0.0022   -0.0000   -0.0032   -0.0000    0.0006   -0.0006    0.0003
     0.0000   -0.0019   -0.0000   -0.0007    0.0005    0.0005   -0.0010
    -0.0033    0.0002   -0.0022    0.0000   -0.0006    0.0006    0.0001
     0.0000    0.0041   -0.0000    0.0043    0.0037    0.0037    0.0030
    -1.2031    0.1162    1.5831   -0.9076   -0.6308   -0.0248   -0.0055
     1.2031    0.1077   -1.5831   -0.9076   -0.0248   -0.6308   -0.0045
    -0.9555    0.1172   -1.4393   -0.7320   -0.0038   -0.0050   -0.6950
     0.9555    0.1182    1.4393   -0.7320   -0.0050   -0.0038   -0.0036

  Columns 8 through 14

          0         0         0    0.0001         0         0         0
          0         0         0         0    0.0001         0         0
          0         0         0         0         0    0.0001         0
          0         0         0         0         0         0    0.0001
          0         0         0         0         0         0         0
          0         0         0         0         0         0         0
          0         0         0         0         0         0         0
          0         0         0         0         0         0         0
     0.0011   -0.0000   -0.0000         0   -0.0000         0         0
     0.0018    0.0000   -0.0003         0    0.0001         0         0
    -0.0003    0.0000    0.0000         0    0.0000         0         0
    -0.0010   -0.0000    0.0000         0   -0.0003         0         0
    -0.0001    0.0000   -0.0000         0   -0.0000         0         0
     0.0030   -0.0000   -0.0001         0    0.0000         0         0
    -0.0045   -0.0000   -0.0029         0   -0.0041         0         0
    -0.0055    0.0000   -0.0027         0   -0.0038         0         0
    -0.0036   -0.0000   -0.0029         0    0.0046         0         0
    -0.6950    0.0000   -0.0030         0    0.0046         0         0

  Columns 15 through 18

          0         0         0         0
          0         0         0         0
          0         0         0         0
          0         0         0         0
```

```
      0.0001         0         0         0
           0    0.0001         0         0
           0         0    0.0001         0
           0         0         0    0.0001
     -0.0000    0.0000   -0.0000    0.0000
      0.0000    0.0000    0.0000    0.0000
      0.0000   -0.0000    0.0001   -0.0001
      0.0000    0.0000   -0.0000   -0.0000
     -0.0000    0.0000    0.0001   -0.0001
      0.0001    0.0001    0.0002    0.0002
     -0.0033   -0.0000   -0.0003   -0.0003
     -0.0000   -0.0033   -0.0003   -0.0003
     -0.0001   -0.0001   -0.0054    0.0000
     -0.0001   -0.0001    0.0000   -0.0054

>> NW5_x

NW5_x =

FTL(1)
FTL(2)
FTL(3)
FTL(4)
Q(3)
Q(4)
Q(5)
Q(6)
Q(7)
Q(8)
Q(9)
Q(10)
U(1)
U(2)
U(3)
U(4)
U(5)
U(6)
U(7)
U(8)
U(9)
U(10)

>> NW5_a

NW5_a =

  Columns 1 through 6

  -1.6667e+01            0            0            0            0   1.1516e+06
            0  -1.6667e+01            0            0            0   1.1516e+06
            0            0  -1.6667e+01            0            0   1.0894e+06
            0            0            0  -1.6667e+01            0   1.0894e+06
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
   2.6193e-05  -2.7358e-05            0            0  -3.1479e+01   6.0648e-02
   3.5856e-04   3.5972e-04   5.6811e-04   5.6811e-04   1.7462e-06   2.6339e-02
  -8.9407e-04   8.1956e-04            0            0  -2.2283e+01  -9.0741e-01
   2.1068e-04   2.2029e-04  -4.4752e-04  -4.4725e-04   8.9551e-06  -4.9762e-03
   7.4506e-04  -7.4506e-04            0            0  -3.3300e+01   2.2970e+00
  -2.1057e-03  -1.9348e-03  -1.3515e-07  -1.3681e-07  -4.2973e-05   7.5536e-02
```



```
   6.9141e-02            0            0            0  -1.2031e+04   4.4188e+01
            0   6.6757e-02            0            0   1.2031e+04  -4.4458e+01
   2.3842e-03   1.7881e-03            0            0  -9.5554e+03  -3.8803e+00
   2.3842e-03   2.3842e-03            0            0   9.5554e+03   3.6782e+00

  Columns 7 through 12

            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
  -3.3719e+01  -1.5716e-05   1.8552e-01  -1.8552e-01  -1.0591e+01   1.0591e+01
  -6.4028e-06   6.1076e+01   6.6156e+00   6.6156e+00   1.8327e+01   1.8327e+01
  -3.2299e+01   2.9057e-03   5.6822e+00  -5.6806e+00   3.4535e+00  -3.4536e+00
  -3.7605e-05  -6.6767e+00   4.7634e+00   4.7635e+00  -1.0441e+01  -1.0441e+01
  -2.1955e+01            0  -6.3981e+00   6.3980e+00   9.8348e-01  -9.8348e-01
  -2.5541e-04   4.3305e+01   3.6514e+01   3.6515e+01   2.9534e+01   2.9534e+01
   1.5831e+04  -9.0756e+03  -6.3082e+03  -2.4773e+02  -5.4550e+01  -4.5400e+01
  -1.5831e+04  -9.0756e+03  -2.4772e+02  -6.3082e+03  -4.5395e+01  -5.4555e+01
  -1.4393e+04  -7.3197e+03  -3.8078e+01  -5.0112e+01  -6.9498e+03  -3.6166e+01
   1.4393e+04  -7.3196e+03  -5.0111e+01  -3.8078e+01  -3.6166e+01  -6.9498e+03

  Columns 13 through 18

            0  -2.3032e+05            0  -3.0402e+05            0            0
            0  -2.3032e+05            0  -3.0402e+05            0            0
            0  -2.1788e+05            0   3.2683e+05            0            0
            0  -2.1788e+05            0   3.2683e+05            0            0
            0            0   1.0000e+00            0            0            0
            0            0            0   1.0000e+00            0            0
            0            0            0            0   1.0000e+00            0
            0            0            0            0            0   1.0000e+00
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
  -2.7048e-03  -1.2157e-02            0  -6.4471e-03            0            0
            0  -5.4459e-03            0   9.3132e-06            0            0
   2.4438e-03   1.8038e-01            0   2.4371e-01            0            0
   1.5678e-08   1.0079e-03            0   1.3652e-05            0            0
   9.5367e-04  -4.6074e-01            0  -1.9088e-01            0            0
   6.1308e-09  -1.4082e-02            0   2.0853e-04            0            0
  -2.8610e-03  -8.8239e+00            0  -1.0560e+01            0            0
   2.8610e-03   8.8668e+00            0   1.0552e+01            0            0
  -5.9605e-03   7.7963e-01            0   7.0333e-02            0            0
   5.9605e-03  -7.4089e-01            0  -7.1526e-02            0            0

  Columns 19 through 22

            0            0            0            0
            0            0            0            0
            0            0            0            0
            0            0            0            0
            0            0            0            0
            0            0            0            0
            0            0            0            0
            0            0            0            0
   1.0000e+00            0            0            0
```

```
                0   1.0000e+00            0            0
                0            0   1.0000e+00            0
                0            0            0   1.0000e+00
      -9.2585e-03   9.2591e-03  -6.5693e-02   6.5691e-02
       2.8025e-02   2.8018e-02   1.1801e-01   1.1802e-01
       3.7573e-01  -3.7558e-01   7.3850e-01  -7.3858e-01
       2.9238e-02   2.9228e-02  -9.1055e-02  -9.1066e-02
      -2.6405e-01   2.6405e-01   5.9560e-01  -5.9560e-01
       9.7577e-01   9.7577e-01   1.9293e+00   1.9295e+00
      -3.2980e+01  -1.4305e-01  -3.1066e+00  -3.2735e+00
      -1.3828e-01  -3.2973e+01  -3.2735e+00  -3.1090e+00
      -1.1247e+00  -1.2070e+00  -5.4254e+01   2.3842e-02
      -1.2076e+00  -1.1247e+00   2.4438e-02  -5.4254e+01

>> NW10_x

NW10_x =

FTL(1)
FTL(2)
FTL(3)
FTL(4)
Q(3)
Q(4)
Q(5)
Q(6)
Q(7)
Q(8)
Q(9)
Q(10)
U(1)
U(2)
U(3)
U(4)
U(5)
U(6)
U(7)
U(8)
U(9)
U(10)

>> NW10_a

NW10_a =

  Columns 1 through 6

  -3.3333e+01            0            0            0            0   2.3032e+06
            0  -3.3333e+01            0            0            0   2.3032e+06
            0            0  -3.3333e+01            0            0   2.1788e+06
            0            0            0  -3.3333e+01            0   2.1788e+06
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
   9.3132e-06  -9.3132e-06            0            0  -3.1479e+01   6.0471e-02
   3.5623e-04   3.6205e-04   5.6811e-04   5.6811e-04  -1.1933e-05   8.7586e-02
  -8.1956e-04   8.1956e-04            0            0  -2.2283e+01  -8.6226e-01
   2.0348e-04   2.1864e-04  -4.4728e-04  -4.4752e-04  -2.5015e-05  -1.9999e-02
   4.4703e-04  -2.9802e-04            0            0  -3.3300e+01   2.2987e+00
  -2.0345e-03  -1.8623e-03  -1.3667e-07  -1.3515e-07  -2.7326e-04   2.9679e-01
   6.9141e-02            0            0            0  -1.2031e+04   4.3719e+01
```



```
            0   6.4373e-02            0            0   1.2031e+04  -4.4744e+01
   2.9802e-03   2.3842e-03            0            0  -9.5554e+03  -4.2462e+00
   2.3842e-03   2.3842e-03            0            0   9.5554e+03   3.4493e+00

  Columns 7 through 12

            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
  -3.3700e+01  -1.8626e-05   1.8551e-01  -1.8551e-01  -1.0591e+01   1.0591e+01
   9.3132e-06   6.1057e+01   6.6154e+00   6.6154e+00   1.8326e+01   1.8326e+01
  -3.2277e+01   2.9802e-03   5.6820e+00  -5.6806e+00   3.4538e+00  -3.4536e+00
   2.5696e-05  -6.6698e+00   4.7633e+00   4.7633e+00  -1.0441e+01  -1.0441e+01
  -2.1955e+01  -1.4901e-04  -6.3977e+00   6.3977e+00   9.8377e-01  -9.8392e-01
   1.9358e-04   4.3306e+01   3.6514e+01   3.6514e+01   2.9534e+01   2.9534e+01
   1.5831e+04  -9.0756e+03  -6.3081e+03  -2.4772e+02  -5.4555e+01  -4.5400e+01
  -1.5831e+04  -9.0756e+03  -2.4773e+02  -6.3081e+03  -4.5397e+01  -5.4553e+01
  -1.4393e+04  -7.3197e+03  -3.8080e+01  -5.0112e+01  -6.9498e+03  -3.6165e+01
   1.4393e+04  -7.3196e+03  -5.0110e+01  -3.8076e+01  -3.6165e+01  -6.9498e+03

  Columns 13 through 18

            0  -2.3032e+05            0  -3.0402e+05            0            0
            0  -2.3032e+05            0  -3.0402e+05            0            0
            0  -2.1788e+05            0   3.2683e+05            0            0
            0  -2.1788e+05            0   3.2683e+05            0            0
            0            0   1.0000e+00            0            0            0
            0            0            0   1.0000e+00            0            0
            0            0            0            0   1.0000e+00            0
            0            0            0            0            0   1.0000e+00
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
  -5.4119e-03  -6.0815e-03            0  -3.2317e-03            0            0
  -1.1642e-07  -1.0895e-02            0            0            0            0
   5.1409e-03   8.7991e-02            0   1.2249e-01            0            0
  -2.3871e-07   1.9871e-03            0  -1.1519e-05            0            0
   1.8477e-03  -2.3067e-01            0  -9.6112e-02            0            0
  -7.4486e-06  -2.8277e-02            0  -3.4279e-06            0            0
  -5.7220e-03  -4.3416e+00            0  -5.3072e+00            0            0
   5.9605e-03   4.4417e+00            0   5.3072e+00            0            0
  -1.2457e-02   4.2140e-01            0   3.5167e-02            0            0
   1.2457e-02  -3.4690e-01            0  -3.5167e-02            0            0

  Columns 19 through 22

            0            0            0            0
            0            0            0            0
            0            0            0            0
            0            0            0            0
            0            0            0            0
            0            0            0            0
            0            0            0            0
            0            0            0            0
   1.0000e+00            0            0            0
            0   1.0000e+00            0            0
```

```
              0            0   1.0000e+00            0
              0            0            0   1.0000e+00
    -9.2480e-03   9.2573e-03  -6.5686e-02   6.5705e-02
     2.8004e-02   2.8008e-02   1.1802e-01   1.1801e-01
     3.7573e-01  -3.7566e-01   7.3865e-01  -7.3865e-01
     2.9216e-02   2.9203e-02  -9.1044e-02  -9.1058e-02
    -2.6405e-01   2.6420e-01   5.9575e-01  -5.9560e-01
     9.7541e-01   9.7539e-01   1.9292e+00   1.9290e+00
    -3.2971e+01  -1.3590e-01  -3.1090e+00  -3.2663e+00
    -1.3590e-01  -3.2969e+01  -3.2687e+00  -3.1042e+00
    -1.1235e+00  -1.2076e+00  -5.4254e+01   2.4438e-02
    -1.2064e+00  -1.1241e+00   2.4438e-02  -5.4255e+01

>> NW15_x

NW15_x =

FTL(1)
FTL(2)
FTL(3)
FTL(4)
Q(3)
Q(4)
Q(5)
Q(6)
Q(7)
Q(8)
Q(9)
Q(10)
U(1)
U(2)
U(3)
U(4)
U(5)
U(6)
U(7)
U(8)
U(9)
U(10)

>> NW15_a

NW15_a =

  Columns 1 through 6

  -5.0000e+01            0            0            0            0   3.4547e+06
            0  -5.0000e+01            0            0            0   3.4547e+06
            0            0  -5.0000e+01            0            0   3.2683e+06
            0            0            0  -5.0000e+01            0   3.2683e+06
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
   3.7253e-05  -3.7253e-05            0            0  -3.1480e+01   6.0275e-02
   3.6438e-04   3.6322e-04   5.6811e-04   5.6811e-04   8.7311e-06   1.8966e-01
  -9.6858e-04   8.1956e-04            0            0  -2.2280e+01  -7.9602e-01
   2.1815e-04   2.1472e-04  -4.4726e-04  -4.4726e-04  -3.7254e-06  -4.4929e-02
   4.4703e-04  -4.4703e-04            0            0  -3.3302e+01   2.3036e+00
  -1.9180e-03  -2.0124e-03  -1.3674e-07  -1.3671e-07   1.7783e-04   6.6905e-01
   6.6757e-02            0            0            0  -1.2031e+04   4.2944e+01
            0   6.6757e-02            0            0   1.2031e+04  -4.5269e+01
```



```
   1.7881e-03   2.3842e-03            0            0  -9.5554e+03  -4.8524e+00
   2.3842e-03   2.3842e-03            0            0   9.5554e+03   3.0589e+00

  Columns 7 through 12

            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
  -3.3668e+01   1.8626e-05   1.8546e-01  -1.8546e-01  -1.0591e+01   1.0591e+01
  -2.6193e-06   6.1025e+01   6.6150e+00   6.6150e+00   1.8325e+01   1.8325e+01
  -3.2244e+01  -2.9802e-03   5.6822e+00  -5.6834e+00   3.4513e+00  -3.4512e+00
   8.7588e-06  -6.6584e+00   4.7630e+00   4.7630e+00  -1.0440e+01  -1.0440e+01
  -2.1956e+01            0  -6.3977e+00   6.3977e+00   9.8273e-01  -9.8273e-01
  -1.3379e-04   4.3306e+01   3.6513e+01   3.6513e+01   2.9534e+01   2.9534e+01
   1.5831e+04  -9.0756e+03  -6.3081e+03  -2.4772e+02  -5.4548e+01  -4.5409e+01
  -1.5831e+04  -9.0756e+03  -2.4772e+02  -6.3081e+03  -4.5397e+01  -5.4553e+01
  -1.4393e+04  -7.3196e+03  -3.8078e+01  -5.0105e+01  -6.9499e+03  -3.6169e+01
   1.4393e+04  -7.3197e+03  -5.0108e+01  -3.8080e+01  -3.6169e+01  -6.9499e+03

  Columns 13 through 18

            0  -2.3032e+05            0  -3.0402e+05            0            0
            0  -2.3032e+05            0  -3.0402e+05            0            0
            0  -2.1788e+05            0   3.2683e+05            0            0
            0  -2.1788e+05            0   3.2683e+05            0            0
            0            0   1.0000e+00            0            0            0
            0            0            0   1.0000e+00            0            0
            0            0            0            0   1.0000e+00            0
            0            0            0            0            0   1.0000e+00
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
  -8.1186e-03  -3.9674e-03            0  -2.1420e-03            0            0
  -1.5522e-07  -1.6339e-02            0   2.3283e-06            0            0
   7.6741e-03   5.0887e-02            0   8.2329e-02            0            0
  -8.1087e-07   3.0027e-03            0  -1.1588e-05            0            0
   2.6623e-03  -1.5318e-01            0  -6.3479e-02            0            0
  -4.0145e-06  -4.2206e-02            0   1.0831e-04            0            0
  -8.9010e-03  -2.8133e+00            0  -3.5596e+00            0            0
   8.9010e-03   2.9540e+00            0   3.5596e+00            0            0
  -1.8398e-02   3.2306e-01            0   2.0862e-02            0            0
   1.8358e-02  -2.0862e-01            0  -2.2054e-02            0            0

  Columns 19 through 22

            0            0            0            0
            0            0            0            0
            0            0            0            0
            0            0            0            0
            0            0            0            0
            0            0            0            0
            0            0            0            0
            0            0            0            0
   1.0000e+00            0            0            0
            0   1.0000e+00            0            0
            0            0   1.0000e+00            0
```

```
              0            0            0   1.0000e+00
    -9.2387e-03   9.2201e-03  -6.5658e-02   6.5640e-02
     2.8015e-02   2.8020e-02   1.1801e-01   1.1800e-01
     3.8177e-01  -3.8177e-01   7.3552e-01  -7.3567e-01
     2.9236e-02   2.9218e-02  -9.1060e-02  -9.1053e-02
    -2.6271e-01   2.6256e-01   5.9590e-01  -5.9575e-01
     9.7577e-01   9.7565e-01   1.9293e+00   1.9292e+00
    -3.2985e+01  -1.3590e-01  -3.0971e+00  -3.2735e+00
    -1.3113e-01  -3.2983e+01  -3.2806e+00  -3.0971e+00
    -1.1373e+00  -1.1939e+00  -5.4250e+01   2.0862e-02
    -1.1945e+00  -1.1367e+00   2.0266e-02  -5.4250e+01

>> NW20_x

NW20_x =

FTL(1)
FTL(2)
FTL(3)
FTL(4)
Q(3)
Q(4)
Q(5)
Q(6)
Q(7)
Q(8)
Q(9)
Q(10)
U(1)
U(2)
U(3)
U(4)
U(5)
U(6)
U(7)
U(8)
U(9)
U(10)

>> NW20_a

NW20_a =

  Columns 1 through 6

  -6.6667e+01            0            0            0            0   4.6063e+06
            0  -6.6667e+01            0            0            0   4.6063e+06
            0            0  -6.6667e+01            0            0   4.3577e+06
            0            0            0  -6.6667e+01            0   4.3577e+06
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0  -3.1479e+01   5.9940e-02
   3.6554e-04   3.5856e-04   5.6811e-04   5.6811e-04            0   3.3249e-01
  -8.1956e-04   8.1956e-04            0            0  -2.2280e+01  -6.9968e-01
   2.1648e-04   2.1068e-04  -4.4726e-04  -4.4739e-04  -5.2973e-06  -7.9947e-02
   5.9605e-04  -5.9605e-04            0            0  -3.3300e+01   2.3083e+00
  -1.8022e-03  -2.0126e-03  -1.3674e-07  -1.3595e-07   1.0514e-04   1.1912e+00
   6.4373e-02            0            0            0  -1.2031e+04   4.1859e+01
            0   6.9141e-02            0            0   1.2031e+04  -4.5974e+01
   2.3842e-03   2.3842e-03            0            0  -9.5554e+03  -5.7018e+00
```



```
   2.9802e-03   2.3842e-03            0            0   9.5554e+03   2.5094e+00

  Columns 7 through 12

            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
  -3.3624e+01            0   1.8544e-01  -1.8544e-01  -1.0591e+01   1.0591e+01
   1.7462e-06   6.0981e+01   6.6144e+00   6.6144e+00   1.8324e+01   1.8324e+01
  -3.2195e+01  -3.1665e-03   5.6817e+00  -5.6832e+00   3.4522e+00  -3.4520e+00
   1.4410e-05  -6.6423e+00   4.7625e+00   4.7625e+00  -1.0439e+01  -1.0439e+01
  -2.1955e+01  -1.4901e-04  -6.3968e+00   6.3966e+00   9.8541e-01  -9.8571e-01
   1.8339e-04   4.3306e+01   3.6512e+01   3.6512e+01   2.9534e+01   2.9534e+01
   1.5831e+04  -9.0756e+03  -6.3081e+03  -2.4772e+02  -5.4550e+01  -4.5416e+01
  -1.5831e+04  -9.0756e+03  -2.4772e+02  -6.3081e+03  -4.5402e+01  -5.4550e+01
  -1.4393e+04  -7.3196e+03  -3.8078e+01  -5.0103e+01  -6.9499e+03  -3.6160e+01
   1.4393e+04  -7.3197e+03  -5.0105e+01  -3.8081e+01  -3.6160e+01  -6.9499e+03

  Columns 13 through 18

            0  -2.3032e+05            0  -3.0402e+05            0            0
            0  -2.3032e+05            0  -3.0402e+05            0            0
            0  -2.1788e+05            0   3.2683e+05            0            0
            0  -2.1788e+05            0   3.2683e+05            0            0
            0            0   1.0000e+00            0            0            0
            0            0            0   1.0000e+00            0            0
            0            0            0            0   1.0000e+00            0
            0            0            0            0            0   1.0000e+00
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
  -1.0820e-02  -2.9802e-03            0  -1.5646e-03            0            0
  -1.1642e-07  -2.1707e-02            0   4.6566e-06            0            0
   1.0138e-02   3.2522e-02            0   6.0350e-02            0            0
  -1.8996e-07   3.9853e-03            0  -1.6450e-06            0            0
   3.6955e-03  -1.1384e-01            0  -4.6492e-02            0            0
  -5.6067e-06  -6.1210e-02            0   1.1107e-04            0            0
  -1.0848e-02  -2.0289e+00            0  -2.6155e+00            0            0
   1.1086e-02   2.2411e+00            0   2.6131e+00            0            0
  -2.4676e-02   2.8670e-01            0   1.5497e-02            0            0
   2.4706e-02  -1.2457e-01            0  -1.6689e-02            0            0

  Columns 19 through 22

            0            0            0            0
            0            0            0            0
            0            0            0            0
            0            0            0            0
            0            0            0            0
            0            0            0            0
            0            0            0            0
            0            0            0            0
   1.0000e+00            0            0            0
            0   1.0000e+00            0            0
            0            0   1.0000e+00            0
            0            0            0   1.0000e+00
```

```
   -9.2387e-03   9.2387e-03  -6.5640e-02   6.5677e-02
    2.8010e-02   2.8012e-02   1.1801e-01   1.1802e-01
    3.8177e-01  -3.8177e-01   7.3545e-01  -7.3548e-01
    2.9205e-02   2.9215e-02  -9.1072e-02  -9.1061e-02
   -2.6420e-01   2.6405e-01   5.9575e-01  -5.9560e-01
    9.7550e-01   9.7559e-01   1.9291e+00   1.9292e+00
   -3.2985e+01  -1.2636e-01  -3.0923e+00  -3.2783e+00
   -1.2398e-01  -3.2985e+01  -3.2806e+00  -3.0971e+00
   -1.1331e+00  -1.1981e+00  -5.4250e+01   2.0266e-02
   -1.1975e+00  -1.1331e+00   2.0862e-02  -5.4250e+01

>> NW40_x

NW40_x =

FTL(1)
FTL(2)
FTL(3)
FTL(4)
Q(3)
Q(4)
Q(5)
Q(6)
Q(7)
Q(8)
Q(9)
Q(10)
U(1)
U(2)
U(3)
U(4)
U(5)
U(6)
U(7)
U(8)
U(9)
U(10)

>> NW40_a

NW40_a =

  Columns 1 through 6

  -1.3333e+02            0            0            0            0   9.2127e+06
            0  -1.3333e+02            0            0            0   9.2127e+06
            0            0  -1.3333e+02            0            0   8.7154e+06
            0            0            0  -1.3333e+02            0   8.7154e+06
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0  -3.1480e+01   5.7369e-02
   3.6089e-04   3.6322e-04   5.6345e-04   5.6345e-04   4.6566e-06   1.3123e+00
  -6.7055e-04   6.7055e-04            0            0  -2.2280e+01  -4.1276e-02
   2.1619e-04   2.1034e-04  -4.4741e-04  -4.4743e-04   8.9915e-06  -3.1979e-01
   1.4901e-04  -5.9605e-04            0            0  -3.3302e+01   2.3401e+00
  -2.0486e-03  -2.0871e-03  -1.3587e-07  -1.3573e-07   4.1295e-06   4.7550e+00
   6.6757e-02   2.3842e-03            0            0  -1.2031e+04   3.4430e+01
   2.3842e-03   6.6757e-02            0            0   1.2031e+04  -5.0850e+01
   3.5763e-03   2.3842e-03            0            0  -9.5554e+03  -1.1503e+01
   2.3842e-03   2.3842e-03            0            0   9.5554e+03  -1.2410e+00
```



```

  Columns 7 through 12

            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
  -3.3320e+01            0   1.8537e-01  -1.8537e-01  -1.0593e+01   1.0593e+01
            0   6.0677e+01   6.6106e+00   6.6106e+00   1.8314e+01   1.8314e+01
  -3.1875e+01  -2.9802e-03   5.6774e+00  -5.6791e+00   3.4569e+00  -3.4570e+00
  -1.1632e-06  -6.5323e+00   4.7596e+00   4.7596e+00  -1.0431e+01  -1.0431e+01
  -2.1955e+01   4.4703e-04  -6.3935e+00   6.3936e+00   9.8869e-01  -9.8869e-01
  -7.3829e-05   4.3306e+01   3.6504e+01   3.6504e+01   2.9530e+01   2.9530e+01
   1.5831e+04  -9.0756e+03  -6.3079e+03  -2.4772e+02  -5.4543e+01  -4.5400e+01
  -1.5831e+04  -9.0756e+03  -2.4772e+02  -6.3079e+03  -4.5400e+01  -5.4545e+01
  -1.4393e+04  -7.3196e+03  -3.8067e+01  -5.0093e+01  -6.9501e+03  -3.6135e+01
   1.4393e+04  -7.3197e+03  -5.0094e+01  -3.8069e+01  -3.6137e+01  -6.9501e+03

  Columns 13 through 18

            0  -2.3032e+05            0  -3.0402e+05            0            0
            0  -2.3032e+05            0  -3.0402e+05            0            0
            0  -2.1788e+05            0   3.2683e+05            0            0
            0  -2.1788e+05            0   3.2683e+05            0            0
            0            0   1.0000e+00            0            0            0
            0            0            0   1.0000e+00            0            0
            0            0            0            0   1.0000e+00            0
            0            0            0            0            0   1.0000e+00
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
  -2.1644e-02  -1.1921e-03            0  -5.9605e-04            0            0
   1.1642e-07  -4.3507e-02            0            0            0            0
   2.0282e-02   2.0117e-03            0   2.9504e-02            0            0
  -2.1658e-07   7.9911e-03            0  -4.5370e-06            0            0
   7.3239e-03  -5.7518e-02            0  -2.3842e-02            0            0
  -2.7165e-08  -1.1783e-01            0  -4.1956e-05            0            0
  -2.2352e-02  -8.6308e-01            0  -1.2851e+00            0            0
   2.2411e-02   1.2708e+00            0   1.2803e+00            0            0
  -4.9204e-02   2.8372e-01            0   1.0729e-02            0            0
   4.9144e-02   3.4571e-02            0  -9.5367e-03            0            0

  Columns 19 through 22

            0            0            0            0
            0            0            0            0
            0            0            0            0
            0            0            0            0
            0            0            0            0
            0            0            0            0
            0            0            0            0
            0            0            0            0
   1.0000e+00            0            0            0
            0   1.0000e+00            0            0
            0            0   1.0000e+00            0
            0            0            0   1.0000e+00
  -9.3877e-03   9.3877e-03  -6.5714e-02   6.5714e-02
```

```
   2.8014e-02   2.8000e-02   1.1801e-01   1.1802e-01
   3.7849e-01  -3.7864e-01   7.3552e-01  -7.3567e-01
   2.9225e-02   2.9235e-02  -9.1055e-02  -9.1050e-02
  -2.6241e-01   2.6256e-01   5.9590e-01  -5.9545e-01
   9.7547e-01   9.7558e-01   1.9292e+00   1.9294e+00
  -3.2973e+01  -1.3590e-01  -3.0971e+00  -3.2759e+00
  -1.3828e-01  -3.2978e+01  -3.2759e+00  -3.1018e+00
  -1.1313e+00  -1.1981e+00  -5.4250e+01   2.0266e-02
  -1.1992e+00  -1.1313e+00   2.0266e-02  -5.4251e+01

>> who

Your variables are:

LO10_a      LO40_x      LW20_a      NO10_x      NO5_a       NW20_x
LO10_x      LO5_a       LW20_x      NO15_a      NO5_x       NW40_a
LO15_a      LO5_x       LW40_a      NO15_x      NW10_a      NW40_x
LO15_x      LW10_a      LW40_x      NO20_a      NW10_x      NW5_a
LO20_a      LW10_x      LW5_a       NO20_x      NW15_a      NW5_x
LO20_x      LW15_a      LW5_x       NO40_a      NW15_x      ans
LO40_a      LW15_x      NO10_a      NO40_x      NW20_a

>> cond(LO5_a)

ans =

   1.8362e+09

>> cond(LO10_a)

ans =

   1.3112e+09

>> cond(LO15_a)

ans =

   3.4768e+08

>> cond(LO20_a)

ans =

   2.3365e+08

>> cond(LO40_a)

ans =

   4.8900e+07

>> cond(LW5_a)

ans =

   4.5439e+11

>> cond(LW10_a)

ans =

   4.0712e+11

>> cond(LW15_a)
```



```
ans =

   3.9601e+11

>> cond(LW20_a)

ans =

   3.7064e+11

>> cond(LW40_a)

ans =

   3.8141e+11

>> cond(NO5_a)

ans =

   6.1335e+09

>> cond(NO10_a)

ans =

   2.2979e+09

>> cond(NO15_a)

ans =

   2.0711e+09

>> cond(NO20_a)

ans =

   2.6359e+08

>> cond(NO40_a)

ans =

   6.9808e+07

>> cond(NW5_a)

ans =

   8.4528e+12

>> cond(NW10_a)

ans =

   3.3929e+11

>> cond(NW15_a)

ans =

   3.4116e+11

>> cond(NW20_a)
```

```
ans =

   3.0550e+11

>> cond(MM40_a)

ans =

   3.1524e+11

>> diary off

>>  SUSP6G_u

SUSP6G_u =

US(1)
US(2)
US(3)
US(4)
UZ(1)
UZ(2)
UZ(3)
UZ(4)
UFD(1)
UFD(2)
UFD(3)
UFD(4)
UFA(3)
UFA(6)
UFA(9)
UFA(12)
UW

>> SUSP6G_b

SUSP6G_b =

  Columns 1 through 6

   3.8395e+03            0            0            0            0            0
            0   3.8395e+03            0            0            0            0
            0            0   4.0722e+03            0            0            0
            0            0            0   4.0722e+03            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0   3.9611e-01   3.9611e-01
            0            0            0            0  -1.7218e+01   1.7218e+01
            0            0            0            0   6.3372e+00   6.3375e+00
            0            0            0            0  -1.1958e+01   1.1958e+01
            0            0            0            0  -5.4446e+00  -5.4446e+00
            0            0            0            0  -6.3011e+00   6.3011e+00
            0            0            0            0   1.1496e+04  -4.7164e+01
            0            0            0            0   4.7164e+01  -1.1496e+04
            0            0            0            0   1.1778e+01  -5.8114e+00
            0            0            0            0   5.8109e+00  -1.1778e+01

  Columns 7 through 12
```



```

            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
   1.4927e+01   1.4927e+01   5.5507e-04   5.5507e-04   5.5134e-04   5.4762e-04
  -2.9454e+01   2.9454e+01   1.8335e-06  -1.8190e-06   1.9354e-05  -1.9369e-05
   1.0917e+01   1.0917e+01  -4.4238e-06  -4.1910e-06   1.0594e-04   1.0571e-04
   1.7075e+01  -1.7075e+01  -2.1770e-04   2.1770e-04  -2.3124e-04   2.3122e-04
   6.3521e+00   6.3521e+00  -3.1199e-04  -3.1199e-04   1.5832e-05   1.7695e-05
  -6.8329e+00   6.8329e+00   3.1433e-06  -3.1433e-06   2.8999e-04  -2.8976e-04
   7.7558e+00  -1.8335e+01  -7.5717e-04  -8.9873e-04  -5.7276e-04   2.3190e-04
   1.8335e+01  -7.7557e+00   8.9966e-04   7.5810e-04  -2.3190e-04   5.7276e-04
   9.6513e+03  -5.1447e+01   6.9663e-04   8.7172e-04  -7.8939e-03   3.3528e-04
   5.1447e+01  -9.6513e+03  -8.7544e-04  -6.9663e-04  -3.3900e-04   7.8976e-03

  Columns 13 through 17

            0            0            0            0            0
            0            0            0            0            0
            0            0            0            0            0
            0            0            0            0            0
            0            0            0            0            0
            0            0            0            0            0
            0            0            0            0            0
            0            0            0            0            0
            0            0            0            0            0
            0            0            0            0            0
            0            0            0            0            0
            0            0            0            0            0
            0            0            0            0            0
            0            0            0            0            0
   1.1176e-05   1.1176e-05   4.8429e-05   4.8429e-05  -3.3528e-05
  -3.9872e-05   3.9858e-05  -8.7675e-05   8.7690e-05   2.1739e-02
  -5.4250e-04  -5.4250e-04  -5.4645e-04  -5.4622e-04   1.0827e-02
  -4.1568e-05   4.1575e-05   6.7645e-05  -6.7652e-05  -3.9965e-03
   5.3458e-04   5.3458e-04  -4.4331e-04  -4.4331e-04   6.6124e-04
  -1.3881e-03   1.3879e-03  -1.4333e-03   1.4334e-03   5.9316e-02
   4.7362e-02   2.5146e-04   2.3022e-03  -2.4345e-03  -1.2306e-01
  -2.5053e-04  -4.7361e-02   2.4345e-03  -2.3022e-03  -8.1797e-02
   1.2591e-03  -2.0601e-03   4.0304e-02   1.4901e-05  -9.6645e-02
   2.0601e-03  -1.2554e-03  -1.4901e-05  -4.0308e-02  -6.2320e-02


>> SUSP6G_y

SUSP6G_y =

Q(1)
Q(2)
Q(3)
Q(4)
Q(5)
Q(6)
Q(7)
Q(8)
```

```
Q(9)
Q(10)
VBB(1)
VBB(2)
VBB(3)
U(4)
U(5)
U(6)
U(7)
U(8)
U(9)
U(10)


>> SUSP6G_c

SUSP6G_c =

  Columns 1 through 6

            0            0            0            0   1.0000e+00            0
            0            0            0            0            0   1.0000e+00
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0

  Columns 7 through 12

            0            0            0            0            0            0
            0            0            0            0            0            0
   1.0000e+00            0            0            0            0            0
            0   1.0000e+00            0            0            0            0
            0            0   1.0000e+00            0            0            0
            0            0            0   1.0000e+00            0            0
            0            0            0            0   1.0000e+00            0
            0            0            0            0            0   1.0000e+00
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0  -1.9999e+01   1.5524e-13  -6.5370e-05            0            0
            0            0   1.9999e+01            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0

  Columns 13 through 18

```



```
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
   1.0000e+00            0            0            0            0            0
            0   1.0000e+00            0            0            0            0
            0            0   1.0005e+00            0            0            0
            0            0   4.5403e-16   1.0000e+00   7.7927e-15            0
            0            0   1.0660e-06            0   1.0000e+00            0
            0            0            0            0            0   1.0000e+00
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0

  Columns 19 through 24

            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
   1.0000e+00            0            0            0            0            0
            0   1.0000e+00            0            0            0            0
            0            0   1.0000e+00            0            0            0
            0            0            0   1.0000e+00            0            0
            0            0            0            0   1.0000e+00            0
            0            0            0            0            0   1.0000e+00

>> diary jac6
>> exit

 No flops.
>> SUSP6G_u

SUSP6G_u =

US(1)
US(2)
US(3)
US(4)
UZ(1)
UZ(2)
UZ(3)
UZ(4)
UFD(1)
UFD(2)
UFD(3)
UFD(4)
UFA(3)
UFA(6)
```

```
UFA(9)
UFA(12)
UW

>> SUSP6G_b

SUSP6G_b =

  Columns 1 through 6

           0            0            0            0            0            0
           0            0            0            0            0            0
           0            0            0            0            0            0
           0            0            0            0            0            0
           0            0            0            0            0            0
           0            0            0            0            0            0
           0            0            0            0            0            0
           0            0            0            0            0            0
           0            0            0            0            0            0
           0            0            0            0            0            0
  3.1665e-04  -3.1665e-04   1.6093e-03  -1.6093e-03   3.9611e-01   3.9611e-01
  2.2504e-02   2.2504e-02   3.0768e-02   3.0768e-02  -1.7218e+01   1.7218e+01
 -1.2958e-02   1.2959e-02  -1.8034e-02   1.8034e-02   6.3372e+00   6.3375e+00
  1.5337e-02   1.5337e-02  -2.4620e-02  -2.4620e-02  -1.1958e+01   1.1958e+01
  8.9658e-03  -8.9658e-03  -1.4607e-02   1.4607e-02  -5.4446e+00  -5.4446e+00
  6.1138e-03   6.1136e-03   9.1910e-03   9.1910e-03  -6.3011e+00   6.3011e+00
  1.0427e+00  -7.2723e-02  -1.6764e-02  -1.2415e-02   1.1496e+04  -4.7164e+01
 -7.2724e-02   1.0427e+00  -1.2414e-02  -1.6764e-02   4.7164e+01  -1.1496e+04
 -1.0159e-02  -7.9535e-03   1.2417e+00  -8.8807e-02   1.1778e+01  -5.8114e+00
 -7.9572e-03  -1.0159e-02  -8.8803e-02   1.2418e+00   5.8109e+00  -1.1778e+01

  Columns 7 through 12

           0            0            0            0            0            0
           0            0            0            0            0            0
           0            0            0            0            0            0
           0            0            0            0            0            0
           0            0            0            0            0            0
           0            0            0            0            0            0
           0            0            0            0            0            0
           0            0            0            0            0            0
           0            0            0            0            0            0
           0            0            0            0            0            0
  1.4927e+01   1.4927e+01   5.5507e-04   5.5507e-04   5.5134e-04   5.4762e-04
 -2.9454e+01   2.9454e+01   1.8335e-06  -1.8190e-06   1.9354e-05  -1.9369e-05
  1.0917e+01   1.0917e+01  -4.4238e-06  -4.1910e-06   1.0594e-04   1.0571e-04
  1.7075e+01  -1.7075e+01  -2.1770e-04   2.1770e-04  -2.3124e-04   2.3122e-04
  6.3521e+00   6.3521e+00  -3.1199e-04  -3.1199e-04   1.5832e-05   1.7695e-05
 -6.8329e+00   6.8329e+00   3.1433e-06  -3.1433e-06   2.8999e-04  -2.8976e-04
  7.7558e+00  -1.8335e+01  -7.5717e-04  -8.9873e-04  -5.7276e-04   2.3190e-04
  1.8335e+01  -7.7557e+00   8.9966e-04   7.5810e-04  -2.3190e-04   5.7276e-04
  9.6513e+03  -5.1447e+01   6.9663e-04   8.7172e-04  -7.8939e-03   3.3528e-04
  5.1447e+01  -9.6513e+03  -8.7544e-04  -6.9663e-04  -3.3900e-04   7.8976e-03

  Columns 13 through 17

           0            0            0            0            0
           0            0            0            0            0
           0            0            0            0            0
           0            0            0            0            0
           0            0            0            0            0
           0            0            0            0            0
           0            0            0            0            0
           0            0            0            0            0
           0            0            0            0            0
           0            0            0            0            0
```



```
  1.1176e-05   1.1176e-05   4.8429e-05   4.8429e-05  -3.3528e-05
 -3.9872e-05   3.9858e-05  -8.7675e-05   8.7690e-05   2.1739e-02
 -5.4250e-04  -5.4250e-04  -5.4645e-04  -5.4622e-04   1.0827e-02
 -4.1575e-05   4.1575e-05   6.7645e-05  -6.7652e-05  -3.9965e-03
  5.3458e-04   5.3458e-04  -4.4331e-04  -4.4331e-04   6.6124e-04
 -1.3881e-03   1.3879e-03  -1.4333e-03   1.4334e-03   5.9316e-02
  4.7362e-02   2.5146e-04   2.3022e-03  -2.4345e-03  -1.2306e-01
 -2.5053e-04  -4.7361e-02   2.4345e-03  -2.3022e-03  -8.1797e-02
  1.2591e-03  -2.0601e-03   4.0304e-02   1.4901e-05  -9.6645e-02
  2.0601e-03  -1.2554e-03  -1.4901e-05  -4.0308e-02  -6.2320e-02

>> SUSP6G_y

SUSP6G_y =

Q(1)
Q(2)
Q(3)
Q(4)
Q(5)
Q(6)
Q(7)
Q(8)
Q(9)
Q(10)
VBB(1)
VBB(2)
VBB(3)
U(4)
U(5)
U(6)
U(7)
U(8)
U(9)
U(10)

>> SUSP6G_c

SUSP6G_c =

  Columns 1 through 6

  1.0000e+00            0            0            0            0            0
           0   1.0000e+00            0            0            0            0
           0            0   1.0000e+00            0            0            0
           0            0            0   1.0000e+00            0            0
           0            0            0            0   1.0000e+00            0
           0            0            0            0            0   1.0000e+00
           0            0            0            0            0            0
           0            0            0            0            0            0
           0            0            0            0            0            0
           0            0            0            0            0            0
           0            0            0            0            0            0
           0            0            0  -1.9999e+01   1.5524e-13  -6.5370e-05
           0            0            0            0   1.9999e+01            0
           0            0            0            0            0            0
           0            0            0            0            0            0
           0            0            0            0            0            0
           0            0            0            0            0            0
           0            0            0            0            0            0
           0            0            0            0            0            0
           0            0            0            0            0            0

  Columns 7 through 12

           0            0            0            0            0            0
```

```
         0          0          0          0          0          0
         0          0          0          0          0          0
         0          0          0          0          0          0
         0          0          0          0          0          0
         0          0          0          0          0          0
1.0000e+00          0          0          0          0          0
         0 1.0000e+00          0          0          0          0
         0          0 1.0000e+00          0          0          0
         0          0          0 1.0000e+00          0          0
         0          0          0          0 1.0005e+00          0
         0          0          0          0 4.5403e-16 1.0000e+00
         0          0          0          0 1.0660e-06          0
         0          0          0          0          0          0
         0          0          0          0          0          0
         0          0          0          0          0          0
         0          0          0          0          0          0
         0          0          0          0          0          0
         0          0          0          0          0          0
         0          0          0          0          0          0

Columns 13 through 18

         0          0          0          0          0          0
         0          0          0          0          0          0
         0          0          0          0          0          0
         0          0          0          0          0          0
         0          0          0          0          0          0
         0          0          0          0          0          0
         0          0          0          0          0          0
         0          0          0          0          0          0
         0          0          0          0          0          0
         0          0          0          0          0          0
         0          0          0          0          0          0
7.7927e-15          0          0          0          0          0
1.0000e+00          0          0          0          0          0
         0 1.0000e+00          0          0          0          0
         0          0 1.0000e+00          0          0          0
         0          0          0 1.0000e+00          0          0
         0          0          0          0 1.0000e+00          0
         0          0          0          0          0 1.0000e+00
         0          0          0          0          0          0
         0          0          0          0          0          0

Columns 19 through 20

         0          0
         0          0
         0          0
         0          0
         0          0
         0          0
         0          0
         0          0
         0          0
         0          0
         0          0
         0          0
         0          0
         0          0
         0          0
         0          0
         0          0
         0          0
1.0000e+00          0
         0 1.0000e+00

```



```
>> SUSP6G_d

SUSP6G_d =

  Columns 1 through 12

     0     0     0     0     0     0     0     0     0     0     0     0
     0     0     0     0     0     0     0     0     0     0     0     0
     0     0     0     0     0     0     0     0     0     0     0     0
     0     0     0     0     0     0     0     0     0     0     0     0
     0     0     0     0     0     0     0     0     0     0     0     0
     0     0     0     0     0     0     0     0     0     0     0     0
     0     0     0     0     0     0     0     0     0     0     0     0
     0     0     0     0     0     0     0     0     0     0     0     0
     0     0     0     0     0     0     0     0     0     0     0     0
     0     0     0     0     0     0     0     0     0     0     0     0
     0     0     0     0     0     0     0     0     0     0     0     0
     0     0     0     0     0     0     0     0     0     0     0     0
     0     0     0     0     0     0     0     0     0     0     0     0
     0     0     0     0     0     0     0     0     0     0     0     0
     0     0     0     0     0     0     0     0     0     0     0     0
     0     0     0     0     0     0     0     0     0     0     0     0
     0     0     0     0     0     0     0     0     0     0     0     0
     0     0     0     0     0     0     0     0     0     0     0     0
     0     0     0     0     0     0     0     0     0     0     0     0
     0     0     0     0     0     0     0     0     0     0     0     0

  Columns 13 through 17

     0     0     0     0     0
     0     0     0     0     0
     0     0     0     0     0
     0     0     0     0     0
     0     0     0     0     0
     0     0     0     0     0
     0     0     0     0     0
     0     0     0     0     0
     0     0     0     0     0
     0     0     0     0     0
     0     0     0     0     0
     0     0     0     0     0
     0     0     0     0     0
     0     0     0     0     0
     0     0     0     0     0
     0     0     0     0     0
     0     0     0     0     0
     0     0     0     0     0
     0     0     0     0     0
     0     0     0     0     0

>> diary jac6
>> exit

 No flops.

```

D.2.2 Cornering Trim Conditions

```
>> CL011_x

CL011_x =

Q(3)
Q(4)
Q(5)
Q(6)
Q(7)
Q(8)
Q(9)
Q(10)
U(1)
U(2)
U(3)
U(4)
U(5)
U(6)
U(7)
U(8)
U(9)
U(10)

>> CL011_a

CL011_a =

  Columns 1 through 6

           0            0            0            0            0            0
           0            0            0            0            0            0
           0            0            0            0            0            0
           0            0            0            0            0            0
           0            0            0            0            0            0
           0            0            0            0            0            0
           0            0            0            0            0            0
           0            0            0            0            0            0
 -3.0626e+01  -8.6641e+00  -3.3806e+01  -4.1100e+00  -1.5168e-01  -8.4495e-01
 -1.8646e+00   1.2207e+02  -2.5269e+00   6.0040e+01   6.3691e+00   6.1744e+00
 -3.4306e+01  -2.0810e-01  -1.6161e+01  -6.8564e-02   2.8930e+00  -2.8622e+00
  2.6146e-01  -2.1266e+01  -2.5455e-01  -7.6352e+00   4.4916e+00   4.3179e+00
 -1.9069e+00   1.7907e-01  -3.3269e+01   2.5724e-02  -1.6189e+00   1.5984e+00
 -1.7736e-01   3.7107e+01  -8.9872e-02   1.9838e+01   2.9274e+01   2.9086e+01
 -1.1427e+04   1.0762e+03   1.5111e+04  -8.6760e+03  -6.0528e+03  -2.5428e+02
  1.1447e+04   1.0589e+03  -1.5141e+04  -8.6874e+03  -2.5446e+02  -6.0599e+03
 -9.5976e+03   1.2902e+03  -1.4383e+04  -7.2891e+03  -3.6003e+01  -3.4720e+01
  9.6145e+03   1.2817e+03   1.4403e+04  -7.2956e+03  -3.4893e+01  -3.5754e+01

  Columns 7 through 12

           0            0            0            0   1.0000e+00            0
           0            0            0            0            0   1.0000e+00
           0            0            0            0            0            0
           0            0            0            0            0            0
           0            0            0            0            0            0
           0            0            0            0            0            0
           0            0            0            0            0            0
           0            0            0            0            0            0
 -1.1663e+01   9.3141e+00  -4.4660e+00   5.5370e-01   8.2888e-04  -1.7689e-01
  1.7640e+01   1.8933e+01   5.1957e-01  -1.2208e+01  -1.1623e-02   3.7633e+00
  3.4813e+00  -3.4733e+00  -4.8705e-01  -1.2355e-02  -7.7518e-04   9.7496e-03
 -1.0504e+01  -1.0461e+01  -1.0592e-01   2.1253e+00   1.9136e-03  -1.3128e+01
  4.4093e+00  -4.4106e+00   7.8041e-01   3.0119e-02   5.3762e-04  -5.6474e-02
  2.9577e+01   2.9547e+01   2.4392e-01  -3.7046e+00  -1.0838e-03   1.3594e+00
 -4.5307e+01  -5.4634e+01   1.0662e+01  -1.0721e+02  -5.3489e-02  -1.5266e+02
 -5.4631e+01  -4.5205e+01   4.3506e+00  -1.0583e+02  -1.2021e-01  -1.4931e+02
```



```
 -6.9551e+03  -2.8527e+01   2.3819e+01  -1.2779e+02  -1.4126e-01   2.0002e+02
 -2.8604e+01  -6.9620e+03  -7.7786e+00  -1.2908e+02  -1.4112e-01   1.9814e+02

  Columns 13 through 18

           0            0            0            0            0            0
           0            0            0            0            0            0
  1.0000e+00            0            0            0            0            0
           0   1.0000e+00            0            0            0            0
           0            0   1.0000e+00            0            0            0
           0            0            0   1.0000e+00            0            0
           0            0            0            0   1.0000e+00            0
           0            0            0            0            0   1.0000e+00
 -1.9626e-04   1.4063e-03  -1.0254e-02   6.8825e-03  -7.2680e-02   5.7863e-02
  8.9407e-04  -1.4901e-04   2.5779e-02   2.7269e-02   1.1295e-01   1.2219e-01
  1.0938e-03   1.0640e-03   3.8216e-01  -3.8268e-01   7.4283e-01  -7.3674e-01
  1.8092e-04   1.5717e-04   2.8005e-02   2.8138e-02  -9.0984e-02  -9.1046e-02
  8.8488e-04  -3.8718e-03  -2.6295e-01   2.6319e-01   5.9678e-01  -5.9839e-01
  2.8317e-02  -2.8739e-03   9.7499e-01   9.7610e-01   1.9320e+00   1.9292e+00
 -4.6948e-02  -7.9654e-03  -3.2993e+01  -1.2264e-01  -3.1133e+00  -3.2804e+00
 -4.7160e-02   1.7428e-02  -1.2237e-01  -3.3035e+01  -3.2648e+00  -3.0897e+00
 -3.7677e-02   1.0866e-02  -1.1353e+00  -1.1950e+00  -5.4339e+01   2.8742e-02
  4.3009e-02   6.9708e-02  -1.1937e+00  -1.1365e+00   3.0890e-02  -5.4330e+01

>> CL021_x

CL021_x =

Q(3)
Q(4)
Q(5)
Q(6)
Q(7)
Q(8)
Q(9)
Q(10)
U(1)
U(2)
U(3)
U(4)
U(5)
U(6)
U(7)
U(8)
U(9)
U(10)

>> CL021_a

CL021_a =

  Columns 1 through 6

           0            0            0            0            0            0
           0            0            0            0            0            0
           0            0            0            0            0            0
           0            0            0            0            0            0
           0            0            0            0            0            0
           0            0            0            0            0            0
           0            0            0            0            0            0
           0            0            0            0            0            0
 -3.0518e+01  -1.2688e+01  -3.3665e+01  -5.9181e+00  -3.8687e-01  -1.0480e+00
 -2.9212e+00   1.2191e+02  -3.8263e+00   5.9791e+01   6.4594e+00   6.1160e+00
 -3.4552e+01  -6.8932e-02  -1.6219e+01  -2.9698e-01   2.8620e+00  -2.8506e+00
  2.4909e-01  -2.1316e+01  -9.2126e-02  -7.6274e+00   4.5677e+00   4.2968e+00
 -1.9730e+00   1.6283e-01  -3.3596e+01   1.0318e-01  -1.5986e+00   1.5973e+00
```

```
   -1.0458e+00   3.7701e+01   4.0613e-01   1.9587e+01   2.9306e+01   2.9051e+01
   -1.1401e+04   1.0773e+03   1.5077e+04  -8.6567e+03  -6.0368e+03  -2.5406e+02
    1.1473e+04   1.0559e+03  -1.5175e+04  -8.6965e+03  -2.5460e+02  -6.0708e+03
   -9.5826e+03   1.2899e+03  -1.4365e+04  -7.2747e+03  -3.6071e+01  -3.4712e+01
    9.6254e+03   1.2806e+03   1.4420e+04  -7.2978e+03  -3.4934e+01  -3.5694e+01

  Columns 7 through 12

             0            0            0            0   1.0000e+00            0
             0            0            0            0            0   1.0000e+00
             0            0            0            0            0            0
             0            0            0            0            0            0
             0            0            0            0            0            0
             0            0            0            0            0            0
             0            0            0            0            0            0
             0            0            0            0            0            0
   -1.2197e+01   8.7168e+00  -4.4514e+00   1.9381e-01   1.6019e-03  -1.4547e-01
    1.7344e+01   1.9186e+01   1.4756e-01  -6.1197e+00  -1.7881e-02   1.8784e+00
    3.4365e+00  -3.4191e+00  -4.7947e-01  -3.9063e-02   1.8728e-03  -3.7986e-03
   -1.0572e+01  -1.0448e+01  -8.5839e-02   1.0639e+00   2.9856e-03  -6.5615e+00
    4.3743e+00  -4.3728e+00   7.8371e-01   6.4663e-02   3.3957e-04  -2.9955e-02
    2.9595e+01   2.9508e+01   1.7614e-01  -1.8784e+00  -5.9449e-03   6.9071e-01
   -4.5427e+01  -5.4607e+01   8.2511e+00  -5.3440e+01  -1.7170e-01  -7.6410e+01
   -5.4685e+01  -4.5176e+01   1.9835e+00  -5.2928e+01  -1.0491e-01  -7.4884e+01
   -6.9420e+03  -2.8744e+01   2.1530e+01  -6.2920e+01  -2.1001e-01   1.0039e+02
   -2.8968e+01  -6.9665e+03  -9.9436e+00  -6.5291e+01  -1.2953e-01   9.8886e+01

  Columns 13 through 18

             0            0            0            0            0            0
             0            0            0            0            0            0
    1.0000e+00            0            0            0            0            0
             0   1.0000e+00            0            0            0            0
             0            0   1.0000e+00            0            0            0
             0            0            0   1.0000e+00            0            0
             0            0            0            0   1.0000e+00            0
             0            0            0            0            0   1.0000e+00
   -1.8626e-04   2.2352e-03  -1.1623e-02   6.0722e-03  -7.5847e-02   5.3830e-02
    1.7881e-03  -2.9802e-04   2.6226e-02   2.8312e-02   1.1176e-01   1.2547e-01
   -7.7736e-04   7.3858e-04   3.7752e-01  -3.7573e-01   7.3726e-01  -7.3518e-01
   -2.7667e-04   5.8922e-05   2.8964e-02   2.8842e-02  -9.1680e-02  -9.2353e-02
    5.3719e-04  -6.8736e-03  -2.6542e-01   2.6093e-01   5.9143e-01  -5.9553e-01
    3.8757e-02   2.0095e-03   9.8247e-01   9.7202e-01   1.9274e+00   1.9314e+00
   -7.4753e-04  -8.4096e-02  -3.3126e+01  -1.3987e-01  -3.1129e+00  -3.2825e+00
   -6.7487e-02   1.6861e-02  -1.3974e-01  -3.2948e+01  -3.2550e+00  -3.0990e+00
   -4.7308e-02   1.2199e-02  -1.1329e+00  -1.1957e+00  -5.4168e+01   1.3443e-02
   -4.7146e-02  -1.6856e-02  -1.2148e+00  -1.1268e+00   1.3242e-02  -5.4252e+01

>> CLO31_x

CLO31_x =

Q(3)
Q(4)
Q(5)
Q(6)
Q(7)
Q(8)
Q(9)
Q(10)
U(1)
U(2)
U(3)
U(4)
U(5)
U(6)
```



```
U(7)
U(8)
U(9)
U(10)

>> CLO31_a

CLO31_a =

  Columns 1 through 6

             0            0            0            0            0            0
             0            0            0            0            0            0
             0            0            0            0            0            0
             0            0            0            0            0            0
             0            0            0            0            0            0
             0            0            0            0            0            0
             0            0            0            0            0            0
             0            0            0            0            0            0
   -3.0460e+01  -1.3434e+01  -3.3560e+01  -6.2831e+00  -4.7375e-01  -1.0976e+00
   -3.2389e+00   1.2213e+02  -4.2838e+00   5.9638e+01   6.5738e+00   6.1131e+00
   -3.4979e+01   2.1434e-01  -1.6331e+01  -4.7741e-01   2.8347e+00  -2.8215e+00
    2.4074e-01  -2.1400e+01   6.7311e-02  -7.6221e+00   4.6586e+00   4.3008e+00
   -2.0777e+00   1.0373e-01  -3.4129e+01   1.8708e-01  -1.5769e+00   1.5818e+00
   -1.8069e+00   3.8680e+01   8.3426e-01   1.9324e+01   2.9349e+01   2.9017e+01
   -1.1381e+04   1.0770e+03   1.5045e+04  -8.6425e+03  -6.0230e+03  -2.5402e+02
    1.1498e+04   1.0528e+03  -1.5204e+04  -8.7074e+03  -2.5474e+02  -6.0779e+03
   -9.5722e+03   1.2877e+03  -1.4344e+04  -7.2651e+03  -3.6168e+01  -3.4707e+01
    9.6400e+03   1.2804e+03   1.4432e+04  -7.2981e+03  -3.4989e+01  -3.5656e+01

  Columns 7 through 12

             0            0            0            0   1.0000e+00            0
             0            0            0            0            0   1.0000e+00
             0            0            0            0            0            0
             0            0            0            0            0            0
             0            0            0            0            0            0
             0            0            0            0            0            0
             0            0            0            0            0            0
             0            0            0            0            0            0
   -1.2303e+01   8.5688e+00  -4.4356e+00  -9.7865e-03   2.1607e-03  -1.0461e-01
    1.7344e+01   1.9234e+01  -5.5335e-02  -4.1062e+00  -1.9670e-02   1.2523e+00
    3.3543e+00  -3.3264e+00  -4.7236e-01  -5.2366e-02  -1.4028e-03  -6.1102e-03
   -1.0653e+01  -1.0457e+01  -6.2061e-02   7.1226e-01   2.9929e-03  -4.3686e+00
    4.3145e+00  -4.3148e+00   7.9085e-01   7.0832e-02  -2.3078e-03  -2.3041e-02
    2.9594e+01   2.9472e+01   1.2736e-01  -1.2866e+00  -5.1557e-03   4.5578e-01
   -4.5546e+01  -5.4626e+01   6.5952e+00  -3.5516e+01  -1.7256e-01  -5.0881e+01
   -5.4733e+01  -4.5177e+01   3.5894e-01  -3.5279e+01  -1.0507e-01  -4.9904e+01
   -6.9275e+03  -2.9170e+01   1.9727e+01  -4.1325e+01  -1.2584e-01   6.7228e+01
   -2.9512e+01  -6.9661e+03  -1.1664e+01  -4.4042e+01  -2.8720e-01   6.5823e+01

  Columns 13 through 18

             0            0            0            0            0            0
             0            0            0            0            0            0
    1.0000e+00            0            0            0            0            0
             0   1.0000e+00            0            0            0            0
             0            0   1.0000e+00            0            0            0
             0            0            0   1.0000e+00            0            0
             0            0            0            0   1.0000e+00            0
             0            0            0            0            0   1.0000e+00
   -2.2352e-04   2.2724e-03  -1.2852e-02   6.1840e-03  -7.6629e-02   5.2415e-02
    2.9802e-03   5.9605e-04   2.8014e-02   3.0994e-02   1.1325e-01   1.2875e-01
   -1.0937e-03   2.9578e-03   3.8245e-01  -3.8144e-01   7.3061e-01  -7.3934e-01
    3.3757e-05   2.8278e-04   3.0296e-02   3.0200e-02  -9.3115e-02  -9.4294e-02
   -8.8106e-04  -9.5222e-03  -2.6616e-01   2.6221e-01   5.9557e-01  -5.9243e-01
```

```
      4.5937e-02  -8.0773e-03   9.7965e-01   9.7531e-01   1.9339e+00   1.9270e+00
     -7.6724e-02  -1.8636e-02  -3.3076e+01  -1.3564e-01  -3.1044e+00  -3.2621e+00
     -7.6559e-02   4.5309e-02  -1.2709e-01  -3.2971e+01  -3.2929e+00  -3.1119e+00
     -5.2275e-02   1.0064e-01  -1.1361e+00  -1.1951e+00  -5.4178e+01   1.1051e-02
     -1.3268e-01   6.6288e-02  -1.2063e+00  -1.1366e+00  -8.2277e-04  -5.4174e+01

>> CL014_x

CL014_x =

Q(3)
Q(4)
Q(5)
Q(6)
Q(7)
Q(8)
Q(9)
Q(10)
U(1)
U(2)
U(3)
U(4)
U(5)
U(6)
U(7)
U(8)
U(9)
U(10)

>> CL014_a

CL014_a =

  Columns 1 through 6

               0            0            0            0            0            0
               0            0            0            0            0            0
               0            0            0            0            0            0
               0            0            0            0            0            0
               0            0            0            0            0            0
               0            0            0            0            0            0
               0            0            0            0            0            0
               0            0            0            0            0            0
     -2.9566e+01  -3.8348e+01  -3.2008e+01  -1.8192e+01  -1.9579e+00  -2.4384e+00
     -8.2588e+00   1.1619e+02  -1.1154e+01   5.7325e+01   6.3187e+00   5.4854e+00
     -3.4325e+01  -1.1104e+00  -1.6134e+01  -2.9820e-01   2.9416e+00  -2.7984e+00
      1.1488e+00  -2.1291e+01  -1.1690e+00  -7.6517e+00   4.7879e+00   4.0155e+00
     -1.9027e+00   7.8890e-01  -3.3340e+01   9.4190e-02  -1.6378e+00   1.5686e+00
     -8.1979e-01   3.7063e+01  -4.0170e-01   1.9769e+01   2.9542e+01   2.8786e+01
     -1.1390e+04   1.1058e+03   1.5062e+04  -8.6529e+03  -6.0356e+03  -2.5398e+02
      1.1486e+04   1.0277e+03  -1.5189e+04  -8.7022e+03  -2.5460e+02  -6.0763e+03
     -9.5742e+03   1.3052e+03  -1.4351e+04  -7.2751e+03  -3.6397e+01  -3.4478e+01
      9.6370e+03   1.2666e+03   1.4437e+04  -7.3094e+03  -3.5088e+01  -3.5323e+01

  Columns 7 through 12

               0            0            0            0   1.0000e+00            0
               0            0            0            0            0   1.0000e+00
               0            0            0            0            0            0
               0            0            0            0            0            0
               0            0            0            0            0            0
               0            0            0            0            0            0
               0            0            0            0            0            0
               0            0            0            0            0            0
     -1.5314e+01   4.8985e+00  -5.1450e+00   2.3114e+00   1.5944e-02  -7.9617e-01
      1.4688e+01   2.0456e+01   2.1943e+00  -1.1524e+01  -5.1856e-02   3.7026e+00
```



```
      3.4871e+00  -3.4692e+00  -4.9898e-01  -3.7398e-02   1.8938e-03   5.6947e-02
     -1.0576e+01  -1.0396e+01  -4.7483e-01   2.0827e+00   9.4879e-03  -1.3131e+01
      4.4064e+00  -4.4139e+00   7.7212e-01   1.5431e-01   3.4186e-04  -2.5708e-01
      2.9610e+01   2.9517e+01   1.0795e+00  -3.5498e+00  -2.3335e-02   1.3617e+00
     -4.5439e+01  -5.4655e+01   3.7434e+01  -1.0423e+02  -5.0051e-01  -1.5869e+02
     -5.4610e+01  -4.5077e+01   2.9008e+01  -9.8619e+01  -4.3253e-01  -1.4329e+02
     -6.9389e+03  -2.8409e+01   5.1247e+01  -1.2086e+02  -5.4277e-01   2.0333e+02
     -2.8728e+01  -6.9788e+03   2.0007e+01  -1.2643e+02  -4.6222e-01   1.9519e+02

  Columns 13 through 18

               0            0            0            0            0            0
               0            0            0            0            0            0
      1.0000e+00            0            0            0            0            0
               0   1.0000e+00            0            0            0            0
               0            0   1.0000e+00            0            0            0
               0            0            0   1.0000e+00            0            0
               0            0            0            0   1.0000e+00            0
               0            0            0            0            0   1.0000e+00
     -1.4901e-03   6.4075e-03  -1.5646e-02   7.4506e-04  -9.5218e-02   2.9951e-02
      4.1723e-03   2.3842e-03   2.3842e-02   2.8610e-02   9.5367e-02   1.3232e-01
      1.3097e-06  -1.2203e-03   3.8257e-01  -3.8218e-01   7.3286e-01  -7.3468e-01
      8.4087e-05  -1.4107e-04   2.8063e-02   2.8060e-02  -9.0734e-02  -9.1332e-02
     -5.3842e-06  -2.2289e-02  -2.6501e-01   2.6427e-01   5.9582e-01  -5.9852e-01
      1.2955e-01   2.8430e-03   9.7645e-01   9.7691e-01   1.9298e+00   1.9337e+00
     -2.1575e-01  -6.2751e-02  -3.3037e+01  -1.2258e-01  -3.0953e+00  -3.2983e+00
     -2.1486e-01   5.3067e-02  -1.1851e-01  -3.2998e+01  -3.2736e+00  -3.0888e+00
     -1.5670e-01   4.8868e-02  -1.1340e+00  -1.1999e+00  -5.4247e+01   2.0113e-02
     -1.5655e-01  -1.2968e-01  -1.1992e+00  -1.1341e+00   1.7393e-02  -5.4406e+01

>> CL019_x

CL019_x =

Q(3)
Q(4)
Q(5)
Q(6)
Q(7)
Q(8)
Q(9)
Q(10)
U(1)
U(2)
U(3)
U(4)
U(5)
U(6)
U(7)
U(8)
U(9)
U(10)

>> CL019_a

CL019_a =

  Columns 1 through 6

               0            0            0            0            0            0
               0            0            0            0            0            0
               0            0            0            0            0            0
               0            0            0            0            0            0
               0            0            0            0            0            0
               0            0            0            0            0            0
               0            0            0            0            0            0
```

```
           0            0            0            0            0            0
 -2.6323e+01  -7.2728e+01  -2.6629e+01  -3.4622e+01  -4.2021e+00  -4.0519e+00
 -1.5892e+01   9.8358e+01  -2.1095e+01   4.9137e+01   5.5230e+00   4.0424e+00
 -3.4391e+01  -2.2718e+00  -1.6081e+01  -5.9116e-01   2.9834e+00  -2.7120e+00
  2.3002e+00  -2.1382e+01  -2.3242e+00  -7.6851e+00   5.1615e+00   3.6224e+00
 -1.8762e+00   1.5685e+00  -3.3536e+01   1.9873e-01  -1.6635e+00   1.5219e+00
 -1.5711e+00   3.6956e+01  -7.9533e-01   1.9607e+01   2.9877e+01   2.8369e+01
 -1.1346e+04   1.1428e+03   1.4993e+04  -8.6276e+03  -6.0116e+03  -2.5356e+02
  1.1528e+04   9.8679e+02  -1.5253e+04  -8.7257e+03  -2.5465e+02  -6.0959e+03
 -9.5439e+03   1.3250e+03  -1.4308e+04  -7.2577e+03  -3.6858e+01  -3.4113e+01
  9.6657e+03   1.2478e+03   1.4477e+04  -7.3247e+03  -3.5356e+01  -3.4744e+01

 Columns 7 through 12

           0            0            0            0   1.0000e+00            0
           0            0            0            0            0   1.0000e+00
           0            0            0            0            0            0
           0            0            0            0            0            0
           0            0            0            0            0            0
           0            0            0            0            0            0
           0            0            0            0            0            0
           0            0            0            0            0            0
 -1.8904e+01  -1.0443e+00  -7.0214e+00   3.7837e+00   6.0201e-02  -1.5861e+00
  9.8348e+00   2.0921e+01   3.5586e+00  -9.6558e+00  -8.9407e-02   3.5083e+00
  3.4992e+00  -3.4601e+00  -5.3310e-01  -8.8373e-02   3.5044e-03   1.1222e-01
 -1.0671e+01  -1.0315e+01  -9.3967e-01   1.9502e+00   1.8919e-02  -1.3137e+01
  4.4032e+00  -4.4123e+00   7.4319e-01   3.1641e-01  -3.9338e-03  -5.0899e-01
  2.9659e+01   2.9468e+01   2.0563e+00  -3.0747e+00  -3.1488e-02   1.3849e+00
 -4.5622e+01  -5.4653e+01   7.0281e+01  -9.0571e+01  -1.0401e+00  -1.6628e+02
 -5.4579e+01  -4.4902e+01   5.5753e+01  -8.1565e+01  -8.1661e-01  -1.3564e+02
 -6.9170e+03  -2.8271e+01   8.3525e+01  -1.0361e+02  -1.0933e+00   2.0760e+02
 -2.8890e+01  -6.9987e+03   5.3388e+01  -1.1479e+02  -1.0273e+00   1.9135e+02

 Columns 13 through 18

           0            0            0            0            0            0
           0            0            0            0            0            0
  1.0000e+00            0            0            0            0            0
           0   1.0000e+00            0            0            0            0
           0            0   1.0000e+00            0            0            0
           0            0            0   1.0000e+00            0            0
           0            0            0            0   1.0000e+00            0
           0            0            0            0            0   1.0000e+00
 -2.9802e-03   1.1921e-02  -1.9670e-02  -5.3644e-03  -1.1623e-01  -8.3447e-03
  7.1526e-03   4.7684e-03   1.7881e-02   2.5034e-02   6.1989e-02   1.3590e-01
  3.6962e-05   2.7890e-03   3.8903e-01  -3.8827e-01   7.3905e-01  -7.3448e-01
  1.7551e-04  -4.1538e-04   2.7740e-02   2.8092e-02  -9.0604e-02  -9.1504e-02
 -1.2302e-03  -4.6423e-02  -2.6876e-01   2.6407e-01   5.9576e-01  -5.9778e-01
  2.4166e-01   4.9971e-04   9.7593e-01   9.7446e-01   1.9332e+00   1.9280e+00
 -2.8485e-01  -7.1939e-02  -3.3099e+01  -1.0641e-01  -3.1177e+00  -3.2941e+00
 -3.3612e-01   1.3047e-01  -9.3484e-02  -3.3008e+01  -3.2580e+00  -3.0759e+00
 -6.8750e-02   1.7420e-01  -1.1349e+00  -1.1879e+00  -5.4261e+01   2.4281e-02
 -2.2070e-01  -1.7550e-01  -1.1970e+00  -1.1396e+00   2.2904e-02  -5.4397e+01

>> CLO24_x

CLO24_x =

Q(3)
Q(4)
Q(5)
Q(6)
Q(7)
Q(8)
Q(9)
Q(10)
```



```
U(1)
U(2)
U(3)
U(4)
U(5)
U(6)
U(7)
U(8)
U(9)
U(10)

>> CLO24_a

CLO24_a =

 Columns 1 through 6

           0            0            0            0            0            0
           0            0            0            0            0            0
           0            0            0            0            0            0
           0            0            0            0            0            0
           0            0            0            0            0            0
           0            0            0            0            0            0
           0            0            0            0            0            0
           0            0            0            0            0            0
 -2.7888e+01  -5.5209e+01  -2.9656e+01  -2.5808e+01  -3.0470e+00  -3.1495e+00
 -1.2763e+01   1.0953e+02  -1.6549e+01   5.4256e+01   6.2358e+00   4.8041e+00
 -3.4584e+01  -1.0799e+00  -1.6218e+01  -1.3118e+00   2.9092e+00  -2.8185e+00
  1.1227e+00  -2.1298e+01  -4.2373e-01  -7.6332e+00   5.0350e+00   3.8142e+00
 -1.9996e+00   7.0129e-01  -3.3791e+01   4.8138e-01  -1.6095e+00   1.5907e+00
 -4.6865e+00   3.7716e+01   1.8037e+00   1.9542e+01   2.9726e+01   2.8588e+01
 -1.1272e+04   1.1166e+03   1.4900e+04  -8.5863e+03  -5.9777e+03  -2.5324e+02
  1.1598e+04   1.0169e+03  -1.5341e+04  -8.7666e+03  -2.5526e+02  -6.1281e+03
 -9.5066e+03   1.3072e+03  -1.4267e+04  -7.2374e+03  -3.6716e+01  -3.4306e+01
  9.6993e+03   1.2627e+03   1.4515e+04  -7.3386e+03  -3.5260e+01  -3.5007e+01

 Columns 7 through 12

           0            0            0            0   1.0000e+00            0
           0            0            0            0            0   1.0000e+00
           0            0            0            0            0            0
           0            0            0            0            0            0
           0            0            0            0            0            0
           0            0            0            0            0            0
           0            0            0            0            0            0
           0            0            0            0            0            0
 -1.7291e+01   2.0856e+00  -4.7393e+00   7.3235e-01   3.3975e-02  -6.4254e-01
  1.2650e+01   2.0760e+01   5.7774e-01  -5.8349e+00  -7.0333e-02   1.7750e+00
  3.4572e+00  -3.4003e+00  -4.5976e-01  -1.4662e-01   8.4226e-04  -1.4358e-02
 -1.0795e+01  -1.0253e+01  -3.7868e-01   9.9998e-01   1.4750e-02  -6.5689e+00
  4.3719e+00  -4.3827e+00   7.3306e-01   2.8885e-01  -5.2343e-04  -1.4099e-01
  2.9715e+01   2.9408e+01   7.7193e-01  -1.7262e+00  -2.3351e-02   6.8119e-01
 -4.5761e+01  -5.4554e+01   2.6238e+01  -4.9571e+01  -7.3873e-01  -7.9141e+01
 -5.4761e+01  -4.4769e+01   1.8482e+01  -4.7560e+01  -6.6943e-01  -7.2287e+01
 -6.8954e+03  -2.8385e+01   4.0217e+01  -5.3785e+01  -8.2293e-01   1.0330e+02
 -2.9348e+01  -7.0145e+03   1.0562e+01  -6.4593e+01  -8.2244e-01   9.6215e+01

 Columns 13 through 18

           0            0            0            0            0            0
           0            0            0            0            0            0
  1.0000e+00            0            0            0            0            0
           0   1.0000e+00            0            0            0            0
           0            0   1.0000e+00            0            0            0
           0            0            0   1.0000e+00            0            0
           0            0            0            0   1.0000e+00            0
```

```
            0            0            0            0            0   1.0000e+00
  -2.9802e-03   8.9407e-03  -1.9073e-02  -2.9802e-03  -1.0610e-01   1.1921e-02
   5.9605e-03   7.1526e-03   2.0266e-02   2.7418e-02   8.1062e-02   1.3709e-01
   2.6371e-03  -3.1394e-03   3.8169e-01  -3.8290e-01   7.2860e-01  -7.3599e-01
   2.6859e-04  -1.5329e-04   2.7677e-02   2.8489e-02  -9.0823e-02  -9.3499e-02
  -1.9281e-04  -3.4540e-02  -2.6430e-01   2.6160e-01   5.9461e-01  -5.9648e-01
   1.7468e-01  -1.0256e-02   9.7662e-01   9.7260e-01   1.9342e+00   1.9270e+00
  -2.9729e-01  -9.5903e-03  -3.2976e+01  -1.2552e-01  -3.1050e+00  -3.2845e+00
  -1.6198e-01   1.0411e-01  -1.2027e-01  -3.3043e+01  -3.2821e+00  -3.0805e+00
  -1.4131e-01   2.4168e-01  -1.1361e+00  -1.1888e+00  -5.4172e+01   2.0365e-02
  -5.9711e-02  -7.0575e-02  -1.1981e+00  -1.1345e+00   1.4342e-03  -5.4322e+01

>> CNW11_x

CNW11_x =

FTL(1)
FTL(2)
FTL(3)
FTL(4)
Q(3)
Q(4)
Q(5)
Q(6)
Q(7)
Q(8)
Q(9)
Q(10)
U(1)
U(2)
U(3)
U(4)
U(5)
U(6)
U(7)
U(8)
U(9)
U(10)

>> CNW11_a

CNW11_a =

  Columns 1 through 6

  -3.3302e+01            0            0            0  -9.6953e+04   2.3537e+06
            0  -3.3457e+01            0            0  -7.8035e+04   2.3757e+06
            0            0  -3.3320e+01            0  -1.3923e+05   2.1565e+06
            0            0            0  -3.3457e+01  -1.2495e+05   2.2000e+06
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
  -2.7712e-05  -3.3270e-05  -9.6764e-06  -5.7544e-05  -3.1492e+01  -3.8177e-01
   3.9294e-04   3.9476e-04   5.0726e-04   5.0434e-04  -2.0678e+00   8.7619e-02
  -2.0875e-04   2.1145e-04  -2.8564e-04   2.8700e-04  -2.0710e+01  -9.4510e-01
   2.6273e-04   2.5959e-04  -4.1229e-04  -4.1319e-04   5.5850e-01  -6.3488e-02
   1.5587e-04  -1.5688e-04  -2.3219e-04   2.2864e-04  -3.5241e+01   2.4095e+00
   1.3292e-04   1.1059e-04   1.7141e-04   1.8956e-04  -1.2965e+00   3.9749e-01
   1.7940e-02  -1.1988e-03  -2.9780e-04  -2.4508e-04  -1.2026e+04   4.4280e+01
  -1.2372e-03   1.8177e-02  -1.9923e-04  -3.4809e-04   1.2105e+04  -5.0055e+01
  -2.3411e-04  -1.2396e-04   1.9649e-02  -1.4804e-03  -9.5397e+03  -4.2346e+00
```



```
  -1.4637e-04  -2.0608e-04  -1.4653e-03   1.9321e-02   9.5624e+03   2.7905e+00

  Columns 7 through 12

   1.2492e+05  -8.7521e+04  -4.5030e+04            0            0            0
   9.9939e+04   4.3846e+04            0   3.6223e+04            0            0
  -2.1162e+05  -1.1776e+05            0            0  -8.9266e+04            0
  -1.9019e+05   7.9916e+04            0            0            0   8.0094e+04
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
  -3.3312e+01  -4.3536e+00  -2.1679e-01  -8.2004e-01  -1.1759e+01   9.2896e+00
  -2.4545e+00   6.0649e+01   6.2774e+00   6.0992e+00   1.7666e+01   1.9040e+01
  -3.4880e+01  -6.0868e-01   5.8783e+00  -6.1364e+00   3.3543e+00  -3.3929e+00
  -2.6092e-01  -7.0340e+00   4.4999e+00   4.2562e+00  -1.0480e+01  -1.0526e+01
  -1.9693e+01   6.4262e-01  -6.6451e+00   6.9478e+00   9.8664e-01  -8.8619e-01
   9.9377e-01   4.5162e+01   3.6707e+01   3.6948e+01   2.9392e+01   2.9456e+01
   1.5804e+04  -9.0897e+03  -6.3203e+03  -2.4583e+02  -5.3492e+01  -4.5047e+01
  -1.5905e+04  -9.1470e+03  -2.4670e+02  -6.3550e+03  -4.5285e+01  -5.3863e+01
  -1.4397e+04  -7.3080e+03  -3.8054e+01  -5.1254e+01  -6.9395e+03  -3.6923e+01
   1.4421e+04  -7.3186e+03  -5.0577e+01  -3.8045e+01  -3.6694e+01  -6.9517e+03

  Columns 13 through 18

   1.7356e+04  -2.3442e+05  -2.9483e+02  -3.1133e+05  -1.4067e-01   4.7684e-03
   1.7512e+04  -2.3662e+05  -2.9614e+02  -3.1193e+05  -1.5259e-01  -2.3842e-03
   1.4112e+04  -2.1492e+05  -2.5543e+02   3.2326e+05  -8.1062e-02            0
   1.4398e+04  -2.1925e+05  -2.6039e+02   3.2917e+05  -8.8215e-02  -2.3842e-03
            0            0   1.0000e+00            0            0            0
            0            0            0   1.0000e+00            0            0
            0            0            0            0   1.0000e+00            0
            0            0            0            0            0   1.0000e+00
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
  -4.4119e+00  -3.2042e-01   4.2235e-02  -1.5553e-03  -1.1176e-04   1.7229e-03
  -2.9657e-01  -3.3528e-02   2.9802e-03  -2.9802e-04   1.3411e-03  -5.9605e-04
  -4.6910e-01   5.9366e-02   4.2142e-03   1.3227e-01  -5.0059e-05  -2.7381e-03
  -3.2332e-04   6.4715e-03   2.2701e-05   1.4435e-04   2.8929e-04   4.8894e-04
   8.1023e-01  -1.8025e-01  -7.9762e-03  -1.1876e-01   8.4401e-05  -6.0641e-03
   3.7327e-03  -3.5491e-02   1.2609e-04  -6.1840e-03   3.1056e-02   2.4768e-03
   3.5225e+00  -4.1905e+00  -3.5879e-02  -5.4471e+00  -4.7674e-02  -1.1460e-02
  -3.5744e+00   4.6668e+00   3.1716e-02   6.0584e+00  -5.2303e-02  -1.1565e-01
   1.5706e+01   1.5624e+00  -1.2896e-01   5.7645e-03  -3.7611e-02   1.6213e-01
  -1.5723e+01  -1.3913e+00   1.2864e-01  -1.3778e-01  -3.7292e-02   5.3267e-02

  Columns 19 through 22

   2.3842e-03            0            0            0
            0  -2.3842e-03            0            0
            0            0  -1.7881e-02            0
            0            0            0   1.9073e-02
            0            0            0            0
            0            0            0            0
            0            0            0            0
            0            0            0            0
   1.0000e+00            0            0            0
            0   1.0000e+00            0            0
            0            0   1.0000e+00            0
            0            0            0   1.0000e+00
```

```
   -5.3458e-03   2.7940e-03  -6.9151e-02   5.3095e-02
    1.8477e-02   1.8924e-02   1.1653e-01   1.2517e-01
    3.8496e-01  -3.8618e-01   7.3474e-01  -7.3361e-01
    2.1720e-02   2.1824e-02  -9.4962e-02  -9.4840e-02
   -2.6527e-01   2.6602e-01   5.9511e-01  -5.9413e-01
    9.7096e-01   9.7890e-01   1.9322e+00   1.9312e+00
   -3.3107e+01  -9.4073e-02  -3.0801e+00  -3.2658e+00
   -7.8585e-02  -3.3224e+01  -3.2646e+00  -3.0789e+00
   -1.1298e+00  -1.2006e+00  -5.4255e+01   4.2888e-04
   -1.1910e+00  -1.1388e+00   3.4319e-03  -5.4176e+01

>> CNW21_x

CNW21_x =

FTL(1)
FTL(2)
FTL(3)
FTL(4)
Q(3)
Q(4)
Q(5)
Q(6)
Q(7)
Q(8)
Q(9)
Q(10)
U(1)
U(2)
U(3)
U(4)
U(5)
U(6)
U(7)
U(8)
U(9)
U(10)

>> CNW21_a

CNW21_a =

  Columns 1 through 6

   -6.6514e+01            0            0            0  -8.6896e+05   4.7640e+06
             0  -6.6751e+01            0            0  -3.3979e+05   4.9195e+06
             0            0  -6.6552e+01            0  -1.1512e+06   4.2235e+06
             0            0            0  -6.6765e+01  -7.8736e+05   4.5626e+06
             0            0            0            0            0            0
             0            0            0            0            0            0
             0            0            0            0            0            0
             0            0            0            0            0            0
             0            0            0            0            0            0
             0            0            0            0            0            0
             0            0            0            0            0            0
             0            0            0            0            0            0
   -4.4590e-05  -4.9885e-05  -3.0456e-05  -7.9353e-05  -3.1375e+01  -7.8604e-01
    3.9156e-04   3.9234e-04   5.0324e-04   4.9595e-04  -3.5989e+00   3.6359e-01
   -2.1178e-04   2.1424e-04  -2.9796e-04   2.9577e-04  -2.0431e+01  -7.6238e-01
    2.6252e-04   2.5830e-04  -4.1106e-04  -4.1187e-04   1.3488e+00  -2.3385e-01
    1.4904e-04  -1.4544e-04  -2.4193e-04   2.4076e-04  -3.6375e+01   2.3838e+00
    1.2292e-04   1.2571e-04   1.6330e-04   1.6105e-04  -4.1950e+00   1.5130e+00
    1.8082e-02  -1.2113e-03  -2.8732e-04  -2.2242e-04  -1.1989e+04   3.9838e+01
   -1.2028e-03   1.7865e-02  -2.3535e-04  -3.0114e-04   1.2189e+04  -5.4973e+01
   -2.0272e-04  -1.7210e-04   2.0105e-02  -1.4877e-03  -9.5134e+03  -7.7399e+00
   -1.5356e-04  -1.9501e-04  -1.4986e-03   2.0013e-02   9.5881e+03   2.0147e+00
```



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  Columns 7 through 12

    1.1259e+06  -6.8471e+05  -4.0338e+05            0            0            0
    4.2803e+05   2.2306e+05            0   1.5776e+05            0            0
   -1.7450e+06  -8.9012e+05            0            0  -7.3759e+05            0
   -1.2003e+06   5.5921e+05            0            0            0   5.0419e+05
             0            0            0            0            0            0
             0            0            0            0            0            0
             0            0            0            0            0            0
             0            0            0            0            0            0
             0            0            0            0            0            0
             0            0            0            0            0            0
             0            0            0            0            0            0
             0            0            0            0            0            0
   -3.2887e+01  -6.8653e+00  -5.4203e-01  -1.0971e+00  -1.2482e+01   8.4762e+00
   -3.8865e+00   6.0377e+01   6.3851e+00   6.0192e+00   1.7239e+01   1.9382e+01
   -3.5637e+01  -1.4331e+00   5.8319e+00  -6.3887e+00   3.2610e+00  -3.3613e+00
   -1.1927e-01  -6.9592e+00   4.7021e+00   4.1359e+00  -1.0449e+01  -1.0610e+01
   -1.9292e+01   1.8196e+00  -6.5788e+00   7.3768e+00   1.0063e+00  -6.4365e-01
    3.0931e+00   4.5899e+01   3.6734e+01   3.7518e+01   2.9300e+01   2.9519e+01
    1.5753e+04  -9.0655e+03  -6.3007e+03  -2.4507e+02  -5.3222e+01  -4.4568e+01
   -1.6012e+04  -9.2012e+03  -2.4741e+02  -6.3926e+03  -4.5442e+01  -5.4556e+01
   -1.4365e+04  -7.2929e+03  -3.8176e+01  -5.2588e+01  -6.9205e+03  -3.7752e+01
    1.4453e+04  -7.3258e+03  -5.0483e+01  -3.8175e+01  -3.6881e+01  -6.9604e+03

  Columns 13 through 18

    2.6254e+04  -2.3666e+05  -9.7454e+02  -3.1461e+05  -4.9591e-01            0
    2.7110e+04  -2.4438e+05  -9.7822e+02  -3.2463e+05  -5.3406e-01            0
    2.1783e+04  -2.0997e+05  -8.3244e+02   3.1779e+05  -2.9564e-01   1.9073e-02
    2.3539e+04  -2.2683e+05  -9.0692e+02   3.4117e+05  -3.3379e-01            0
             0            0   1.0000e+00            0            0            0
             0            0            0   1.0000e+00            0            0
             0            0            0            0   1.0000e+00            0
             0            0            0            0            0   1.0000e+00
             0            0            0            0            0            0
             0            0            0            0            0            0
             0            0            0            0            0            0
             0            0            0            0            0            0
   -4.3833e+00  -4.9154e-01   4.1984e-02  -1.4901e-04  -3.7253e-05   2.7940e-03
   -4.7836e-01  -7.6197e-02   4.4703e-03  -5.9605e-04   2.0862e-03  -5.9605e-04
   -4.5791e-01  -1.2841e-02   5.1525e-03   6.4387e-02   1.9150e-03  -2.2538e-03
   -8.4623e-04   1.1703e-02  -5.4715e-05   1.1909e-03   6.7521e-04   3.8999e-04
    8.1012e-01  -3.0744e-02  -6.0076e-03  -5.8373e-02  -2.5524e-03  -8.0792e-03
    1.0643e-02  -7.5288e-02   3.6765e-04  -1.9517e-02   5.4716e-02  -2.5985e-03
    3.3973e+00  -1.6285e+00  -2.3078e-02  -2.4439e+00  -1.0259e-01  -1.3412e-01
   -3.5042e+00   2.3831e+00   2.5644e-02   3.2458e+00  -8.0131e-02  -1.0249e-01
    1.5613e+01   2.1047e+00  -2.0857e-01   1.3397e-01   1.0291e-02   1.7193e-01
   -1.5648e+01  -1.7972e+00   1.3412e-01  -1.1883e-02  -6.8764e-02  -1.8172e-02

  Columns 19 through 22

    1.9073e-02            0            0            0
             0            0            0            0
             0            0  -5.7220e-02            0
             0            0            0   8.5831e-02
             0            0            0            0
             0            0            0            0
             0            0            0            0
             0            0            0            0
    1.0000e+00            0            0            0
             0   1.0000e+00            0            0
             0            0   1.0000e+00            0
             0            0            0   1.0000e+00
   -6.7800e-03   2.0489e-03  -7.3761e-02   4.7125e-02
```

```
     1.8477e-02   1.9670e-02   1.1504e-01   1.2934e-01
     3.8177e-01  -3.8161e-01   7.3813e-01  -7.3189e-01
     2.2452e-02   2.2246e-02  -9.5832e-02  -9.6111e-02
    -2.6984e-01   2.6986e-01   5.9124e-01  -5.9445e-01
     9.7489e-01   9.6999e-01   1.9279e+00   1.9262e+00
    -3.3124e+01  -7.5726e-02  -3.0956e+00  -3.2675e+00
    -8.5873e-02  -3.3092e+01  -3.2374e+00  -3.0656e+00
    -1.1206e+00  -1.2059e+00  -5.4101e+01  -6.7055e-04
    -1.2103e+00  -1.1131e+00   9.2565e-04  -5.4095e+01

>> CNW31_x
>> CCNW31_x

CCNW31_x =

FTL(1)
FTL(2)
FTL(3)
FTL(4)
Q(3)
Q(4)
Q(5)
Q(6)
Q(7)
Q(8)
Q(9)
Q(10)
U(1)
U(2)
U(3)
U(4)
U(5)
U(6)
U(7)
U(8)
U(9)
U(10)

>> CCNW31_a

CCNW31_a =

  Columns 1 through 6

  -9.9630e+01            0            0            0  -5.7416e+06   6.8171e+06
            0  -1.0006e+02            0            0  -5.2155e+04   7.6835e+06
            0            0  -9.9693e+01            0  -5.8523e+06   5.5823e+06
            0            0            0  -1.0013e+02  -2.3160e+06   7.2466e+06
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
  -7.2544e-05  -7.8398e-05  -6.5006e-05  -1.1278e-04  -3.1029e+01  -9.5055e-01
   3.8662e-04   3.8590e-04   4.9609e-04   4.8857e-04  -6.1333e+00   8.7023e-01
  -2.1955e-04   2.1295e-04  -3.0056e-04   2.9631e-04  -1.9743e+01  -3.9602e-01
   2.6226e-04   2.5586e-04  -4.0980e-04  -4.1270e-04   2.9638e+00  -5.4508e-01
   1.5342e-04  -1.4552e-04  -2.4432e-04   2.4017e-04  -3.8586e+01   2.2404e+00
   1.1226e-04   1.2721e-04   1.5079e-04   1.6614e-04  -9.9338e+00   3.4214e+00
   1.8295e-02  -1.2114e-03  -2.8838e-04  -2.3315e-04  -1.1916e+04   3.1350e+01
  -1.2224e-03   1.7737e-02  -2.0756e-04  -3.0532e-04   1.2359e+04  -6.4467e+01
  -1.8819e-04  -1.7299e-04   2.0189e-02  -1.5014e-03  -9.4679e+03  -1.6248e+01
  -1.4176e-04  -1.9923e-04  -1.5031e-03   1.9746e-02   9.6350e+03   3.2293e+00
```



```

  Columns 7 through 12

   7.5035e+06  -4.3743e+06  -2.6646e+06            0            0            0
  -3.1146e+03  -1.5879e+04            0   2.3390e+04            0            0
  -8.8344e+06  -4.4274e+06            0            0  -3.7456e+06            0
  -3.5461e+06   1.6759e+06            0            0            0   1.4803e+06
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            C            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
  -3.2030e+01  -1.1031e+01  -1.1356e+00  -1.5666e+00  -1.3636e+01   7.1082e+00
  -6.2168e+00   5.9701e+01   6.5613e+00   5.8508e+00   1.6500e+01   1.9869e+01
  -3.7113e+01  -3.1587e+00   5.7182e+00  -6.9124e+00   3.0512e+00  -3.2974e+00
   5.8006e-02  -6.8829e+00   5.1032e+00   3.8875e+00  -1.0406e+01  -1.0798e+01
  -1.8374e+01   4.2383e+00  -6.4161e+00   8.2529e+00   1.0215e+00  -1.5034e-01
   7.2491e+00   4.7403e+01   3.6732e+01   3.8699e+01   2.9114e+01   2.9636e+01
   1.5658e+04  -9.0121e+03  -6.2625e+03  -2.4344e+02  -5.2656e+01  -4.3586e+01
  -1.6225e+04  -9.3151e+03  -2.4812e+02  -6.4712e+03  -4.5741e+01  -5.5911e+01
  -1.4315e+04  -7.2720e+03  -3.8391e+01  -5.5334e+01  -6.8867e+03  -3.9489e+01
   1.4522e+04  -7.3402e+03  -5.0188e+01  -3.8495e+01  -3.7398e+01  -6.9768e+03

  Columns 13 through 18

   3.8926e+04  -2.2411e+05  -2.1292e+03  -2.9900e+05  -2.0981e-01   9.5367e-03
   4.3878e+04  -2.5257e+05  -2.4428e+03  -3.3998e+05  -2.5749e-01  -2.8610e-02
   3.0534e+04  -1.8375e+05  -1.7339e+03   2.8151e+05  -1.1921e-01  -9.5367e-03
   3.9652e+04  -2.3853e+05  -2.3209e+03   3.6022e+05  -1.5259e-01            0
            0            0   1.0000e+00            0            0            0
            0            0            0   1.0000e+00            0            0
            0            0            0            0   1.0000e+00            0
            0            0            0            0            0   1.0000e+00
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
  -4.3043e+00  -7.7599e-01   4.1723e-02   4.4703e-04  -2.9802e-04   4.6194e-03
  -7.6933e-01  -1.7304e-01   9.5367e-03  -2.3842e-03   1.1921e-03   1.1921e-03
  -4.4332e-01  -6.2069e-02   5.2131e-03   4.4878e-02   1.9663e-03  -6.8708e-03
  -2.2266e-03   1.8043e-02  -1.4028e-04   1.2486e-03   1.2096e-03   1.3364e-03
   8.1587e-01   6.2936e-02  -7.7696e-03  -4.3166e-02  -9.6334e-04  -1.4008e-02
   2.7035e-02  -1.1079e-01   2.8532e-03  -1.7423e-02   6.8113e-02  -1.3467e-02
   3.3139e+00  -4.9836e-01  -3.6177e-02  -1.4601e+00  -1.2105e-01   3.2783e-03
  -3.5730e+00   1.5831e+00   2.8554e-02   2.5485e+00  -1.0291e-01  -7.5735e-02
   1.5400e+01   3.1559e+00  -2.0799e-01  -2.0541e-02   6.4184e-02   3.5261e-01
  -1.5491e+01  -2.7503e+00   1.2667e-01  -1.5668e-01  -7.6326e-03   1.2265e-01

  Columns 19 through 22

   9.5367e-03            0            0            0
            0  -2.8610e-02            0            0
            0            0  -2.8610e-02            0
            0            0            0   3.8147e-02
            0            0            0            0
            0            0            0            0
            0            0            0            0
            0            0            0            0
   1.0000e+00            0            0            0
            0   1.0000e+00            0            0
            0            0   1.0000e+00            0
            0            0            0   1.0000e+00
  -9.2387e-03  -2.9802e-04  -8.1360e-02   3.6806e-02
```

```
     1.9073e-02   2.2650e-02   1.1325e-01   1.3947e-01
     3.8442e-01  -3.8414e-01   7.2878e-01  -7.3162e-01
     2.3981e-02   2.3856e-02  -9.8324e-02  -1.0055e-01
    -2.6609e-01   2.6598e-01   5.8903e-01  -5.9255e-01
     9.7526e-01   9.7248e-01   1.9327e+00   1.9319e+00
    -3.3124e+01  -1.0164e-01  -3.0998e+00  -3.2756e+00
    -1.0187e-01  -3.3081e+01  -3.2608e+00  -3.0781e+00
    -1.1356e+00  -1.1983e+00  -5.3946e+01  -2.5469e-02
    -1.1987e+00  -1.1285e+00  -3.3468e-02  -5.4175e+01

>> CNW14_x

CNW14_x =

FTL(1)
FTL(2)
FTL(3)
FTL(4)
Q(3)
Q(4)
Q(5)
Q(6)
Q(7)
Q(8)
Q(9)
Q(10)
U(1)
U(2)
U(3)
U(4)
U(5)
U(6)
U(7)
U(8)
U(9)
U(10)

>> CNW14_a

CNW14_a =

  Columns 1 through 6

  -3.3066e+01            0            0            0  -5.8390e+05   2.3978e+06
            0  -3.3704e+01            0            0  -1.6103e+05   2.4973e+06
            0            0  -3.3072e+01            0  -7.6262e+05   2.1023e+06
            0            0            0  -3.3685e+01  -4.6201e+05   2.3230e+06
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
  -1.2912e-04  -1.3361e-04  -1.2353e-04  -1.7035e-04  -3.0302e+01  -1.6697e+00
   3.7448e-04   3.7296e-04   4.8689e-04   4.7518e-04  -9.0027e+00  -3.8803e-01
  -2.1429e-04   2.0687e-04  -2.9848e-04   2.9284e-04  -1.9715e+01  -1.0342e+00
   2.6706e-04   2.5377e-04  -4.1129e-04  -4.1298e-04   2.4348e+00  -2.1241e-01
   1.5288e-04  -1.3915e-04  -2.4044e-04   2.3715e-04  -3.7168e+01   2.5585e+00
   1.1309e-04   1.3647e-04   1.5531e-04   1.5944e-04  -5.8012e+00   7.8025e-01
   1.8371e-02  -1.1845e-03  -2.7438e-04  -2.1297e-04  -1.1935e+04   3.8381e+01
  -1.1993e-03   1.7465e-02  -2.2574e-04  -2.8567e-04   1.2285e+04  -6.1122e+01
  -1.9595e-04  -1.9199e-04   2.0225e-02  -1.4711e-03  -9.5044e+03  -4.2767e+00
  -1.3321e-04  -2.0446e-04  -1.4849e-03   1.9782e-02   9.5947e+03   1.7181e-01
```



```
  Columns 7 through 12

   7.5687e+05  -4.5477e+05  -2.7111e+05            0            0            0
   1.9909e+05   1.0477e+05            0   7.4754e+04            0            0
  -1.1554e+06  -5.8546e+05            0            0  -4.8885e+05            0
  -7.0565e+05   3.3069e+05            0            0            0   2.9609e+05
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
  -3.1681e+01  -1.8870e+01  -2.0176e+00  -2.4331e+00  -1.5485e+01   4.7389e+00
  -1.0633e+01   5.7806e+01   6.1601e+00   5.4348e+00   1.4614e+01   2.0587e+01
  -3.6236e+01  -2.7085e+00   5.6087e+00  -6.8487e+00   3.2912e+00  -3.4203e+00
  -1.1151e+00  -7.0501e+00   4.8863e+00   3.8376e+00  -1.0411e+01  -1.0617e+01
  -1.8385e+01   2.9736e+00  -6.4375e+00   7.8365e+00   1.0196e+00  -5.6034e-01
   4.5344e+00   4.6702e+01   3.6689e+01   3.7937e+01   2.9278e+01   2.9557e+01
   1.5684e+04  -9.0206e+03  -6.2745e+03  -2.4380e+02  -5.3181e+01  -4.4264e+01
  -1.6136e+04  -9.2760e+03  -2.4793e+02  -6.4403e+03  -4.5312e+01  -5.4817e+01
  -1.4357e+04  -7.2949e+03  -3.8104e+01  -5.3537e+01  -6.9192e+03  -3.7786e+01
   1.4467e+04  -7.3324e+03  -5.0332e+01  -3.8304e+01  -3.6681e+01  -6.9695e+03

  Columns 13 through 18

   7.6557e+04  -2.2690e+05  -1.2920e+03  -3.2109e+05  -1.2636e-01   7.1526e-03
   7.9605e+04  -2.3636e+05  -1.3375e+03  -3.2395e+05  -1.5497e-01   4.7684e-03
   5.9891e+04  -2.0125e+05  -1.0847e+03   3.1623e+05  -5.8413e-02            0
   6.6209e+04  -2.2236e+05  -1.1827e+03   3.4666e+05  -7.2718e-02  -2.3842e-03
            0            0   1.0000e+00            0            0            0
            0            0            0   1.0000e+00            0            0
            0            0            0            0   1.0000e+00            0
            0            0            0            0            0   1.0000e+00
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
  -4.0530e+00  -1.2442e+00   2.0266e-02   2.9802e-04  -1.4901e-03   6.7055e-03
  -1.2350e+00  -3.9337e-01   7.7486e-03  -1.7881e-03   4.7684e-03   1.7881e-03
  -4.7248e-01  -4.2135e-02   1.2573e-03   1.3228e-01  -7.4622e-04  -3.4925e-04
  -5.9192e-03   2.0353e-02  -9.0222e-05   6.0332e-04   9.8691e-04   4.4325e-04
   8.3948e-01  -1.5313e-03  -4.2119e-03  -1.2290e-01  -5.1688e-04  -2.6154e-02
   2.7122e-02  -7.3292e-02  -1.0579e-04  -3.4826e-02   1.2353e-01   9.2801e-03
   4.2800e+00  -2.6743e+00  -1.1371e-02  -4.5694e+00  -1.4308e-01  -2.6923e-01
  -5.0301e+00   4.8256e+00   1.2843e-02   7.0966e+00  -8.3465e-02  -6.1160e-02
   1.4892e+01   5.2105e+00  -6.2646e-02   4.9967e-02  -1.4691e-01   1.2217e-01
  -1.5018e+01  -4.8170e+00   6.2438e-02  -3.3313e-01  -1.5123e-01  -7.0549e-02

  Columns 19 through 22

            0            0            0            0
            0  -2.3842e-03            0            0
            0            0  -1.4305e-02            0
            0            0            0   1.5497e-02
            0            0            0            0
            0            0            0            0
            0            0            0            0
            0            0            0            0
   1.0000e+00            0            0            0
            0   1.0000e+00            0            0
            0            0   1.0000e+00            0
            0            0            0   1.0000e+00
  -9.2387e-03  -2.3842e-03  -9.3281e-02   2.3097e-02
   1.6093e-02   1.8477e-02   9.7752e-02   1.3530e-01
```

3.8320e-01	-3.8424e-01	7.3463e-01	-7.3730e-01
2.1338e-02	2.1139e-02	-9.5517e-02	-9.5714e-02
-2.6958e-01	2.6622e-01	5.9655e-01	-5.9665e-01
9.7938e-01	9.7027e-01	1.9355e+00	1.9306e+00
-3.3228e+01	-7.9949e-02	-3.0804e+00	-3.2684e+00
-8.5207e-02	-3.3044e+01	-3.2688e+00	-3.0652e+00
-1.1286e+00	-1.1946e+00	-5.4263e+01	9.1941e-03
-1.2127e+00	-1.1250e+00	1.1217e-03	-5.4260e+01

```
>> CNN19_x

CNN19_x =

FTL(1)
FTL(2)
FTL(3)
FTL(4)
Q(3)
Q(4)
Q(5)
Q(6)
Q(7)
Q(8)
Q(9)
Q(10)
U(1)
U(2)
U(3)
U(4)
U(5)
U(6)
U(7)
U(8)
U(9)
U(10)

>> CNN19_a

CNN19_a =
```

Columns 1 through 6

-3.2769e+01	0	0	0	-1.9853e+06	2.2952e+06
0	-3.4036e+01	0	0	1.3154e+05	2.5779e+06
0	0	-3.2734e+01	0	-2.1359e+06	1.8540e+06
0	0	0	-3.3975e+01	-7.1982e+05	2.4530e+06
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
-2.4333e-04	-2.4886e-04	-2.6028e-04	-2.9859e-04	-2.6612e+01	-2.9373e+00
3.1174e-04	3.1124e-04	4.3037e-04	4.0388e-04	-1.7245e+01	-1.8442e+00
-2.1550e-04	2.0534e-04	-2.9800e-04	2.9430e-04	-1.8064e+01	-1.1984e+00
2.7221e-04	2.4593e-04	-4.1130e-04	-4.1361e-04	4.9680e+00	-4.1716e-01
1.5951e-04	-1.3413e-04	-2.3926e-04	2.3788e-04	-3.9923e+01	2.7887e+00
1.0560e-04	1.3806e-04	1.5880e-04	1.6424e-04	-1.2216e+01	1.3162e+00
1.8725e-02	-1.1661e-03	-2.9238e-04	-2.2971e-04	-1.1816e+04	3.1390e+01
-1.2018e-03	1.7223e-02	-2.3153e-04	-3.1077e-04	1.2550e+04	-7.7936e+01
-1.9952e-04	-2.0812e-04	2.0070e-02	-1.4843e-03	-9.4596e+03	-4.1065e+00
-1.0596e-04	-1.9803e-04	-1.4916e-03	1.9733e-02	9.6444e+03	-3.5227e+00

Columns 7 through 12

2.5919e+06	-1.5138e+06	-9.2237e+05	0	0	0
-2.0048e+05	-1.1793e+05	0	-6.1011e+04	0	0
-3.2245e+06	-1.6169e+06	0	0	-1.3686e+06	0
-1.1062e+06	5.1916e+05	0	0	0	4.6083e+05
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
-2.6709e+01	-3.5647e+01	-4.2105e+00	-4.0799e+00	-1.9088e+01	-1.3566e+00
-1.9987e+01	4.9262e+01	5.2893e+00	4.0281e+00	9.6381e+00	2.1013e+01
-3.8380e+01	-5.7488e+00	5.2348e+00	-7.9084e+00	3.1730e+00	-3.4348e+00
-2.2476e+00	-7.1620e+00	5.3890e+00	3.2714e+00	-1.0336e+01	-1.0771e+01
-1.6415e+01	6.3036e+00	-6.1299e+00	9.1338e+00	1.0441e+00	-1.0594e-01
9.7800e+00	4.8929e+01	3.6615e+01	3.9381e+01	2.9109e+01	2.9684e+01
1.5524e+04	-8.9307e+03	-6.2189e+03	-2.4024e+02	-5.2719e+01	-4.3229e+01
-1.6471e+04	-9.4625e+03	-2.4920e+02	-6.5626e+03	-4.5350e+01	-5.6052e+01
-1.4314e+04	-7.2797e+03	-3.8163e+01	-5.6834e+01	-6.8920e+03	-3.9127e+01
1.4531e+04	-7.3533e+03	-4.9902e+01	-3.8742e+01	-3.6844e+01	-6.9915e+03

Columns 13 through 18

1.3815e+05	-1.8305e+05	-2.5004e+03	-3.1189e+05	-7.0333e-02	4.7684e-03
1.5471e+05	-2.0591e+05	-2.7628e+03	-3.2907e+05	-1.1325e-01	4.7684e-03
1.0062e+05	-1.5557e+05	-1.8878e+03	2.8010e+05	3.3379e-02	5.9605e-04
1.3325e+05	-2.0575e+05	-2.4893e+03	3.6483e+05	3.6955e-02	0
0	0	1.0000e+00	0	0	0
0	0	0	9.9987e-01	0	0
0	0	0	0	1.0000e+00	0
0	0	0	0	0	1.0000e+00
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
-3.0654e+00	-2.0468e+00	3.6359e-02	1.7881e-03	-4.1723e-03	1.0729e-02
-2.0381e+00	-1.3830e+00	2.3842e-02	-2.3842e-03	8.3447e-03	4.7684e-03
-4.4597e-01	-1.6716e-01	6.4634e-03	1.3418e-01	7.8697e-04	6.3796e-04
-2.3106e-02	3.4699e-02	1.8663e-04	3.1492e-03	4.3862e-03	1.6284e-04
8.1962e-01	2.3473e-01	-8.6927e-03	-1.4350e-01	-1.7509e-03	-4.4315e-02
8.0976e-02	-1.0318e-01	2.3571e-03	-7.7356e-02	2.4917e-01	-8.5570e-03
4.5145e+00	-6.6469e-01	-4.6561e-02	-3.6487e+00	-4.2268e-01	-1.7005e-01
-7.3346e+00	4.3648e+00	4.0261e-02	9.1444e+00	-4.0904e-01	1.4016e-01
1.2665e+01	9.3910e+00	-2.0672e-01	2.9250e-01	-3.2983e-03	1.8327e-01
-1.3107e+01	-8.7751e+00	2.0137e-01	-6.1731e-01	-1.5478e-01	-1.5863e-02

Columns 19 through 22

2.3842e-03	0	0	0
0	-1.1921e-03	0	0
0	0	8.3447e-03	0
0	0	0	-8.3447e-03
0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0
1.0000e+00	0	0	0
0	1.0000e+00	0	0
0	0	1.0000e+00	0
0	0	0	1.0000e+00
-1.3709e-02	-7.7486e-03	-1.1623e-01	-1.7881e-02
1.0729e-02	1.5497e-02	7.1526e-02	1.3947e-01
3.8651e-01	-3.9117e-01	7.3520e-01	-7.3249e-01

```
     2.0630e-02   2.0702e-02  -9.6705e-02  -9.7386e-02
    -2.6825e-01   2.6822e-01   5.9558e-01  -5.9368e-01
     9.8151e-01   9.7146e-01   1.9368e+00   1.9227e+00
    -3.3270e+01  -6.0378e-02  -3.0976e+00  -3.2651e+00
    -9.4762e-02  -3.3127e+01  -3.2550e+00  -3.0465e+00
    -1.1406e+00  -1.1914e+00  -5.4119e+01  -1.0504e-03
    -1.2076e+00  -1.1312e+00  -5.8603e-03  -5.4164e+01

>> CNW24_x

CNW24_x =

FTL(1)
FTL(2)
FTL(3)
FTL(4)
Q(3)
Q(4)
Q(5)
Q(6)
Q(7)
Q(8)
Q(9)
Q(10)
U(1)
U(2)
U(3)
U(4)
U(5)
U(6)
U(7)
U(8)
U(9)
U(10)

>> CNW24_a

CNW24_a =

  Columns 1 through 6

  -6.6061e+01            0            0            0  -1.4830e+07   2.8777e+06
            0  -6.7073e+01            0            0   1.2456e+06   5.0914e+06
            0            0  -6.6098e+01            0  -1.0635e+07   2.0761e+06
            0            0            0  -6.7073e+01  -1.5766e+06   5.1432e+06
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
  -1.8590e-04  -1.9071e-04  -1.9914e-04  -2.4195e-04  -2.8183e+01  -2.7937e+00
   3.4683e-04   3.4633e-04   4.5822e-04   4.3875e-04  -1.4625e+01  -7.8201e-01
  -2.1986e-04   2.0978e-04  -3.0272e-04   2.9749e-04  -1.6659e+01  -9.0936e-01
   2.6623e-04   2.4940e-04  -4.0887e-04  -4.1351e-04   5.8457e+00  -7.4829e-01
   1.5698e-04  -1.4079e-04  -2.4571e-04   2.4162e-04  -4.3034e+01   2.5647e+00
   1.0277e-04   1.3510e-04   1.5267e-04   1.6705e-04  -1.8135e+01   2.5994e+00
   1.8634e-02  -1.1992e-03  -2.8610e-04  -2.2815e-04  -1.1830e+04   2.9324e+01
  -1.2076e-03   1.7432e-02  -2.1664e-04  -3.1150e-04   1.2688e+04  -8.2277e+01
  -1.8363e-04  -1.9259e-04   2.0218e-02  -1.5135e-03  -9.4212e+03  -1.3938e+01
  -1.1890e-04  -2.0371e-04  -1.5209e-03   1.9562e-02   9.7116e+03   1.2899e-03

  Columns 7 through 12

```



```
   1.9494e+07  -1.1228e+07  -6.8842e+06            0            0            0
  -1.7202e+06  -9.8079e+05            0  -5.7922e+05            0            0
  -1.5989e+07  -7.9949e+06            0            0  -6.8029e+06            0
  -2.4497e+06   1.1247e+06            0            0            0   1.0064e+06
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
  -2.9110e+01  -2.7668e+01  -3.2693e+00  -3.2729e+00  -1.7682e+01   1.4126e+00
  -1.5501e+01   5.4144e+01   6.0618e+00   4.7815e+00   1.2263e+01   2.0971e+01
  -4.0338e+01  -6.3586e+00   5.5336e+00  -8.1415e+00   2.8646e+00  -3.3228e+00
  -5.9684e-01  -7.2402e+00   5.5893e+00   3.2255e+00  -1.0338e+01  -1.1083e+01
  -1.4463e+01   8.2005e+00  -6.3061e+00   9.9791e+00   9.9056e-01   5.8210e-01
   1.3695e+01   5.1537e+01   3.6890e+01   4.0773e+01   2.8903e+01   2.9874e+01
   1.5539e+04  -8.9518e+03  -6.2186e+03  -2.3939e+02  -5.2114e+01  -4.1891e+01
  -1.6640e+04  -9.5475e+03  -2.4899e+02  -6.6210e+03  -4.5725e+01  -5.8046e+01
  -1.4278e+04  -7.2576e+03  -3.8551e+01  -6.0135e+01  -6.8586e+03  -4.1756e+01
   1.4630e+04  -7.3745e+03  -5.0199e+01  -3.9061e+01  -3.7911e+01  -7.0153e+03

  Columns 13 through 18

   6.3910e+04  -1.2916e+05  -2.1780e+03  -1.9038e+05  -6.9141e-02            0
   1.1306e+05  -2.2855e+05  -4.1753e+03  -3.3640e+05  -1.5259e-01  -1.4305e-02
   4.3490e+04  -9.4435e+04  -1.6140e+03   1.5805e+05  -7.3016e-03   2.9802e-04
   1.0783e+05  -2.3393e+05  -4.0538e+03   3.8110e+05  -3.5763e-02  -2.3842e-03
            0            0   1.0000e+00            0            0            0
            0            0            0   1.0000e+00            0            0
            0            0            0            0   1.0000e+00            0
            0            0            0            0            0   1.0000e+00
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
  -3.5748e+00  -1.7502e+00   3.8147e-02   5.9605e-04  -2.3842e-03   1.0133e-02
  -1.7433e+00  -8.8434e-01   1.9073e-02   1.1921e-03   5.9605e-03   2.3842e-03
  -4.2045e-01  -1.5126e-01   5.6648e-03   7.0992e-02   1.5111e-03  -3.3225e-03
  -1.4913e-02   3.4434e-02   4.4944e-04   3.5160e-03   4.3246e-03   5.3020e-04
   7.9676e-01   2.3855e-01  -7.1776e-03  -8.4483e-02  -1.2986e-03  -3.7888e-02
   6.7442e-02  -1.1127e-01  -1.1595e-03  -5.9209e-02   1.9417e-01  -1.3468e-02
   3.5328e+00   5.5292e-02  -3.0694e-02  -1.8659e+00  -3.8713e-01  -1.4042e-02
  -4.8062e+00   2.2627e+00   2.6028e-02   5.0611e+00  -3.1684e-01  -5.5297e-03
   1.3850e+01   7.3943e+00  -1.6781e-01   1.9403e-01  -1.2653e-02   4.3791e-01
  -1.4172e+01  -6.7839e+00   2.0753e-01  -5.2647e-01  -1.5985e-01   1.6589e-05

  Columns 19 through 22

   2.3842e-03            0            0            0
            0  -4.7684e-03            0            0
            0            0  -1.7881e-03            0
            0            0            0   9.5367e-03
            0            0            0            0
            0            0            0            0
            0            0            0            0
            0            0            0            0
   1.0000e+00            0            0            0
            0   1.0000e+00            0            0
            0            0   1.0000e+00            0
            0            0            0   1.0000e+00
  -1.3113e-02  -7.1526e-03  -1.0848e-01  -5.3644e-03
   1.4305e-02   1.9073e-02   8.8215e-02   1.4544e-01
   3.8433e-01  -3.7783e-01   7.2513e-01  -7.3310e-01
   2.2529e-02   2.2363e-02  -1.0082e-01  -1.0317e-01
```

```
  -2.6964e-01   2.6569e-01   5.8766e-01  -5.9074e-01
   9.7961e-01   9.6222e-01   1.9309e+00   1.9231e+00
  -3.3117e+01  -9.9770e-02  -3.0975e+00  -3.2631e+00
  -8.5072e-02  -3.3010e+01  -3.2631e+00  -3.0628e+00
  -1.1346e+00  -1.1964e+00  -5.3757e+01  -2.7761e-02
  -1.2124e+00  -1.1082e+00  -4.8485e-02  -5.3944e+01

>> CNW26_x

CNW26_x =

FTL(1)
FTL(2)
FTL(3)
FTL(4)
Q(3)
Q(4)
Q(5)
Q(6)
Q(7)
Q(8)
Q(9)
Q(10)
U(1)
U(2)
U(3)
U(4)
U(5)
U(6)
U(7)
U(8)
U(9)
U(10)

>> CNW26_a

CNW26_a =

  Columns 1 through 6

  -6.5907e+01            0            0            0  -3.8971e+07  -1.2982e+06
            0  -6.7077e+01            0            0   1.4467e+06   4.8553e+06
            0            0  -6.5915e+01            0  -1.9457e+07  -5.9051e+05
            0            0            0  -6.7143e+01  -1.5217e+06   5.2683e+06
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
  -2.2214e-04  -2.2873e-04  -2.4168e-04  -2.8417e-04  -2.6468e+01  -2.8741e+00
   3.2081e-04   3.1849e-04   4.3545e-04   4.0658e-04  -1.7889e+01  -1.0943e+00
  -2.2947e-04   2.1683e-04  -3.1107e-04   3.0434e-04  -1.4242e+01  -9.7335e-01
   2.6459e-04   2.4300e-04  -4.0575e-04  -4.1222e-04   7.4168e+00  -9.4157e-01
   1.6509e-04  -1.4383e-04  -2.5159e-04   2.4698e-04  -4.6807e+01   2.5918e+00
   8.9451e-05   1.2854e-04   1.4513e-04   1.6526e-04  -2.3586e+01   3.0128e+00
   1.8852e-02  -1.2378e-03  -2.6638e-04  -2.2384e-04  -1.1838e+04   2.6111e+01
  -1.2532e-03   1.7322e-02  -2.0127e-04  -3.0430e-04   1.2952e+04  -9.3921e+01
  -1.7152e-04  -1.9133e-04   2.0285e-02  -1.5505e-03  -9.4149e+03  -1.6709e+01
  -9.7358e-05  -1.9372e-04  -1.5550e-03   1.9493e-02   9.7760e+03  -3.4202e-01

  Columns 7 through 12

   5.1364e+07  -2.9398e+07  -1.8051e+07            0            0            0
```



```
  -1.9878e+06  -1.1237e+06            0  -6.7012e+05            0            0
  -2.9164e+07  -1.4552e+07            0            0  -1.2399e+07            0
  -2.3859e+06   1.0746e+06            0            0            0   9.6705e+05
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
  -2.7205e+01  -3.3340e+01  -4.2379e+00  -3.9089e+00  -1.8847e+01  -7.8678e-01
  -1.8315e+01   5.1010e+01   5.9009e+00   4.5395e+00   1.0555e+01   2.1002e+01
  -4.3288e+01  -8.4078e+00   5.3591e+00  -8.9478e+00   2.4918e+00  -3.0823e+00
  -7.5010e-01  -7.7199e+00   5.9909e+00   3.0420e+00  -1.0448e+01  -1.1420e+01
  -1.1015e+01   1.0837e+01  -6.2654e+00   1.1228e+01   7.9167e-01   1.2012e+00
   1.8361e+01   5.5024e+01   3.7150e+01   4.2573e+01   2.8753e+01   3.0010e+01
   1.5545e+04  -8.9386e+03  -6.2063e+03  -2.3722e+02  -5.2145e+01  -4.1054e+01
  -1.6978e+04  -9.7235e+03  -2.5009e+02  -6.7302e+03  -4.5960e+01  -5.9682e+01
  -1.4287e+04  -7.2506e+03  -3.8798e+01  -6.4162e+01  -6.8315e+03  -4.4603e+01
   1.4709e+04  -7.3945e+03  -5.0621e+01  -3.9920e+01  -3.9666e+01  -7.0242e+03

  Columns 13 through 18

  -3.4442e+04   5.5171e+04   1.1261e+03   8.5680e+04   5.9509e+00  -3.0518e-01
   1.2880e+05  -2.0638e+05  -4.5865e+03  -3.2170e+05  -6.1035e+00  -3.0518e-01
  -1.4791e+04   2.5628e+04   6.0375e+02  -4.5119e+04   7.0572e-01   7.6294e-02
   1.3214e+05  -2.2854e+05  -4.9172e+03   3.8879e+05  -1.6785e+00  -1.1444e-01
            0            0   1.0000e+00            0            0            0
            0            0            0   1.0000e+00            0            0
            0            0            0            0   1.0000e+00            0
            0            0            0            0            0   1.0000e+00
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
  -3.1901e+00  -1.9913e+00   3.5763e-02   4.7684e-03  -3.5763e-03   1.1921e-02
  -1.9766e+00  -1.2703e+00   2.3842e-02            0   7.1526e-03   7.1526e-03
  -4.0285e-01  -1.8613e-01   4.1816e-03   6.9812e-02   2.1886e-03  -5.2992e-03
  -2.2856e-02   4.1201e-02   3.6322e-04   5.8394e-03   3.4738e-03  -9.3132e-05
   7.8972e-01   3.2427e-01  -1.0359e-02  -9.6515e-02  -3.7020e-03  -4.4061e-02
   9.1444e-02  -1.2152e-01  -1.5134e-05  -7.3764e-02   2.3406e-01  -3.6426e-02
   3.4188e+00   5.2977e-01  -4.1351e-02  -1.6865e+00  -2.8567e-01   1.5255e-02
  -5.3231e+00   2.2892e+00   3.8762e-02   5.7187e+00  -2.4486e-01   4.5858e-02
   1.2962e+01   8.7876e+00  -1.2413e-01   3.5260e-01   8.9016e-02   7.0233e-01
  -1.3428e+01  -8.0662e+00   1.2424e-01  -7.5467e-01  -2.8646e-01   3.0439e-01

  Columns 19 through 22

  -1.5259e-01            0            0            0
            0  -3.0518e-01            0            0
            0            0   1.5259e-01            0
            0            0            0   4.5776e-01
            0            0            0            0
            0            0            0            0
            0            0            0            0
            0            0            0            0
   1.0000e+00            0            0            0
            0   1.0000e+00            0            0
            0            0   1.0000e+00            0
            0            0            0   1.0000e+00
  -2.0266e-02  -9.5367e-03  -1.1802e-01  -2.6226e-02
   1.4305e-02   2.6226e-02   8.5831e-02   1.5259e-01
   3.8203e-01  -3.9167e-01   7.1140e-01  -7.2110e-01
   2.8601e-02   2.7157e-02  -1.0687e-01  -1.0985e-01
  -2.6458e-01   2.6637e-01   5.8257e-01  -5.8521e-01
```

```
    9.7870e-01   9.7344e-01   1.9306e+00   1.9178e+00
   -3.2999e+01  -1.0110e-01  -3.1018e+00  -3.2743e+00
   -1.0952e-01  -3.3279e+01  -3.2816e+00  -3.0559e+00
   -1.1444e+00  -1.1935e+00  -5.3402e+01  -7.8268e-02
   -1.2050e+00  -1.1427e+00  -1.0308e-01  -5.3622e+01

>> CNM315_x

CNM315_x =

FTL(1)
FTL(2)
FTL(3)
FTL(4)
Q(3)
Q(4)
Q(5)
Q(6)
Q(7)
Q(8)
Q(9)
Q(10)
U(1)
U(2)
U(3)
U(4)
U(5)
U(6)
U(7)
U(8)
U(9)
U(10)

>> CNM315_a

CNM315_a =

  Columns 1 through 6

  -9.9630e+01            0            0            0  -5.7416e+06   6.8171e+06
            0  -1.0006e+02            0            0  -5.2155e+04   7.6835e+06
            0            0  -9.9693e+01            0  -5.8523e+06   5.5823e+06
            0            0            0  -1.0013e+02  -2.3160e+06   7.2466e+06
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
  -7.2544e-05  -7.8398e-05  -6.5006e-05  -1.1278e-04  -3.1029e+01  -9.5055e-01
   3.8662e-04   3.8590e-04   4.9609e-04   4.8857e-04  -6.1333e+00   8.7023e-01
  -2.1955e-04   2.1295e-04  -3.0056e-04   2.9631e-04  -1.9743e+01  -3.9602e-01
   2.6226e-04   2.5586e-04  -4.0980e-04  -4.1270e-04   2.9638e+00  -5.4508e-01
   1.5342e-04  -1.4552e-04  -2.4432e-04   2.4017e-04  -3.8586e+01   2.2404e+00
   1.1226e-04   1.2721e-04   1.5079e-04   1.6614e-04  -9.9338e+00   3.4214e+00
   1.8295e-02  -1.2114e-03  -2.8838e-04  -2.3315e-04  -1.1916e+04   3.1350e+01
  -1.2224e-03   1.7737e-02  -2.0756e-04  -3.0532e-04   1.2359e+04  -6.4467e+01
  -1.8819e-04  -1.7299e-04   2.0189e-02  -1.5014e-03  -9.4679e+03  -1.6248e+01
  -1.4176e-04  -1.9923e-04  -1.5031e-03   1.9746e-02   9.6350e+03   3.2293e+00

  Columns 7 through 12

   7.5035e+06  -4.3743e+06  -2.6646e+06            0            0            0
  -3.1146e+03  -1.5879e+04            0   2.3390e+04            0            0
```



```
  -8.8344e+06  -4.4274e+06            0            0  -3.7456e+06            0
  -3.5461e+06   1.6759e+06            0            0            0   1.4803e+06
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
  -3.2030e+01  -1.1031e+01  -1.1356e+00  -1.5666e+00  -1.3636e+01   7.1082e+00
  -6.2168e+00   5.9701e+01   6.5613e+00   5.8508e+00   1.6500e+01   1.9869e+01
  -3.7113e+01  -3.1587e+00   5.7182e+00  -6.9124e+00   3.0512e+00  -3.2974e+00
   5.8006e-02  -6.8829e+00   5.1032e+00   3.8875e+00  -1.0406e+01  -1.0798e+01
  -1.8374e+01   4.2383e+00  -6.4161e+00   8.2529e+00   1.0215e+00  -1.5034e-01
   7.2491e+00   4.7403e+01   3.6732e+01   3.8699e+01   2.9114e+01   2.9636e+01
   1.5658e+04  -9.0121e+03  -6.2625e+03  -2.4344e+02  -5.2656e+01  -4.3586e+01
  -1.6225e+04  -9.3151e+03  -2.4812e+02  -6.4712e+03  -4.5741e+01  -5.5911e+01
  -1.4315e+04  -7.2720e+03  -3.8391e+01  -5.5334e+01  -6.8867e+03  -3.9489e+01
   1.4522e+04  -7.3402e+03  -5.0188e+01  -3.8495e+01  -3.7398e+01  -6.9768e+03

  Columns 13 through 18

   3.8926e+04  -2.2411e+05  -2.1292e+03  -2.9900e+05  -2.0981e-01   9.5367e-03
   4.3878e+04  -2.5257e+05  -2.4428e+03  -3.3998e+05  -2.5749e-01  -2.8610e-02
   3.0534e+04  -1.8375e+05  -1.7339e+03   2.8151e+05  -1.1921e-01  -9.5367e-03
   3.9652e+04  -2.3853e+05  -2.3209e+03   3.6022e+05  -1.5259e-01            0
            0            0   1.0000e+00            0            0            0
            0            0            0   1.0000e+00            0            0
            0            0            0            0   1.0000e+00            0
            0            0            0            0            0   1.0000e+00
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
  -4.3043e+00  -7.7599e-01   4.1723e-02   4.4703e-04  -2.9802e-04   4.6194e-03
  -7.6933e-01  -1.7304e-01   9.5367e-03  -2.3842e-03   1.1921e-03   1.1921e-03
  -4.4332e-01  -6.2069e-02   5.2131e-03   4.4878e-02   1.9663e-03  -6.8708e-03
  -2.2266e-03   1.8043e-02  -1.4028e-04   1.2486e-03   1.2096e-03   1.3364e-03
   8.1587e-01   6.2936e-02  -7.7696e-03  -4.3166e-02  -9.6334e-04  -1.4008e-02
   2.7035e-02  -1.1079e-01   2.8532e-03  -1.7423e-02   6.8113e-02  -1.3467e-02
   3.3139e+00  -4.9836e-01  -3.6177e-02  -1.4601e+00  -1.2105e-01   3.2783e-03
  -3.5730e+00   1.5831e+00   2.8554e-02   2.5485e+00  -1.0291e-01  -7.5735e-02
   1.5400e+01   3.1559e+00  -2.0799e-01  -2.0541e-02   6.4184e-02   3.5261e-01
  -1.5491e+01  -2.7503e+00   1.2667e-01  -1.5668e-01  -7.6326e-03   1.2265e-01

  Columns 19 through 22

   9.5367e-03            0            0            0
            0  -2.8610e-02            0            0
            0            0  -2.8610e-02            0
            0            0            0   3.8147e-02
            0            0            0            0
            0            0            0            0
            0            0            0            0
            0            0            0            0
   1.0000e+00            0            0            0
            0   1.0000e+00            0            0
            0            0   1.0000e+00            0
            0            0            0   1.0000e+00
  -9.2387e-03  -2.9802e-04  -8.1360e-02   3.6806e-02
   1.9073e-02   2.2650e-02   1.1325e-01   1.3947e-01
   3.8442e-01  -3.8414e-01   7.2878e-01  -7.3162e-01
   2.3981e-02   2.3856e-02  -9.8324e-02  -1.0055e-01
  -2.6609e-01   2.6598e-01   5.8903e-01  -5.9255e-01
   9.7526e-01   9.7248e-01   1.9327e+00   1.9319e+00
```

```
   -3.3124e+01  -1.0164e-01  -3.0998e+00  -3.2756e+00
   -1.0187e-01  -3.3081e+01  -3.2608e+00  -3.0781e+00
   -1.1356e+00  -1.1983e+00  -5.3946e+01  -2.5469e-02
   -1.1987e+00  -1.1285e+00  -3.3468e-02  -5.4175e+01

>> CNW33_x

CNW33_x =

FTL(1)
FTL(2)
FTL(3)
FTL(4)
Q(3)
Q(4)
Q(5)
Q(6)
Q(7)
Q(8)
Q(9)
Q(10)
U(1)
U(2)
U(3)
U(4)
U(5)
U(6)
U(7)
U(8)
U(9)
U(10)

>> CNW33_a

CNW33_a =

  Columns 1 through 6

  -9.9397e+01            0            0            0  -3.1488e+07   2.5019e+06
            0  -1.0017e+02            0            0   2.4931e+06   7.6018e+06
            0            0  -9.9521e+01            0  -1.9687e+07   1.9244e+06
            0            0            0  -1.0020e+02  -2.6915e+06   7.7607e+06
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
  -1.3516e-04  -1.4222e-04  -1.3999e-04  -1.8595e-04  -2.9659e+01  -1.3947e+00
   3.6910e-04   3.6640e-04   4.7872e-04   4.6243e-04  -1.1500e+01   6.9737e-01
  -2.2619e-04   2.1641e-04  -3.0766e-04   3.0205e-04  -1.6964e+01  -4.0171e-01
   2.6245e-04   2.4952e-04  -4.0760e-04  -4.1253e-04   5.8381e+00  -9.0051e-01
   1.6026e-04  -1.4656e-04  -2.4972e-04   2.4469e-04  -4.3481e+01   2.1704e+00
   9.7183e-05   1.2785e-04   1.4632e-04   1.6524e-04  -1.9554e+01   4.1462e+00
   1.8637e-02  -1.2351e-03  -2.7029e-04  -2.2853e-04  -1.1848e+04   2.4465e+01
  -1.2486e-03   1.7534e-02  -1.9382e-04  -3.0887e-04   1.2714e+04  -8.1302e+01
  -1.7537e-04  -1.7968e-04   2.0268e-02  -1.5339e-03  -9.4217e+03  -2.3567e+01
  -1.1643e-04  -1.9847e-04  -1.5395e-03   1.9566e-02   9.7346e+03   4.8241e+00

  Columns 7 through 12

   4.1437e+07  -2.3779e+07  -1.4591e+07            0            0            0
  -3.4195e+06  -1.9302e+06            0  -1.1616e+06            0            0
  -2.9549e+07  -1.4758e+07            0            0  -1.2561e+07            0
```



```
  -4.1715e+06   1.9295e+06            0            0            0   1.7153e+06
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
  -3.0466e+01  -2.0390e+01  -2.4909e+00  -2.5827e+00  -1.6032e+01   3.9345e+00
  -1.1325e+01   5.7262e+01   6.6042e+00   5.4252e+00   1.4418e+01   2.0677e+01
  -4.0533e+01  -6.3102e+00   5.5263e+00  -8.0425e+00   2.6600e+00  -3.1596e+00
   2.5299e-02  -7.2037e+00   5.7278e+00   3.3983e+00  -1.0398e+01  -1.1214e+01
  -1.4654e+01   8.5759e+00  -6.2532e+00   1.0059e+01   9.2804e-01   7.8848e-01
   1.4699e+01   5.1660e+01   3.6877e+01   4.1108e+01   2.8839e+01   2.9918e+01
   1.5560e+04  -8.9564e+03  -6.2171e+03  -2.4006e+02  -5.2075e+01  -4.1828e+01
  -1.6666e+04  -9.5507e+03  -2.4929e+02  -6.6257e+03  -4.6072e+01  -5.8604e+01
  -1.4275e+04  -7.2500e+03  -3.8692e+01  -6.0901e+01  -6.8419e+03  -4.3001e+01
   1.4649e+04  -7.3751e+03  -5.0116e+01  -3.9264e+01  -3.8767e+01  -7.0137e+03

  Columns 13 through 18

   2.6192e+04  -7.9323e+04  -1.4567e+03  -1.0937e+05  -1.6212e-01            0
   7.9631e+04  -2.4104e+05  -4.1563e+03  -3.3812e+05  -3.6240e-01  -1.9073e-02
   1.9340e+04  -6.1285e+04  -1.2143e+03   9.7872e+04  -6.8545e-03            0
   7.8051e+04  -2.4713e+05  -4.4020e+03   3.8310e+05   1.8120e-01   9.5367e-03
            0            0   1.0000e+00            0            0            0
            0            0            0   1.0000e+00            0            0
            0            0            0            0   1.0000e+00            0
            0            0            0            0            0   1.0000e+00
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
            0            0            0            0            0            0
  -3.9662e+00  -1.3725e+00   4.1127e-02   3.5763e-03  -1.7881e-03   8.3447e-03
  -1.3620e+00  -5.1023e-01   1.4305e-02            0   3.5763e-03            0
  -4.2203e-01  -1.2216e-01   4.5821e-03   4.4573e-02  -8.8243e-03  -3.7346e-03
  -7.9004e-03   2.9115e-02   4.6566e-06   2.7847e-03   2.4773e-03   1.6531e-03
   8.1710e-01   1.8813e-01  -9.5461e-03  -5.4499e-02  -2.0885e-03  -2.7048e-02
   5.7726e-02  -1.2710e-01   1.7908e-03  -4.5037e-02   1.3610e-01  -2.5099e-02
   3.2718e+00   2.0084e-01  -3.8948e-02  -1.1443e+00  -8.8308e-02  -2.4829e-02
  -3.9952e+00   1.5695e+00   2.5574e-02   3.2424e+00  -1.1772e-01  -4.7870e-03
   1.4650e+01   5.5571e+00  -1.6466e-01   1.8551e-01   3.6550e-02   5.3932e-01
  -1.4878e+01  -4.9790e+00   1.2501e-01  -2.9488e-01  -1.8112e-01   2.5848e-01

  Columns 19 through 22

            0            0            0            0
            0            0            0            0
            0            0  -1.7881e-03            0
            0            0            0  -4.7684e-02
            0            0            0            0
            0            0            0            0
            0            0            0            0
            0            0            0            0
   1.0000e+00            0            0            0
            0   1.0000e+00            0            0
            0            0   1.0000e+00            0
            0            0            0   1.0000e+00
  -1.4901e-02  -5.3644e-03  -9.7752e-02   8.9407e-03
   2.1458e-02   2.7418e-02   1.0610e-01   1.4901e-01
   3.8193e-01  -3.8505e-01   7.1926e-01  -7.2214e-01
   2.6994e-02   2.6408e-02  -1.0314e-01  -1.0744e-01
  -2.6277e-01   2.6809e-01   5.8338e-01  -5.8506e-01
   9.7875e-01   9.7442e-01   1.9296e+00   1.9171e+00
  -3.2962e+01  -1.0293e-01  -3.1100e+00  -3.2702e+00
```

```
   -1.1604e-01  -3.3198e+01  -3.2654e+00  -3.0578e+00
   -1.1476e+00  -1.2072e+00  -5.3447e+01  -6.3712e-02
   -1.1998e+00  -1.1285e+00  -7.7267e-02  -5.3693e+01

>> who

Your variables are:

CCNW31_a     CLO19_x     CNW11_a     CNW21_x     CNW315_a
CCNW31_x     CLO21_a     CNW11_x     CNW24_a     CNW315_x
CLO11_a      CLO21_x     CNW14_a     CNW24_x     CNW33_a
CLO11_x      CLO24_a     CNW14_x     CNW26_a     CNW33_x
CLO14_a      CLO24_x     CNW19_a     CNW26_x     NW335_a
CLO14_x      CLO31_a     CNW19_x     CNW30_a     NW335_x
CLO19_a      CLO31_x     CNW21_a     CNW30_x

>> cond(CLO11_a)

ans =

   1.4573e+09

>> cond(CLO14_a)

ans =

   7.4741e+07

>> cond(CLO19_a)

ans =

   1.8668e+07

>> cond(CLO21_a)

ans =

   7.6994e+08

>> cond(CLO24_a)

ans =

   3.7156e+07

>> cond(CLO31_a)

ans =

   1.1220e+09

>> cond(CNW11_a)

ans =

   1.9855e+11

>> cond(CNW14_a)

ans =

   2.5976e+10

>> cond(CNW19_a)

```



```
ans =

   3.5176e+10

>> cond(CNW21_a)

ans =

   7.4478e+10

>> cond(CNW24_a)

ans =

   2.2844e+11

>> cond(CNW26_a)

ans =

   7.2168e+11

>> cond(CCNW31_a)

ans =

   7.2270e+10

>> cond(CNW315_a)

ans =

   7.2270e+10

>> cond(CNW33_a)

ans =

   3.7343e+11

>> diary cjac62
```

D.3 Eigenvalues

D.3.1 Straight Ahead Trim Condition

```
>> who

Your variables are:

LO10_a    LO15_a    LO20_a    LO40_a    LO5_a
LO10_x    LO15_x    LO20_x    LO40_x    LO5_x

>> c2w(eig(LO5_a))

ans =

   81.6950    0.3047
   75.4631    0.1989
   82.5042    0.2917
   78.6324    0.1915
   31.1134    1.0000
    8.4577    0.2665
    6.8797    0.2045
   22.5758    1.0000
   11.3993    0.2659
    0.0027    1.0000
    0.0001    1.0000

>> c2w(eig(LO10_a))

ans =

   81.6950    0.3047
   75.4630    0.1989
   82.0602    0.2991
   78.5465    0.1939
    8.4574    0.2665
    6.8802    0.2044
   11.3759    0.2665
   13.9859    0.9678
    0.0054    1.0000
    0.0002    1.0000

>> c2w(eig(LO15_a))

ans =

   81.6979    0.3047
   75.4597    0.1989
   81.9776    0.3019
   78.5286    0.1948
    8.4568    0.2659
    6.8819    0.2057
   11.3751    0.2674
    9.9887    0.9045
    0.0081    1.0000
    0.0010    1.0000

>> c2w(eig(LO20_a))

ans =

   75.4578    0.1989
   78.5217    0.1952
   81.6980    0.3047
   81.9498    0.3033
    8.4555    0.2660
    6.8825    0.2062
   11.3809    0.2677
    8.1274    0.8342
    0.0108    1.0000
```



```
    0.0017    1.0000


>> c2w(eig(LO30_a))

ans =

   75.4709    0.2005
   78.5559    0.1972
   81.7474    0.3141
   81.9880    0.3142
    8.5719    0.2689
    6.9122    0.2073
   11.4930    0.2699
    6.4801    0.6993
    0.0162    1.0000
    0.0044    1.0000


>> c2w(eig(LO40_a))

ans =

   75.4610    0.1989
   78.5132    0.1959
   81.6993    0.3047
   81.9254    0.3054
    8.4514    0.2661
    6.8882    0.2045
   11.3916    0.2669
    5.7957    0.5884
    0.0216    1.0000
    0.0085    1.0000

>> diary off
>> who

Your variables are:

NO10_a    NO15_a    NO20_a    NO40_a    NO5_a
NO10_x    NO15_x    NO20_x    NO40_x    NO5_x

>> c2w(eig(NO5_a))

ans =

   76.9656    0.1962
   81.6615    0.3041
   80.0997    0.1879
   82.4226    0.2930
   28.9605    1.0000
   23.6222    1.0000
    6.6531    0.1985
    8.5575    0.2691
   11.2740    0.2653
    0.0027    1.0000
    0.0000    1.0000

>> c2w(eig(NO10_a))

ans =

   76.9654    0.1961
   80.0035    0.1904
   81.6614    0.3041
   82.0372    0.3000
```

```
      8.5572    0.2692
      6.6536    0.1985
     11.2492    0.2658
     13.7197    0.9655
      0.0055    1.0000
      0.0001    1.0000

>> c2w(eig(NO15_a))

ans =

     76.9620    0.1962
     79.9832    0.1914
     81.6641    0.3041
     81.9666    0.3026
      8.5554    0.2685
      6.6552    0.2000
     11.2473    0.2668
      9.7256    0.9092
      0.0082    1.0000
      0.0002    1.0000

>> c2w(eig(NO20_a))

ans =

     76.9603    0.1962
     79.9757    0.1918
     81.6645    0.3041
     81.9432    0.3039
      8.5550    0.2685
      6.6564    0.2004
     11.2518    0.2670
      7.8512    0.8453
      0.0109    1.0000
      0.0017    1.0000

>> c2w(eig(NO40_a))

ans =

     76.9636    0.1962
     79.9665    0.1925
     81.6656    0.3041
     81.9227    0.3058
      8.5493    0.2688
      6.6629    0.1985
     11.2602    0.2663
      5.4719    0.6104
      0.0218    1.0000
      0.0067    1.0000

>> diary off
>> who

Your variables are:

LW10_a    LW15_a    LW20_a    LW40_a    LW5_a
LW10_x    LW15_x    LW20_x    LW40_x    LW5_x


>> c2w(eig(LW5_a))

ans =

     78.4996    0.1966
```



```
     81.9450    0.3075
     81.6950    0.3047
     75.4631    0.1989
     23.3819    0.3152
     19.3872    0.4792
     11.3120    0.2701
      6.8797    0.2045
      8.4577    0.2665
      0.0000    1.0000
      0.0027    1.0000
     16.6667    1.0000
     16.6667    1.0000

>> c2w(eig(LW10_a))

ans =

     78.5000    0.1966
     81.9451    0.3075
     81.6950    0.3047
     75.4630    0.1989
     22.3016    0.8716
     21.3097    0.6520
     11.3211    0.2697
      8.4574    0.2665
      6.8802    0.2044
      0.0003    1.0000
      0.0054    1.0000
     33.3333    1.0000
     33.3333    1.0000

>> c2w(eig(LW15_a))

ans =

     78.4999    0.1966
     81.9454    0.3075
     81.6979    0.3047
     75.4597    0.1989
     38.7672    0.9993
     13.1149    0.8589
     11.3203    0.2693
      8.4568    0.2659
      6.8819    0.2057
      0.0009    1.0000
      0.0081    1.0000
     50.0000    1.0000
     50.0000    1.0000

>> c2w(eig(LW20_a))

ans =

     78.4998    0.1966
     81.9524    0.3075
     81.6980    0.3047
     75.4578    0.1989
     59.4008    1.0000
     58.8812    1.0000
      9.3225    0.8072
     11.3234    0.2693
      8.4555    0.2660
      6.8825    0.2062
      0.0019    1.0000
      0.0108    1.0000
     66.6667    1.0000
```

```
     66.6667    1.0000

>> c2w(eig(LW40_a))

ans =

  130.1583    1.0000
  129.6235    1.0000
   78.4984    0.1966
   81.9462    0.3074
   81.6993    0.3047
   75.4610    0.1989
   11.3209    0.2693
    6.1006    0.5667
    8.4514    0.2661
    6.8882    0.2045
    0.0073    1.0000
    0.0216    1.0000
  133.3333    1.0000
  133.3333    1.0000

>> diary off
>> who

Your variables are:

NW10_a    NW15_a    NW20_a    NW40_a    NW5_a
NW10_x    NW15_x    NW20_x    NW40_x    NW5_x

>> c2w(eig(NW5_a))

ans =

   79.5603    0.1929
   76.9656    0.1962
   81.9256    0.3076
   81.6615    0.3041
   22.7353    0.3173
   19.8082    0.4625
   10.4587    0.3266
    6.6531    0.1985
    8.5575    0.2691
    0.0000    1.0000
   16.6667    1.0000
    0.0027    1.0000
   16.6667    1.0000

>> c2w(eig(NW10_a))

ans =

   79.5957    0.1918
   76.9654    0.1961
   81.9250    0.3076
   81.6614    0.3041
   22.4710    0.8816
   20.3651    0.6526
   10.9119    0.3154
    6.6536    0.1985
    8.5572    0.2692
    0.0003    1.0000
    0.0055    1.0000
   33.3333    1.0000
   33.3333    1.0000

>> c2w(eig(NW15_a))
```



```

ans =

   79.6531    0.1910
   76.9620    0.1962
   81.9268    0.3076
   81.6641    0.3041
   39.5713    0.9988
   12.5609    0.8291
   10.9624    0.3035
    6.6552    0.2000
    8.5554    0.2685
    0.0009    1.0000
    0.0082    1.0000
   50.0000    1.0000
   50.0000    1.0000

>> c2w(eig(NW20_a))

ans =

   79.7180    0.1906
   76.9603    0.1962
   81.9266    0.3076
   81.6645    0.3041
   59.7066    1.0000
    9.1876    0.7580
   10.9745    0.2995
    6.6564    0.2004
    8.5550    0.2685
    0.0022    1.0000
    0.0109    1.0000
   66.6667    1.0000
   66.6667    1.0000

>> c2w(eig(NW40_a))

ans =

  130.5625    1.0000
  129.8853    1.0000
   79.8844    0.1908
   76.9636    0.1962
   81.9307    0.3076
   81.6656    0.3041
   11.0112    0.2962
    8.5493    0.2688
    6.6629    0.1985
    6.3025    0.4920
    0.0078    1.0000
    0.0218    1.0000
  133.3333    1.0000
  133.3333    1.0000


>> diary off


>> diary off
```

D.3.2 Cornering Trim Condition

```
>> c2w(eig(CLO11_a))

ans =

   81.6966    0.3048
   82.0635    0.2995
   75.4725    0.1991
   78.5578    0.1941
   14.0005    0.9679
   11.3719    0.2669
    8.4585    0.2689
    6.8599    0.2182
    4.2802    1.0000
    0.0002    1.0000

>> c2w(eig(CLO21_a))

ans =

   81.6638    0.3041
   81.9527    0.3032
   75.4573    0.1994
   78.5230    0.1956
   11.3852    0.2680
    8.4626    0.2675
    6.8650    0.2161
    8.1280    0.8348
    4.2867    1.0000
    0.0005    1.0000

>> c2w(eig(CLO31_a))

ans =

   81.5895    0.3044
   81.9153    0.3041
   75.4233    0.1992
   78.5048    0.1959
   11.4056    0.2680
    8.4720    0.2679
    6.8771    0.2181
    6.4755    0.6994
    4.2919    1.0000
    0.0004    1.0000

>> c2w(eig(CLO14_a))

ans =

   81.6926    0.3046
   82.0795    0.2997
   75.4631    0.1991
   78.5611    0.1941
   14.0016    0.9680
   11.3714    0.2668
    6.8600    0.2186
    8.4604    0.2678
    4.2717    1.0000
    0.0036    1.0000

>> c2w(eig(CLO19_a))

ans =

   81.6067    0.3042
   82.1467    0.3001
```



```
   75.4233    0.1993
   78.5619    0.1942
   14.0123    0.9683
   11.3777    0.2665
    8.4675    0.2680
    6.8617    0.2212
    4.2526    1.0000
    0.0146    1.0000

>> c2w(eig(CLO24_a))

ans =

   81.4244    0.3048
   82.1945    0.3029
   75.3713    0.1993
   78.5887    0.1952
   11.3933    0.2680
    8.4723    0.2668
    6.8683    0.2188
    8.1160    0.8358
    4.2833    1.0000
    0.0110    1.0000

>> diary off
>> c2w(eig(CNW11_a))

ans =

   80.0302    0.1935
   77.1366    0.1970
   81.6602    0.3076
   81.5984    0.3033
   33.3927    1.0000
   33.4114    1.0000
   22.7336    0.8707
   20.4347    0.6733
   11.2139    0.2672
    8.5489    0.2712
    6.5625    0.2165
    0.0002    1.0000
    4.2629    1.0000

>> c2w(eig(CNW21_a))

ans =

   80.1473    0.1913
   77.2745    0.1945
   81.6162    0.3125
   81.7384    0.2926
   59.5551    0.9999
   66.9401    1.0000
   67.3105    1.0000
   11.2542    0.2696
    8.7875    0.8366
    8.5344    0.2740
    6.5456    0.2169
    0.0020    1.0000
    4.2588    1.0000

>> c2w(eig(CNW31_a))

ans =

  100.8807    1.0000
```

```
   95.9926    1.0000
   94.8863    1.0000
  102.0870    1.0000
   80.4161    0.1872
   77.6886    0.1915
   82.6403    0.2789
   81.2067    0.3177
   11.2843    0.2701
    8.5409    0.2789
    6.5128    0.2162
    6.1876    0.7255
    0.0056    1.0000
    4.2559    1.0000

>> c2w(eig(CNW14_a))

ans =

   80.2194    0.1918
   77.1862    0.1942
   81.8557    0.3109
   81.3252    0.2968
   33.6964    1.0000
   33.8410    1.0000
   22.9897    0.8671
   20.6371    0.6708
   11.2092    0.2678
    6.5135    0.2172
    0.0052    1.0000
    4.2516    1.0000
    8.4969    0.2745

>> c2w(eig(CNW24_a))

ans =

   81.8232    0.1142
   81.8347    0.3181
   80.4765    0.1804
   84.1307    0.2068
   77.7497    1.0000
   79.8997    1.0000
   64.2695    1.0000
   59.6949    1.0000
   11.2910    0.2859
    8.3825    0.3704
    5.8726    0.2107
    6.3040    0.7126
    0.0559    1.0000
    4.2233    1.0000

>> c2w(eig(CNW19_a))

ans =

   80.6230    0.1875
   77.2454    0.1833
   82.0567    0.3127
   80.9855    0.2797
   34.9741    1.0000
   35.6293    1.0000
   23.1676    0.8892
   20.0495    0.6928
   11.2040    0.2728
    0.0220    1.0000
    4.2081    1.0000
```



```
    6.3632    0.2203
    8.2742    0.2851

>> c2w(eig(CNW26_a))

ans =

   88.1697    0.0200
   86.9925    0.1483
   81.1357    0.1818
   81.9088    0.3150
   93.8507    1.0000
   90.0844    1.0000
   65.2008    1.0000
   60.1844    1.0000
   11.3321    0.3014
    9.1238    0.3783
    5.0267    0.4731
    4.0850    0.1393
    0.0955    1.0000
    4.3470    1.0000

>> c2w(eig(CNW315_a))

ans =

  104.3426    1.0000
   96.7526    1.0000
  102.2485    1.0000
   94.5420    1.0000
   80.7323    0.1781
   78.4440    0.1856
   83.4510    0.2620
   81.1016    0.3222
   11.2943    0.2727
    8.5182    0.2866
    6.4187    0.2146
    5.9674    0.7133
    0.0119    1.0000
    4.2494    1.0000

>> c2w(eig(CNW33_a))

ans =

  115.4534    1.0000
  116.2101    1.0000
   99.3910    1.0000
   94.4015    1.0000
   86.5556    0.0840
   88.1316    0.1837
   80.8576    0.1800
   81.2391    0.3210
   11.3470    0.2875
    8.7616    0.3230
    5.4108    0.2053
    4.6176    0.5127
    0.0610    1.0000
    4.2475    1.0000

>> diary off
>> c2w(eig(CNW335_a))

ans =

  121.2721    1.0000
```

265	117.1543	1.0000
266	100.2172	1.0000
267	94.9361	1.0000
268	89.0368	0.0519
269	88.5162	0.1886
270	80.1051	0.1750
271	81.5025	0.3283
272	11.5420	0.2931
273	8.7398	0.3212
274	4.9639	0.0690
275	5.7717	0.4892
276	0.0440	1.0000
277	4.3243	1.0000
278		

D.4 Poles and Zeros

D.4.1 Ride Modes

D.4.1.1 Linear Tyre without Delay

```
>> [poles,zeros] = vertical displacement/front road

poles =

            0            0
   8.2003e+01   3.1329e-01
   8.1741e+01   3.1460e-01
   7.8555e+01   1.9596e-01
   7.5466e+01   1.9972e-01
   1.1486e+01   2.7005e-01
   8.1283e+00   8.3418e-01
   8.5728e+00   2.6931e-01
   6.9094e+00   2.0610e-01
   1.0805e-02   1.0000e+00
   2.2056e-03   1.0000e+00


zeros =

            0            0
   7.4323e+13   1.0000e+00
   7.1063e+02   1.0000e+00
   7.8555e+01   1.9596e-01
   8.2003e+01   3.1330e-01
   8.1711e+01   3.1479e-01
   1.5526e+01   1.0000e+00
   1.1484e+01   2.7018e-01
   8.7115e+00   2.7060e-01
   8.1322e+00   8.3477e-01
   9.7669e-03   1.0000e+00
   2.7385e-03   1.0000e+00

>> [poles,zeros] = vertical velocity/front road

poles =

            0            0
   8.2003e+01   3.1329e-01
   8.1741e+01   3.1460e-01
   7.8555e+01   1.9596e-01
   7.5466e+01   1.9972e-01
   1.1486e+01   2.7005e-01
   8.1283e+00   8.3418e-01
   8.5728e+00   2.6931e-01
   6.9094e+00   2.0610e-01
   1.0805e-02   1.0000e+00
   2.2056e-03   1.0000e+00


zeros =

            0            0
   7.1063e+02   1.0000e+00
   7.8555e+01   1.9596e-01
   8.2003e+01   3.1329e-01
   8.1712e+01   3.1480e-01
   1.5526e+01   1.0000e+00
   1.1484e+01   2.7018e-01
   8.7115e+00   2.7060e-01
   8.1321e+00   8.3476e-01
   9.7669e-03   1.0000e+00
   2.7385e-03   1.0000e+00
   5.1487e-12  -1.0000e+00

>> [poles,zeros] = pitch angle/front road
```



```
poles =

            0            0
   8.2003e+01   3.1329e-01
   8.1741e+01   3.1460e-01
   7.8555e+01   1.9596e-01
   7.5466e+01   1.9972e-01
   1.1486e+01   2.7005e-01
   8.1283e+00   8.3418e-01
   8.5728e+00   2.6931e-01
   6.9094e+00   2.0610e-01
   1.0805e-02   1.0000e+00
   2.2056e-03   1.0000e+00


zeros =

            0            0
   1.9156e+14   1.0000e+00
   5.7413e+02   1.0000e+00
   7.8555e+01   1.9596e-01
   8.2004e+01   3.1327e-01
   8.1772e+01   3.1437e-01
   1.6045e+01   1.0000e+00
   1.1487e+01   2.7003e-01
   8.4009e+00   2.6768e-01
   8.1272e+00   8.3421e-01
   1.3890e-02   1.0000e+00

>> [poles,zeros] = pitch rate/front road

poles =

            0            0
   8.2003e+01   3.1329e-01
   8.1741e+01   3.1460e-01
   7.8555e+01   1.9596e-01
   7.5466e+01   1.9972e-01
   1.1486e+01   2.7005e-01
   8.1283e+00   8.3418e-01
   8.5728e+00   2.6931e-01
   6.9094e+00   2.0610e-01
   1.0805e-02   1.0000e+00
   2.2056e-03   1.0000e+00


zeros =

            0            0
   5.7413e+02   1.0000e+00
   7.8555e+01   1.9596e-01
   8.2003e+01   3.1329e-01
   8.1774e+01   3.1435e-01
   1.6045e+01   1.0000e+00
   1.1487e+01   2.7003e-01
   8.4009e+00   2.6768e-01
   8.1272e+00   8.3421e-01
   1.1149e-02   1.0000e+00
   2.7289e-03   1.0000e+00
   9.8603e-12   1.0000e+00

>> [poles,zeros] = roll angle/front road

poles =

            0            0
```

```
    8.2003e+01   3.1329e-01
    8.1741e+01   3.1460e-01
    7.8555e+01   1.9596e-01
    7.5466e+01   1.9972e-01
    1.1486e+01   2.7005e-01
    8.1283e+00   8.3418e-01
    8.5728e+00   2.6931e-01
    6.9094e+00   2.0610e-01
    1.0805e-02   1.0000e+00
    2.2056e-03   1.0000e+00


zeros =

             0            0
    3.6950e+13   1.0000e+00
    1.7927e+03  -1.0000e+00
    8.3285e+01   3.0337e-01
    8.1741e+01   3.1460e-01
    7.5466e+01   1.9972e-01
    2.6149e+01   1.0000e+00
    8.1037e+00   8.6700e-01
    8.5728e+00   2.6931e-01
    6.9094e+00   2.0610e-01
    1.0805e-02   1.0000e+00
    3.8574e-03   1.0000e+00

>> [poles,zeros] = roll rate/front road

poles =

             0            0
    8.2003e+01   3.1329e-01
    8.1741e+01   3.1460e-01
    7.8555e+01   1.9596e-01
    7.5466e+01   1.9972e-01
    1.1486e+01   2.7005e-01
    8.1283e+00   8.3418e-01
    8.5728e+00   2.6931e-01
    6.9094e+00   2.0610e-01
    1.0805e-02   1.0000e+00
    2.2056e-03   1.0000e+00


zeros =

             0            0
    1.7927e+03  -1.0000e+00
    8.3285e+01   3.0337e-01
    8.1741e+01   3.1460e-01
    7.5466e+01   1.9972e-01
    2.6149e+01   1.0000e+00
    8.1037e+00   8.6700e-01
    8.5728e+00   2.6931e-01
    6.9094e+00   2.0610e-01
    1.0805e-02   1.0000e+00
    3.8574e-03   1.0000e+00
    2.4348e-12   1.0000e+00

>> [poles,zeros] - front suspension/front road

poles =

             0            0
    8.2003e+01   3.1329e-01
    8.1741e+01   3.1460e-01
```



```
    7.8555e+01   1.9596e-01
    7.5466e+01   1.9972e-01
    1.1486e+01   2.7005e-01
    8.1283e+00   8.3418e-01
    8.5728e+00   2.6931e-01
    6.9094e+00   2.0610e-01
    1.0805e-02   1.0000e+00
    2.2056e-03   1.0000e+00


zeros =

             0            0
    6.7414e+16  -1.0000e+00
    8.1955e+01   3.1377e-01
    8.1866e+01   3.1420e-01
    7.7007e+01   1.9797e-01
    1.1445e+01  -6.1624e-02
    1.1479e+01   1.0000e+00
    1.3472e+01   5.4124e-01

>> [poles,zeros] = rear suspension/front road

poles =

             0            0
    8.2003e+01   3.1329e-01
    8.1741e+01   3.1460e-01
    7.8555e+01   1.9596e-01
    7.5466e+01   1.9972e-01
    1.1486e+01   2.7005e-01
    8.1283e+00   8.3418e-01
    8.5728e+00   2.6931e-01
    6.9094e+00   2.0610e-01
    1.0805e-02   1.0000e+00
    2.2056e-03   1.0000e+00


zeros =

             0            0
    1.1165e+03  -1.0000e+00
    7.8262e+01  -2.4512e-03
    8.1735e+01   3.1459e-01
    7.5403e+01   1.9993e-01
    2.6618e+01   1.0000e+00
    8.1017e+00   8.5368e-01
    8.2948e+00   2.7276e-01
    7.2285e+00   2.0540e-01
    9.1929e-03   1.0000e+00
    4.3551e-03   1.0000e+00

>> diary rlopozero
```

D.4.1.2 Linear Tyre with Delay

```
>> [poles,zeros] = vertical displacement/front road

poles =

              0            0
     8.1741e+01   3.1460e-01
     8.0997e+01   3.1124e-01
     7.8319e+01   1.9465e-01
     7.5466e+01   1.9972e-01
     6.6663e+01   1.0000e+00
     6.6662e+01   1.0000e+00
     6.0298e+01   9.9997e-01
     1.0009e+01   7.4110e-01
     1.0715e+01   2.9204e-01
     8.5728e+00   2.6931e-01
     6.9094e+00   2.0610e-01
     1.0805e-02   1.0000e+00
     2.5216e-03   1.0000e+00


zeros =

              0            0
     3.7162e+13   1.0000e+00
     7.1063e+02   1.0000e+00
     7.8319e+01   1.9465e-01
     8.1706e+01   3.1478e-01
     8.1002e+01   3.1126e-01
     6.7076e+01   1.0000e+00
     6.6182e+01   1.0000e+00
     6.0330e+01   1.0000e+00
     1.5533e+01   1.0000e+00
     1.0017e+01   7.4125e-01
     1.0712e+01   2.9203e-01
     8.7105e+00   2.7040e-01
     9.7340e-03   1.0000e+00
     3.0513e-03   1.0000e+00

>> [poles,zeros] = vertical velocity/front road

poles =

              0            0
     8.1741e+01   3.1460e-01
     8.0997e+01   3.1124e-01
     7.8319e+01   1.9465e-01
     7.5466e+01   1.9972e-01
     6.6663e+01   1.0000e+00
     6.6662e+01   1.0000e+00
     6.0298e+01   9.9997e-01
     1.0009e+01   7.4110e-01
     1.0715e+01   2.9204e-01
     8.5728e+00   2.6931e-01
     6.9094e+00   2.0610e-01
     1.0805e-02   1.0000e+00
     2.5216e-03   1.0000e+00


zeros =

              0            0
     7.0739e+02   1.0000e+00
     7.8319e+01   1.9465e-01
     8.1721e+01   3.1488e-01
     8.0996e+01   3.1124e-01
     6.6663e+01   1.0000e+00
```



```
     6.6662e+01   1.0000e+00
     6.0297e+01   9.9997e-01
     1.3850e+01  -1.0000e+00
     1.5808e+01   1.0000e+00
     1.0012e+01   7.3911e-01
     1.0716e+01   2.9231e-01
     8.5068e+00   2.5658e-01
     1.1156e-02   1.0000e+00
     2.9968e-03   1.0000e+00

>> [poles,zeros] = pitch angle/front road

poles =

              0            0
     8.1741e+01   3.1460e-01
     8.0997e+01   3.1124e-01
     7.8319e+01   1.9465e-01
     7.5466e+01   1.9972e-01
     6.6663e+01   1.0000e+00
     6.6662e+01   1.0000e+00
     6.0298e+01   9.9997e-01
     1.0009e+01   7.4110e-01
     1.0715e+01   2.9204e-01
     8.5728e+00   2.6931e-01
     6.9094e+00   2.0610e-01
     1.0805e-02   1.0000e+00
     2.5216e-03   1.0000e+00


zeros =

              0            0
     1.0642e+13  -1.0000e+00
     5.7413e+02   1.0000e+00
     7.8319e+01   1.9465e-01
     8.1773e+01   3.1434e-01
     8.0999e+01   3.1126e-01
     6.6672e+01   1.0000e+00
     6.0285e+01   9.9996e-01
     1.6121e+01   1.0000e+00
     9.9949e+00   7.3809e-01
     1.0718e+01   2.9219e-01
     8.4017e+00   2.6793e-01
     1.1157e-02   1.0000e+00
     2.9967e-03   1.0000e+00

>> [poles,zeros] = pitch rate/front road

poles =

              0            0
     8.1741e+01   3.1460e-01
     8.0997e+01   3.1124e-01
     7.8319e+01   1.9465e-01
     7.5466e+01   1.9972e-01
     6.6663e+01   1.0000e+00
     6.6662e+01   1.0000e+00
     6.0298e+01   9.9997e-01
     1.0009e+01   7.4110e-01
     1.0715e+01   2.9204e-01
     8.5728e+00   2.6931e-01
     6.9094e+00   2.0610e-01
     1.0805e-02   1.0000e+00
     2.5216e-03   1.0000e+00
```

```

zeros =

             0            0
    5.7413e+02   1.0000e+00
    7.8319e+01   1.9465e-01
    8.1774e+01   3.1434e-01
    8.0997e+01   3.1125e-01
    6.6662e+01   1.0000e+00
    6.6663e+01   1.0000e+00
    6.0294e+01   9.9997e-01
    1.6121e+01   1.0000e+00
    9.9949e+00   7.3809e-01
    1.0718e+01   2.9219e-01
    8.4017e+00   2.6794e-01
    1.1157e-02   1.0000e+00
    2.9967e-03   1.0000e+00
    4.0683e-13  -1.0000e+00

>> [poles,zeros] = roll angle/front road

poles =

             0            0
    8.1741e+01   3.1460e-01
    8.0997e+01   3.1124e-01
    7.8319e+01   1.9465e-01
    7.5466e+01   1.9972e-01
    6.6663e+01   1.0000e+00
    6.6662e+01   1.0000e+00
    6.0298e+01   9.9997e-01
    1.0009e+01   7.4110e-01
    1.0715e+01   2.9204e-01
    8.5728e+00   2.6931e-01
    6.9094e+00   2.0610e-01
    1.0805e-02   1.0000e+00
    2.5216e-03   1.0000e+00


zeros =

             0            0
    5.5425e+13   1.0000e+00
    1.7864e+03  -1.0000e+00
    8.2269e+01   3.0223e-01
    8.1745e+01   3.1455e-01
    7.5466e+01   1.9972e-01
    6.6711e+01   9.9996e-01
    6.0915e+01   1.0000e+00
    4.5724e+01   9.7856e-01
    9.3221e+00   4.6145e-01
    8.5727e+00   2.6932e-01
    6.9094e+00   2.0610e-01
    1.0805e-02   1.0000e+00
    4.2092e-03   1.0000e+00

>> [poles,zeros] = roll rate/front road

poles =

             0            0
    8.1741e+01   3.1460e-01
    8.0997e+01   3.1124e-01
    7.8319e+01   1.9465e-01
    7.5466e+01   1.9972e-01
    6.6663e+01   1.0000e+00
```



```
    6.6662e+01   1.0000e+00
    6.0298e+01   9.9997e-01
    1.0009e+01   7.4110e-01
    1.0715e+01   2.9204e-01
    8.5728e+00   2.6931e-01
    6.9094e+00   2.0610e-01
    1.0805e-02   1.0000e+00
    2.5216e-03   1.0000e+00


zeros =

             0            0
    1.7864e+03  -1.0000e+00
    8.2273e+01   3.0218e-01
    8.1741e+01   3.1460e-01
    7.5466e+01   1.9972e-01
    6.6663e+01   1.0000e+00
    6.6662e+01   1.0000e+00
    6.1016e+01   1.0000e+00
    4.5719e+01   9.7856e-01
    9.3221e+00   4.6146e-01
    8.5728e+00   2.6931e-01
    6.9094e+00   2.0610e-01
    1.0805e-02   1.0000e+00
    4.2092e-03   1.0000e+00
    3.3433e-12  -1.0000e+00

>> [poles,zeros] = front suspension/front road

poles =

             0            0
    8.1741e+01   3.1460e-01
    8.0997e+01   3.1124e-01
    7.8319e+01   1.9465e-01
    7.5466e+01   1.9972e-01
    6.6663e+01   1.0000e+00
    6.6662e+01   1.0000e+00
    6.0298e+01   9.9997e-01
    1.0009e+01   7.4110e-01
    1.0715e+01   2.9204e-01
    8.5728e+00   2.6931e-01
    6.9094e+00   2.0610e-01
    1.0805e-02   1.0000e+00
    2.5216e-03   1.0000e+00


zeros =

             0            0
    4.0449e+16   1.0000e+00
    7.6900e+01   1.9710e-01
    8.1865e+01   3.1289e-01
    8.0946e+01   3.1326e-01
    6.7557e+01   9.9934e-01
    5.9172e+01   9.9896e-01
    1.1604e+01   8.6027e-01
    1.2130e+01   3.0346e-01
    8.6685e+00  -5.6851e-02

>> [poles,zeros] = rear suspension/front road

poles =

             0            0
```

```
     8.1741e+01   3.1460e-01
     8.0997e+01   3.1124e-01
     7.8319e+01   1.9465e-01
     7.5466e+01   1.9972e-01
     6.6663e+01   1.0000e+00
     6.6662e+01   1.0000e+00
     6.0298e+01   9.9997e-01
     1.0009e+01   7.4110e-01
     1.0715e+01   2.9204e-01
     8.5728e+00   2.6931e-01
     6.9094e+00   2.0610e-01
     1.0805e-02   1.0000e+00
     2.5216e-03   1.0000e+00


zeros =

              0            0
     1.1075e+03  -1.0000e+00
     7.7479e+01  -2.9722e-03
     8.1752e+01   3.1477e-01
     7.5408e+01   1.9986e-01
     6.6663e+01   1.0000e+00
     6.6662e+01   1.0000e+00
     6.3187e+01   1.0000e+00
     4.6306e+01   9.4835e-01
     9.4076e+00   4.4099e-01
     8.2021e+00   2.4840e-01
     7.2191e+00   2.3528e-01
     9.6312e-03   1.0000e+00
     4.5271e-03   1.0000e+00

>> diary rlwpozero
```

D.4.2 Handling Modes

D.4.2.1 Linear Tyre without Delay

```
>> [poles,zeros] = lateral velocity/front steer

poles =

            0            0
   8.2003e+01   3.1329e-01
   8.1741e+01   3.1460e-01
   7.8555e+01   1.9596e-01
   7.5466e+01   1.9972e-01
   1.1486e+01   2.7005e-01
   8.1283e+00   8.3418e-01
   8.5728e+00   2.6931e-01
   6.9094e+00   2.0610e-01
   1.0805e-02   1.0000e+00
   2.2056e-03   1.0000e+00


zeros =

            0            0
   8.2048e+01   3.1695e-01
   8.1741e+01   3.1460e-01
   8.0187e+01   1.9585e-01
   7.5466e+01   1.9972e-01
   1.1522e+01   2.7145e-01
   1.0668e+01   3.9984e-01
   8.5728e+00   2.6931e-01
   6.9094e+00   2.0610e-01
   1.0805e-02   1.0000e+00


>> [poles,zeros] = yaw rate/front steer

poles =

            0            0
   8.2003e+01   3.1329e-01
   8.1741e+01   3.1460e-01
   7.8555e+01   1.9596e-01
   7.5466e+01   1.9972e-01
   1.1486e+01   2.7005e-01
   8.1283e+00   8.3418e-01
   8.5728e+00   2.6931e-01
   6.9094e+00   2.0610e-01
   1.0805e-02   1.0000e+00
   2.2056e-03   1.0000e+00


zeros =

            0            0
   8.1741e+01   3.1460e-01
   8.1998e+01   3.0978e-01
   8.0339e+01   2.0880e-01
   7.5466e+01   1.9972e-01
   1.1508e+01   2.6977e-01
   8.3553e+00   1.0000e+00
   8.5728e+00   2.6931e-01
   6.9094e+00   2.0610e-01
   1.0805e-02   1.0000e+00
   2.4087e-13  -1.0000e+00

>> [poles,zeros] = roll angle/front steer

poles =
```



```

            0            0
   8.2003e+01   3.1329e-01
   8.1741e+01   3.1460e-01
   7.8555e+01   1.9596e-01
   7.5466e+01   1.9972e-01
   1.1486e+01   2.7005e-01
   8.1283e+00   8.3418e-01
   8.5728e+00   2.6931e-01
   6.9094e+00   2.0610e-01
   1.0805e-02   1.0000e+00
   2.2056e-03   1.0000e+00


zeros =

            0            0
   2.6889e+10  -1.0000e+00
   1.0434e+02   8.9351e-01
   8.3398e+01   3.0588e-01
   8.1741e+01   3.1460e-01
   7.5466e+01   1.9972e-01
   1.0588e+01   4.1401e-01
   8.5728e+00   2.6931e-01
   6.9094e+00   2.0610e-01
   2.1934e-02  -1.0000e+00
   1.0805e-02   1.0000e+00

>> [poles,zeros] = roll rate/front steer

poles =

            0            0
   8.2003e+01   3.1329e-01
   8.1741e+01   3.1460e-01
   7.8555e+01   1.9596e-01
   7.5466e+01   1.9972e-01
   1.1486e+01   2.7005e-01
   8.1283e+00   8.3418e-01
   8.5728e+00   2.6931e-01
   6.9094e+00   2.0610e-01
   1.0805e-02   1.0000e+00
   2.2056e-03   1.0000e+00


zeros =

            0            0
   1.0434e+02   8.9351e-01
   8.3398e+01   3.0588e-01
   8.1741e+01   3.1460e-01
   7.5466e+01   1.9972e-01
   1.0588e+01   4.1401e-01
   8.5728e+00   2.6931e-01
   6.9094e+00   2.0610e-01
   2.1934e-02  -1.0000e+00
   1.0805e-02   1.0000e+00
   1.3444e-09   1.0000e+00

>> [poles,zeros] = pitch angle/front steer

poles =

            0            0
   8.2003e+01   3.1329e-01
   8.1741e+01   3.1460e-01
```

```
    7.8555e+01   1.9596e-01
    7.5466e+01   1.9972e-01
    1.1486e+01   2.7005e-01
    8.1283e+00   8.3418e-01
    8.5728e+00   2.6931e-01
    6.9094e+00   2.0610e-01
    1.0805e-02   1.0000e+00
    2.2056e-03   1.0000e+00


zeros =

             0            0
    3.1546e+10  -1.0000e+00
    7.4821e+01   6.9463e-06
    8.2003e+01   3.1329e-01
    8.1773e+01   3.1423e-01
    7.8555e+01   1.9596e-01
    1.1486e+01   2.7005e-01
    8.1245e+00   8.3413e-01
    8.4230e+00   2.6739e-01
    1.1452e-02   1.0000e+00
    5.0444e-03   1.0000e+00

>> [poles,zeros] = pitch rate/front steer

poles =

             0            0
    8.1741e+01   3.1460e-01
    8.0997e+01   3.1124e-01
    7.8319e+01   1.9465e-01
    7.5466e+01   1.9972e-01
    6.6663e+01   1.0000e+00
    6.6662e+01   1.0000e+00
    6.0298e+01   9.9997e-01
    1.0009e+01   7.4110e-01
    1.0715e+01   2.9204e-01
    8.5728e+00   2.6931e-01
    6.9094e+00   2.0610e-01
    1.0805e-02   1.0000e+00
    2.5216e-03   1.0000e+00


zeros =

             0            0
    1.4230e+12   1.0000e+00
    7.4847e+01   4.9563e-03
    7.8319e+01   1.9465e-01
    8.1785e+01   3.1428e-01
    8.0995e+01   3.1124e-01
    6.6662e+01   1.0000e+00
    6.0303e+01   9.9996e-01
    1.0026e+01   7.3897e-01
    1.0715e+01   2.9199e-01
    8.4016e+00   2.6685e-01
    1.1544e-02   1.0000e+00
    3.1524e-03   1.0000e+00



>> [poles,zeros] = front suspension/front steer

poles =

```



```
             0            0
    8.2003e+01   3.1329e-01
    8.1741e+01   3.1460e-01
    7.8555e+01   1.9596e-01
    7.5466e+01   1.9972e-01
    1.1486e+01   2.7005e-01
    8.1283e+00   8.3418e-01
    8.5728e+00   2.6931e-01
    6.9094e+00   2.0610e-01
    1.0805e-02   1.0000e+00
    2.2056e-03   1.0000e+00


zeros =

             0            0
    1.8344e+13  -1.0000e+00
    8.2062e+01   3.1351e-01
    8.1741e+01   3.1460e-01
    7.7038e+01   1.9767e-01
    1.6596e+01  -1.0105e-01
    1.1813e+01   2.9839e-01
    7.7161e+00   9.0736e-01
    8.5638e+00   2.6917e-01
    1.0507e-02   9.8115e-01

>> [poles,zeros] = rear suspension/front steer

poles =

             0            0
    8.2003e+01   3.1329e-01
    8.1741e+01   3.1460e-01
    7.8555e+01   1.9596e-01
    7.5466e+01   1.9972e-01
    1.1486e+01   2.7005e-01
    8.1283e+00   8.3418e-01
    8.5728e+00   2.6931e-01
    6.9094e+00   2.0610e-01
    1.0805e-02   1.0000e+00
    2.2056e-03   1.0000e+00


zeros =

             0            0
    8.9358e+10  -1.0000e+00
    8.4077e+01   8.2234e-01
    8.1728e+01   3.1468e-01
    7.3241e+01  -1.7472e-02
    7.5396e+01   1.9991e-01
    9.8679e+00   5.3161e-01
    9.7155e+00   2.8457e-01
    5.9264e+00   1.4389e-01
    3.2540e-02  -1.0000e+00
    1.2122e-02   1.0000e+00

>> diary hlopozero
```

D.4.2.2 Linear Tyre with Delay

```
>> [poles,zeros] = lateral velocity/front steer

poles =

                0              0
      8.1741e+01    3.1460e-01
      8.0997e+01    3.1124e-01
      7.8319e+01    1.9465e-01
      7.5466e+01    1.9972e-01
      6.6663e+01    1.0000e+00
      6.6662e+01    1.0000e+00
      6.0298e+01    9.9997e-01
      1.0009e+01    7.4110e-01
      1.0715e+01    2.9204e-01
      8.5728e+00    2.6931e-01
      6.9094e+00    2.0610e-01
      1.0805e-02    1.0000e+00
      2.5216e-03    1.0000e+00


zeros =

                0              0
      2.4484e+13   -1.0000e+00
      8.3061e+01    2.0628e-01
      8.3112e+01    3.2786e-01
      8.1735e+01    3.1462e-01
      7.5465e+01    1.9971e-01
      6.6663e+01    1.0000e+00
      6.0367e+01    1.0000e+00
      1.0757e+01    3.0074e-01
      5.3413e+00   -1.0000e+00
      8.5727e+00    2.6931e-01
      6.9094e+00    2.0610e-01
      1.4469e-02    1.0000e+00
      1.0805e-02    1.0000e+00

>> [poles,zeros] = yaw rate/front steer

poles =

                0              0
      8.1741e+01    3.1460e-01
      8.0997e+01    3.1124e-01
      7.8319e+01    1.9465e-01
      7.5466e+01    1.9972e-01
      6.6663e+01    1.0000e+00
      6.6662e+01    1.0000e+00
      6.0298e+01    9.9997e-01
      1.0009e+01    7.4110e-01
      1.0715e+01    2.9204e-01
      8.5728e+00    2.6931e-01
      6.9094e+00    2.0610e-01
      1.0805e-02    1.0000e+00
      2.5216e-03    1.0000e+00


zeros =

                0              0
      1.5249e+13    1.0000e+00
      8.8070e+01    2.3107e-01
      7.5466e+01    1.9971e-01
      8.1744e+01    3.1459e-01
      7.8839e+01    2.9923e-01
      6.6663e+01    1.0000e+00
```



```
      5.8269e+01    1.0000e+00
      1.0018e+01    1.0000e+00
      1.0743e+01    3.0016e-01
      8.5727e+00    2.6931e-01
      6.9094e+00    2.0610e-01
      1.0805e-02    1.0000e+00

>> [poles,zeros] = roll angle/front steer

poles =

                0              0
      8.1741e+01    3.1460e-01
      8.0997e+01    3.1124e-01
      7.8319e+01    1.9465e-01
      7.5466e+01    1.9972e-01
      6.6663e+01    1.0000e+00
      6.6662e+01    1.0000e+00
      6.0298e+01    9.9997e-01
      1.0009e+01    7.4110e-01
      1.0715e+01    2.9204e-01
      8.5728e+00    2.6931e-01
      6.9094e+00    2.0610e-01
      1.0805e-02    1.0000e+00
      2.5216e-03    1.0000e+00


zeros =

                0              0
      4.1236e+06   -2.5439e-06
      7.3049e+01    4.9805e-03
      8.2363e+01    3.0177e-01
      8.1741e+01    3.1460e-01
      7.5466e+01    1.9972e-01
      6.6667e+01    1.0000e+00
      6.1144e+01    1.0000e+00
      1.1284e+01    3.1053e-01
      8.5728e+00    2.6931e-01
      6.9094e+00    2.0610e-01
      1.0805e-02    1.0000e+00
      5.7322e-03    1.0000e+00

>> [poles,zeros] = roll rate/front steer

poles =

                0              0
      8.1741e+01    3.1460e-01
      8.0997e+01    3.1124e-01
      7.8319e+01    1.9465e-01
      7.5466e+01    1.9972e-01
      6.6663e+01    1.0000e+00
      6.6662e+01    1.0000e+00
      6.0298e+01    9.9997e-01
      1.0009e+01    7.4110e-01
      1.0715e+01    2.9204e-01
      8.5728e+00    2.6931e-01
      6.9094e+00    2.0610e-01
      1.0805e-02    1.0000e+00
      2.5216e-03    1.0000e+00


zeros =

                0              0
```

```
    2.7207e+13  -1.0000e+00
    7.3049e+01   4.9805e-03
    8.2363e+01   3.0176e-01
    8.1741e+01   3.1461e-01
    7.5465e+01   1.9972e-01
    6.6721e+01   1.0000e+00
    6.6613e+01   1.0000e+00
    6.1144e+01   1.0000e+00
    1.1284e+01   3.1053e-01
    8.5728e+00   2.6931e-01
    6.9094e+00   2.0610e-01
    1.0805e-02   1.0000e+00
    5.7322e-03   1.0000e+00

>> [poles,zeros] = pitch angle/front steer

poles =

             0            0
    8.1741e+01   3.1460e-01
    8.0997e+01   3.1124e-01
    7.8319e+01   1.9465e-01
    7.5466e+01   1.9972e-01
    6.6663e+01   1.0000e+00
    6.6662e+01   1.0000e+00
    6.0298e+01   9.9997e-01
    1.0009e+01   7.4110e-01
    1.0715e+01   2.9204e-01
    8.5728e+00   2.6931e-01
    6.9094e+00   2.0610e-01
    1.0805e-02   1.0000e+00
    2.5216e-03   1.0000e+00


zeros -

             0            0
    4.3078e+06   3.4136e-05
    7.4847e+01   4.9564e-03
    7.8319e+01   1.9465e-01
    8.1783e+01   3.1427e-01
    8.0997e+01   3.1125e-01
    6.6662e+01   1.0000e+00
    6.0303e+01   9.9996e-01
    1.0026e+01   7.3897e-01
    1.0715e+01   2.9199e-01
    8.4016e+00   2.6685e-01
    1.1544e-02   1.0000e+00
    3.1524e-03   1.0000e+00

>> [poles,zeros] = pitch rate/front steer

poles =

             0            0
    8.1741e+01   3.1460e-01
    8.0997e+01   3.1124e-01
    7.8319e+01   1.9465e-01
    7.5466e+01   1.9972e-01
    6.6663e+01   1.0000e+00
    6.6662e+01   1.0000e+00
    6.0298e+01   9.9997e-01
    1.0009e+01   7.4110e-01
    1.0715e+01   2.9204e-01
    8.5728e+00   2.6931e-01
    6.9094e+00   2.0610e-01
```



```
    1.0805e-02   1.0000e+00
    2.5216e-03   1.0000e+00


zeros =

             0            0
    7.4230e+12   1.0000e+00
    7.4847e+01   4.9563e-03
    7.8319e+01   1.9465e-01
    8.1785e+01   3.1428e-01
    8.0995e+01   3.1124e-01
    6.6662e+01   1.0000e+00
    6.0303e+01   9.9996e-01
    1.0026e+01   7.3897e-01
    1.0715e+01   2.9199e-01
    8.4016e+00   2.6685e-01
    1.1544e-02   1.0000e+00
    3.1524e-03   1.0000e+00

>> [poles,zeros] = front suspension/front steer

poles =

             0            0
    8.1741e+01   3.1460e-01
    8.0997e+01   3.1124e-01
    7.8319e+01   1.9465e-01
    7.5466e+01   1.9972e-01
    6.6663e+01   1.0000e+00
    6.6662e+01   1.0000e+00
    6.0298e+01   9.9997e-01
    1.0009e+01   7.4110e-01
    1.0715e+01   2.9204e-01
    8.5728e+00   2.6931e-01
    6.9094e+00   2.0610e-01
    1.0805e-02   1.0000e+00
    2.5216e-03   1.0000e+00


zeros =

             0            0
    6.7207e+07  -1.0000e+00
    6.7206e+07   1.0000e+00
    8.1741e+01   3.1460e-01
    8.1134e+01   3.1152e-01
    7.6857e+01   1.9711e-01
    6.6662e+01   1.0000e+00
    6.3684e+01   1.0000e+00
    6.0081e+01   1.0000e+00
    2.2844e+01   4.2977e-02
    1.0795e+01   3.3283e-01
    8.8028e+00   4.8050e-01
    8.5592e+00   2.6977e-01
    1.0227e-02   1.0000e+00
    4.1394e-03   1.0000e+00

>> [poles,zeros] = rear suspension/front steer

poles =

             0            0
    8.1741e+01   3.1460e-01
    8.0997e+01   3.1124e-01
    7.8319e+01   1.9465e-01
```

```
     7.5466e+01   1.9972e-01
     6.6663e+01   1.0000e+00
     6.6662e+01   1.0000e+00
     6.0298e+01   9.9997e-01
     1.0009e+01   7.4110e-01
     1.0715e+01   2.9204e-01
     8.5728e+00   2.6931e-01
     6.9094e+00   2.0610e-01
     1.0805e-02   1.0000e+00
     2.5216e-03   1.0000e+00


zeros =

              0            0
     5.6309e+06  -1.0000e+00
     5.6305e+06   1.0000e+00
     7.6651e+01  -1.0022e-01
     8.1758e+01   3.1474e-01
     7.5010e+01   7.2615e-02
     7.5414e+01   2.0004e-01
     6.7302e+01   1.0000e+00
     6.6662e+01   1.0000e+00
     6.4558e+01   1.0000e+00
     1.1243e+01   3.0575e-01
     8.4394e+00   2.2146e-01
     7.1235e+00   2.5058e-01
     7.4207e-03   9.4711e-01

>> diary hlwpozero
```

D.5 Frequency Response Matrix Elements

D.5.1 Ride Modes

D.5.1.1 Linear Tyre without Delay

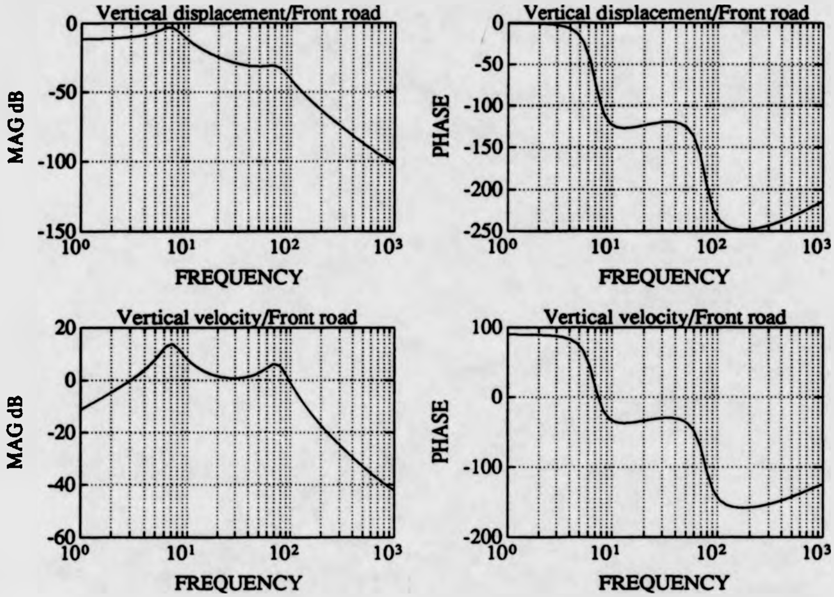

Figure D.37: Linear, without, speed: $20m/s$

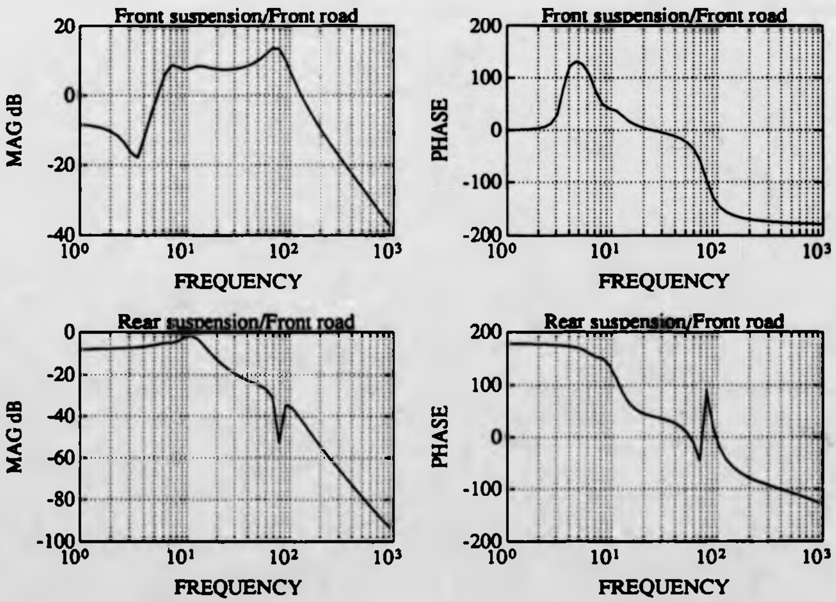

Figure D.38: Linear, without, speed: $20m/s$

D.5.1.2 Linear Tyre with Delay

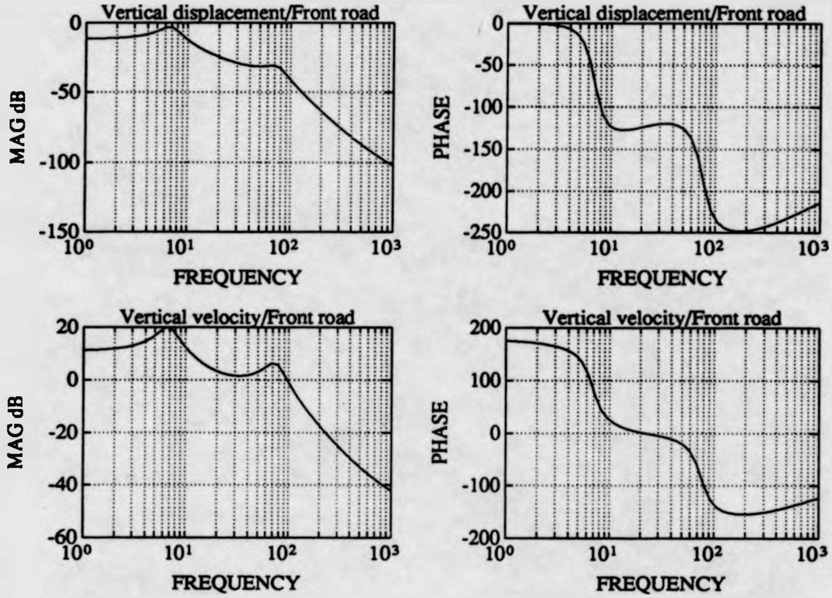

Figure D.39: Linear, with, speed: $20m/s$

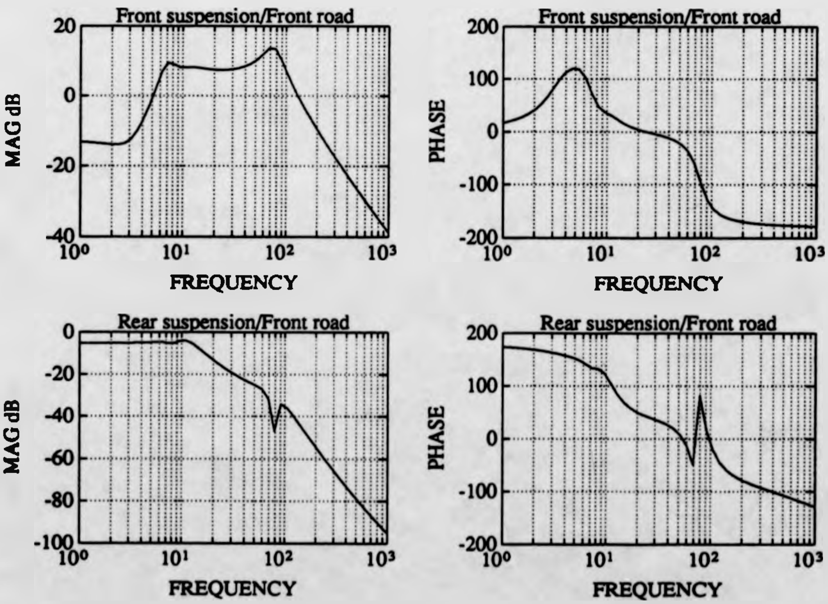

Figure D.40: Linear, with, speed: $20m/s$

D.5.2 Handling Modes

D.5.2.1 Linear Tyre without Delay

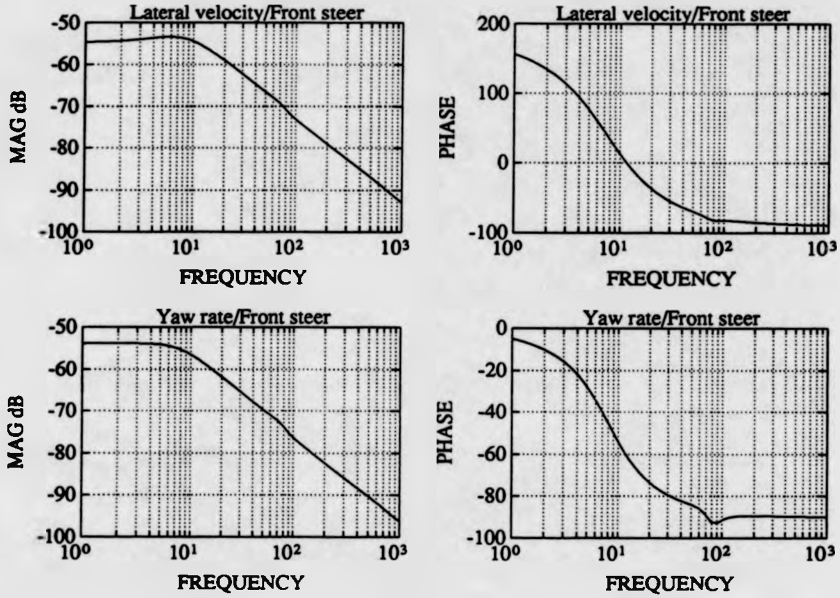

Figure D.41: Linear, without, speed: $20m/s$

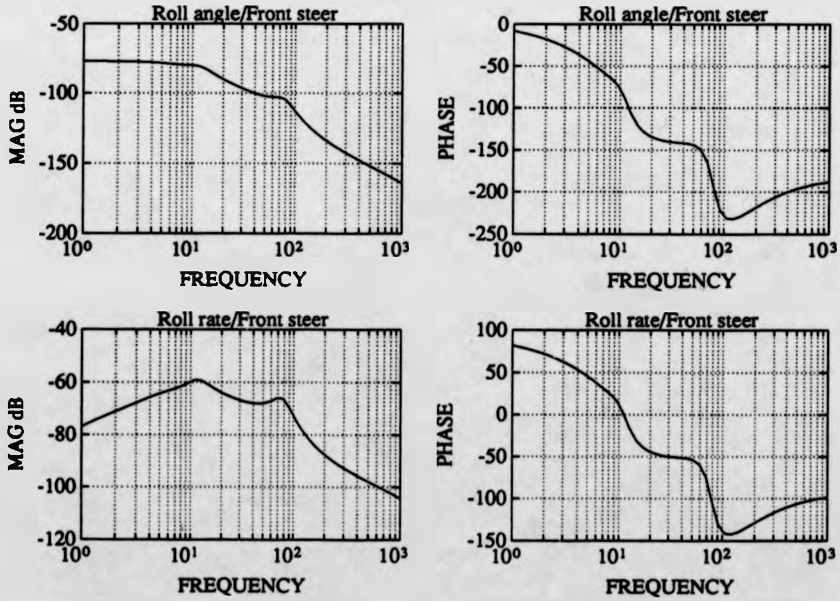

Figure D.42: Linear, without, speed: $20m/s$

D.5.2.2 Linear Tyre with Delay

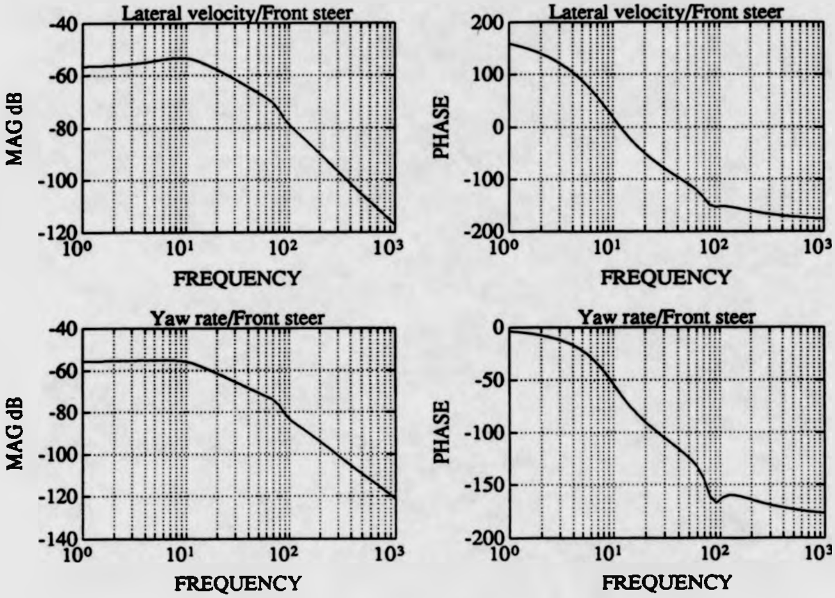

Figure D.43: Linear, with, speed: $20m/s$

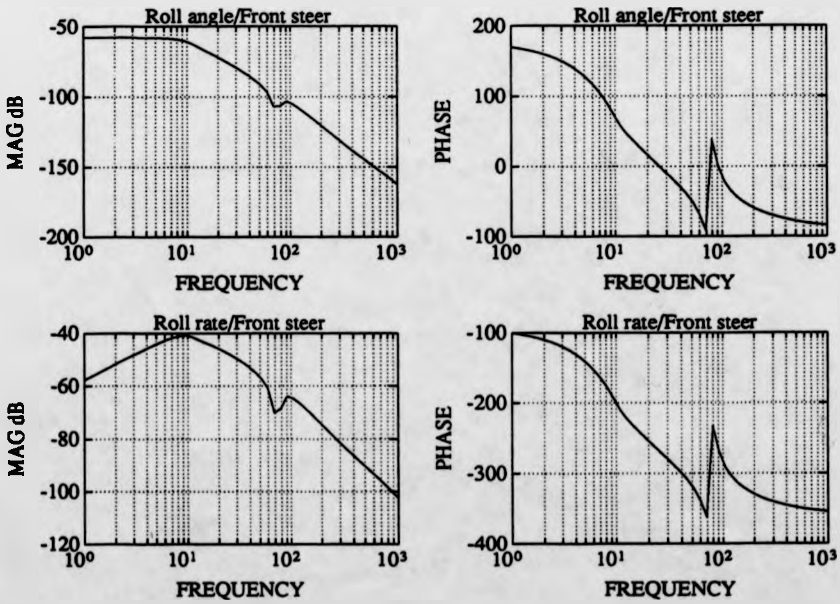

Figure D.44: Linear, with, speed: $20m/s$